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Nonlinear Electrical Conductivity in Graphene under the influence of DC and High-Frequency AC Fields with Hot-Electron Injection

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ABSTRACT
A theoretical model describing hot-electron injection effects on the high-frequency electrical conductivity of graphene under DC-AC fields is presented. The quasiclassical Boltzmann kinetic equation, within the constant relaxation-time approximation, is employed to obtain the nonequilibrium electron distribution function. An explicit analytical expression for the electrical conductivity at arbitrary injection rates is subsequently derived. A systematic analysis of frequency-dependent electrical conductivity is performed across a range of hot-electron injection rates at a constant DC field. At sufficiently high injection rates, a pronounced peak is observed that is well suited to terahertz radiation generation, with negative differential conductivity (NDC) arising from the DC field and domain formation associated with NDC being suppressed by the AC field. These findings indicate that graphene subjected to hot-electron injection may be utilised in high-frequency nanoelectronic device applications.

1. Introduction

The electronic properties and electronic transport in graphene have been extensively examined in theoretical studies [1-24]. Nevertheless, the electrodynamic properties of graphene warrant further study, as they underpin the development of carbon-based devices. As in other reports [20, 21, 24, 25], the kinetic transport equation was utilised to investigate the current-voltage $(I - V)$ characteristics and the high-frequency conductivity of graphene under hot-electron injection in an electric field, where the kinetic equation is valid [24]. The Stark frequency smaller than the conduction-band gap was assumed [24]. In such a quasi-classical scenario, both inter-band transitions and quantum mechanical corrections to the intra-band motion can be neglected. This ensures the use of the semiclassical approach as in [24]. Additionally, the classical regime [26] is considered, in which the photon energy $\hbar\omega$ is much smaller than the Fermi energy E_F, i.e., $\hbar\omega \ll E_F$, ensuring the validity of the Boltzmann kinetic equation for electron dynamics. Where ω is the frequency of the photon and $\hbar$ is the reduced Planck's constant. In this approach, scattering mechanisms (Coulomb, short-range processes) and the screening effects in graphene are neglected, and a constant relaxation time (τ) is assumed.

Despite extensive theoretical research on the electronic properties and transport phenomena of graphene [1-24], studies addressing nonlinear electrical conductivity induced by high-frequency fields and hot-electron injection have received comparatively little attention. The novelty of the present work lies in a semiclassical analysis of nonlinear transport in graphene subjected to combined DC and high-frequency AC fields, incorporating hot-electron injection for advanced nanoelectronic device applications.

2. Theory

Following the approach in [22-29], the motion of an electron is considered in the presence of a high-frequency electric field $E(t)$ with hot electron injection. The electric field $E(t)$ is directed along the graphene growth axis (i.e., along the x axis), and the conductivity is derived using the Boltzmann kinetic equation. Electron transport in graphene is described using distribution functions within the simplest momentum-independent relaxation-time approximation, where τ is assumed constant, equal to the electron mean-free-path time [22-29]. In this case, the Boltzmann kinetic equations for the symmetric f_s and antisymmetric f_a distribution functions are expressed as [24, 27-29]:

$$\frac{\partial f_s}{\partial t} + eE(t)\frac{\partial f_a}{\partial p} = \frac{F - f_s}{\tau} + Q\delta(p - p') - \frac{Q}{n_o} f_s - \frac{Q}{n_o} f_E \quad (1a)$$

$$\frac{\partial f_a}{\partial t} + eE(t)\frac{\partial f_s}{\partial p} = \frac{-f_a}{\tau} + \frac{Q}{n_o} f_a \quad (1b)$$

Where e is the electron charge, p is the electron dynamical momentum, F is the equilibrium distribution function and, f_s and f_a are the symmetric and antisymmetric distribution functions, respectively. $\frac{Q}{n_o}$ is the hot electron pumping frequency;

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$\boldsymbol{Q}$ is the rate of injection of hot electrons; $\boldsymbol{n_o}$ is the particle density; and $\boldsymbol{p'}$ is the hot electron injection momentum. $\boldsymbol{f_E}$ is the stationary homogeneous distribution function in the absence of a hot electron source. In the case of a constant electric field $\boldsymbol{E_o}$ and denoting the stationary distribution function as $\boldsymbol{f_a}^{\boldsymbol{o}} = \boldsymbol{f_a}^{\boldsymbol{E}}$, solving Eq. (1) without the hot source yields,

$$\frac{\partial^2 f_a^o}{\partial \xi^2} - \chi_0^{\,2} f_a^o = \frac{\partial F}{(eEa\tau)\partial \xi} \tag{2}$$

where $\chi_0^{\,2} = \frac{\pi^2}{(eaE\tau)^2}$ and $\xi = \frac{a}{\pi}p$ [24].

The solution of the boundary value problem 2 is

$$f_a^o = \frac{1}{(\omega_E\tau)} \int_{-1}^{1} G(\xi_x, {\xi'}_x) \frac{\partial F}{\partial \xi} d{\xi'}_x \tag{3}$$

where

$$G(\xi_x, {\xi'}_x) = \sum_{n=1}^{\infty} \frac{sinn\pi(\xi_x) sinn\pi({\xi'}_x)}{\chi_0^{\,2} + (n\pi)^2}$$

$$\xi' = \frac{a}{\pi}p'$$

Using $\frac{\partial F}{\partial \xi} = \frac{\partial F}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial \xi} = v\frac{\partial F}{\partial \varepsilon}$, and substituting $G(\xi_x, {\xi'}_x)$, we get

$$f_a^o = \frac{1}{(\omega_E\tau)} v \frac{\partial F}{\partial \varepsilon} \sum_{n=1}^{\infty} \frac{1}{\chi_0^{\,2} + (n\pi)^2} \int_{-1}^{1} sinn\pi(\xi_x) sinn\pi({\xi'}_x) d{\xi'}_x \tag{4}$$

We shall consider the cases where $\xi_x = {\xi'}_x$ and $\xi_x \neq {\xi'}_x$. The second case is vanished, and we obtain for $\xi_x = {\xi'}_x$, the expression

$$f_a^o = \frac{1}{(\omega_E\tau)} v \frac{\partial F}{\partial \varepsilon} \sum_{n=1}^{\infty} \frac{1}{\chi_0^{\,2} + (n\pi)^2} \int_{-1}^{1} sinn\pi(\xi_x) sinn\pi({\xi'}_x) d{\xi'}_x = v\frac{\partial F}{\partial \varepsilon}\frac{\omega_E\tau}{\omega_E^{\,2}\tau^2 + 1} \tag{5}$$

Here $\omega_E = eaE$ is the Bloch frequency

As in [29], in the presence of the hot source, the stationary homogeneous distribution function $\boldsymbol{f_a}^{\boldsymbol{c}}$ is given by $\boldsymbol{f_a}^{\boldsymbol{c}} = \boldsymbol{f_a}^{\boldsymbol{E}} + \boldsymbol{f_a}'$ and solving Eq. (1) for the constant electric field, we obtain,

$$f_a{}' = v\frac{\partial F}{\partial \varepsilon}\frac{\eta\omega_E\tau}{(1+\eta)^2 + \omega_E^{\,2}\tau^2}\left[1 - \frac{\omega_E\tau}{\omega_E^{\,2}\tau^2 + 1}\right] \tag{6}$$

Where $\eta = \frac{Q\tau}{n_o}$ is the dimensionless hot electron pumping or injection rate parameter.

The electron spectrum in graphene is given by [22].

$$|\varepsilon(p)| = \pm\frac{3\gamma_o b}{2\hbar}|p - p_F| \tag{7}$$

Where the tight-binding nearest-neighbour hopping parameter $\gamma_o \approx 2.7$ eV, $b = 0.142$ nm is the distance between the neighbouring carbon atoms in the graphene. + and $-$ signs are related to the conduction and valence bands, respectively. With $\boldsymbol{p_F}$ as the constant quasimomentum corresponding to the particular Fermi point.

The current density of the mobile electron in the first Brillouin zone for graphene is given by the expression [22];

$$j_x = \frac{2e}{(2\pi\hbar)^2}\iint v_x f_a^1 d^2 p \tag{8}$$

and the quasiclassical velocity $\boldsymbol{v_x(p)}$ of an electron moving along the graphene growth axis (i.e., along the $\boldsymbol{x}$ axis) can be expressed as

$$v_x(p) = \frac{\partial \varepsilon}{\partial p} = \frac{a}{\pi}\frac{\partial \varepsilon}{\partial \xi} = \frac{a}{\pi} grad\varepsilon \tag{9}$$

and writing

$$d^2 p = \frac{\pi}{a} d^2\xi = \frac{\pi}{a}\frac{ds}{\left|\frac{a}{\pi} grad\varepsilon\right|} dE \tag{10}$$

$\boldsymbol{ds}$ is the element of the length of the curve.

The effects of combined direct current (DC) causing NDC and alternating current (AC) electric fields are now considered, where the AC field has frequency $\boldsymbol{\omega}$ and wave–vector $\boldsymbol{\kappa}$. It is assumed that the AC field amplitude is much lower than the DC field strength ($\boldsymbol{E \gg E_{\omega,\kappa}}$). Under this assumption, Eqs. (1) are linearised using the perturbations

$$E(t) = \left(E + E_{\omega,\kappa}e^{-i(\omega t + kx)}\right),\ f_s = f_s^c + f_s^1 e^{-i(\omega t + kx)},\ \text{and}\ f_a = f_a^c + f_a^1 e^{-i(\omega t + kx)}.$$

We have

$$\frac{\partial^2 f_a^1}{\partial \xi^2} - \chi_\omega^{\,2} f_a^1 = \pi^2 \frac{(i\omega\tau - 1)}{\omega_E\tau}\frac{\partial f_s^c}{\partial \xi}\frac{E_{\omega k}}{E} - \frac{\partial^2 f_a^c}{\partial \xi^2}\frac{E_{\omega k}}{E} \tag{11}$$

where

$$\chi_\omega^{\,2} = \frac{\{1 - 2i\omega\tau - \omega_E^{\,2}\tau^2\}}{(\omega_E\tau)^2}$$

Using the solution of Eq. (11) together with (10) and (9) into (8) and using (7), we obtain the conductivity $\boldsymbol{\sigma(\omega\tau)}$ expression as $\boldsymbol{\sigma(\omega\tau) = \sigma^0(\omega\tau) + \sigma'(\omega\tau)}$.

Below, expressions are given for the electron conductivity $\boldsymbol{\sigma^0(\omega\tau)}$, the hot-electron conductivity $\boldsymbol{\sigma'(\omega\tau)}$, and the total conductivity $\boldsymbol{\sigma(\omega\tau)}$.

$$\boldsymbol{\sigma^0(\omega\tau) = \sigma_o \frac{1-(\omega_E\tau)^2 - i\omega\tau}{[(\omega_E\tau)^2 - (\omega\tau)^2 - 1 + 2i\omega\tau]((\omega_E\tau)^2 + 1)}} \quad (12)$$

$$\boldsymbol{\sigma'(\omega\tau)} = \left\{\frac{\eta\sigma_o[(1+\eta) - i\omega\tau(1+\eta) - (\omega_E\tau)^2]}{[(1+\eta)^2 + (\omega_E\tau)^2][(\omega_E\tau)^2 - (\omega\tau)^2 - 1 + 2i\omega\tau]}\right\} \times \left[1 - \frac{\omega_E\tau}{(\omega_E\tau)^2 + 1}\right] \quad (13)$$

$$\boldsymbol{\sigma(\omega\tau)} = \boldsymbol{\sigma_o}\left(\frac{1-(\omega_E\tau)^2 - i\omega\tau}{[(\omega_E\tau)^2 - (\omega\tau)^2 - 1 + 2i\omega\tau]((\omega_E\tau)^2 + 1)} + \left\{\frac{\eta[(1+\eta) - i\omega\tau(1+\eta) - (\omega_E\tau)^2]}{[(1+\eta)^2 + (\omega_E\tau)^2][(\omega_E\tau)^2 - (\omega\tau)^2 - 1 + 2i\omega\tau]}\right\} \times \left[1 - \frac{\omega_E\tau}{(\omega_E\tau)^2 + 1}\right]\right) \quad (14)$$

Where $\boldsymbol{\omega_E\tau = eaE\tau}$ is dimensionless Bloch frequency $(\boldsymbol{\omega_E})$ and $\boldsymbol{\sigma_o} = \frac{8In2e^2}{\hbar}\frac{kT\tau}{\hbar}$

3. Results and Discussion

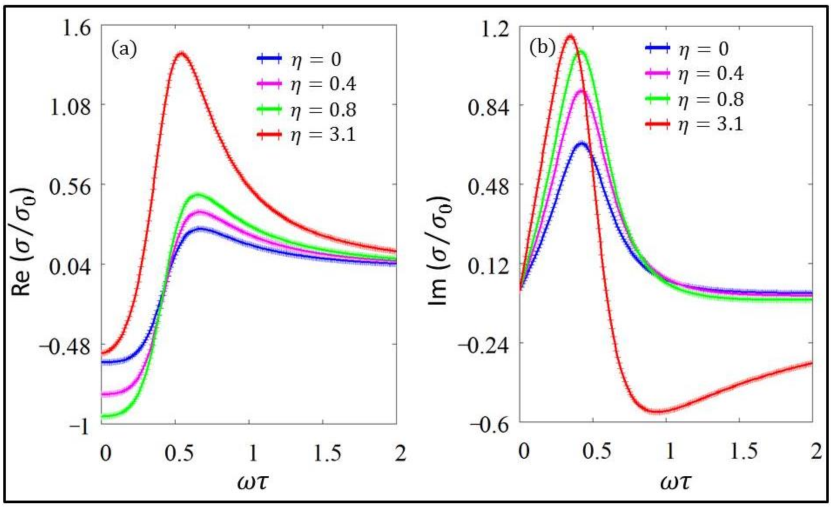

Figure 1: Normalised real (a) and imaginary (b) components of the conductivity, Re (σ/σ_0) and Im (σ/σ_0), plotted as functions of the dimensionless frequency ωτ for graphene at injection rates $\eta = 0$, $0.4, 0.8$, and 3.1.

The resulting plots of normalised real (Re) and imaginary (Im) components of the conductivity $(\boldsymbol{\sigma/\sigma_0})$ as a functions of the normalised frequency of the AC field $(\boldsymbol{\omega\tau})$ reveal a pronounced frequency – dependent behaviour of the conductivity at different dimensionless hot electron injection rates (i.e., $\boldsymbol{\eta}$ = 0 (without hot electrons), 0.4, 0.6, 3.1), as illustrated Fig. 1 (a) and Fig. 1 (b). At sufficiently high injection rate $(\mathbf{i.e.}, \boldsymbol{\eta = 3.1})$, an enhanced conductivity is observed, signifying an optimal regime for terahertz radiation generation. The results indicate that graphene with hot-electron injection constitutes an active medium for Bloch oscillations while remaining free from domain instabilities associated with negative differential conductivity arising from combined DC and High-frequency AC fields [30-31]. The corresponding three-dimensional representations are provided in Fig. 2 (a) and Fig. 2 (b).

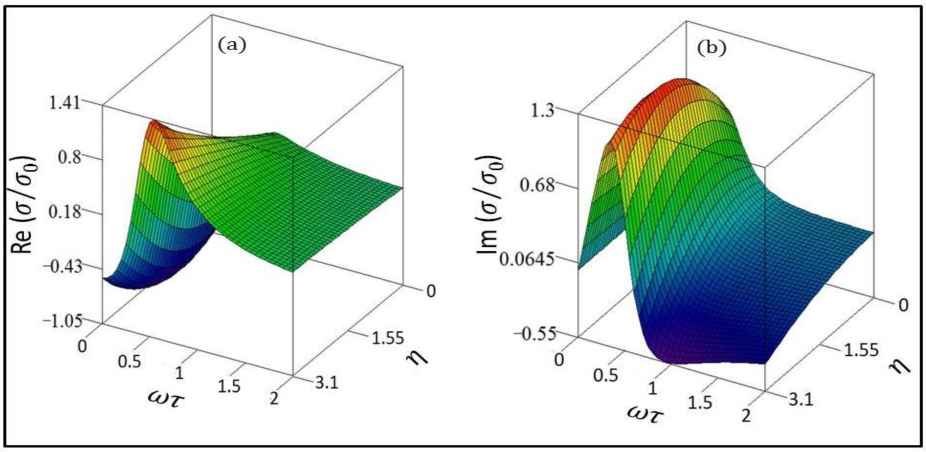

Figure 2: Three-dimensional plots of (a) the normalised real conductivity, Re$(\sigma(\omega)/\sigma_o(\omega))$ and (b) normalised imaginary conductivity, $(\sigma(\omega)/\sigma_o(\omega))$, as functions of the dimensionless frequency ωτ and the hot electron injection rate η in graphene.

4. Conclusion

An explicit analytical expression for high-frequency conductivity has been derived for arbitrary hot–electron injection rates using a simplified tight-binding model coupled with the quasiclassical Boltzmann kinetic equation within the constant relaxation-time approximation. It has been established that high-frequency conductivity exhibits strong sensitivity to the hot electron injection rate in graphene under DC- AC fields, reflecting the significant influence of non-equilibrium carrier dynamics on its transport behaviour. These results suggest that controlled hot-electron injection provides an effective mechanism for modulating the high-frequency response of graphene. As a result, graphene-based systems incorporating hot electron injection are identified as promising candidates for advanced high-frequency and terahertz electronic applications.

Authors' Contributions

All authors contributed equally to all aspects of this work, including conceptualization, investigations, analysis, and manuscript writing.

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