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Solution of Three - Dimensional Mboctara Equation via Triple Kamal Transform

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ABSTRACT

The goal of this study is to suggest a new triple Kamal integral transform. We outline its essential properties and proved some important results, including existence theorem, triple convolution theorem and derivatives properties. Moreover, the proposed new transform is utilized to solve Mboctara partial differential equations.

1. Introduction

Integral transform methods have been proved as one of the most effective means for solving partial differential equations. Their applications extend to many phenomena in mathematical physics, engineering and many other scientific fields. These sciences can be modeled in the form of mathematical equations expressed in terms of PDEs [1–11]. There by, it is feasible to transform PDEs in terms of algebraic equations and thus obtain exact solutions of PDEs. Many researchers and academics have exerted great efforts to develop and perfect these methods and apply them to deal with a wide range of problems in Mathematics. Well known examples of these methods are the Fourier and Laplace transforms among many other methods [12-22].

Recently, triple Laplace transform has widely been applied to solve PDEs with unknown functions of three variables to obtain more satisfactory results compared with numerical methods [27-29]. In addition, researchers have contributed extensions the original triple Laplace transform such as triple Sumudu transform [30], triple Elzaki transform [31], triple Shehu transform [32], triple Aboodh transform [33] and triple Natural transform [34] extensions to the original triple Laplace transform.

Abdelilah [22-26] is introduced a novel integral transform known as the Kamal transform is defined as

$$K[f(t)] = G(v) = \int_0^\infty e^{-\frac{t}{v}}[f(t)]dt \,. \quad t \geq 0 \qquad (1)$$

where v in this transform is used to factor of the variable t in the argument of the function f.

In the present study, we introduce a new notion of triple transform of functions with three variables, known as triple Kamal transform (TKT). First, the definition of the new TKT for functions of three positive variables is provided. Next, we proceed to prove some basic theorems such as existence, triple convolution and other properties. Also, we obtain the TKT of some basic functions. It is established that the novel TKT implies the original triple Laplace transform. Finally, the new TKT is effectively utilized to solve Mboctara partial differential equations.

2. Basic Definitions and Theorems for Single Kamal Transform [22]

In this section, we introduce the basic properties of Kamal transform.

Definition 2.1. Let $f(t)$ be a continuous function of t specified for $t > 0$. Then, Kamal transform of $f(t)$, denoted by $K[f(t)]$, and is defined by

$$K[f(t)] = G(v) = \int_0^\infty e^{-\frac{t}{v}}[f(t)]dt \,. \quad t \geq 0$$

The inverse of Kamal transform is given by

$$K^{-1}[G(v)] = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} e^{\frac{t}{v}} G(v)dv = f(t) \quad , \quad t > 0 \qquad (2)$$

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where c is real number.

Theorem 2.1. (*Existence conditions*).If $f(t)$ is a piecewise continuous function on the interval $[0,\infty)$ and is of exponential order φ. Then $K[f(t)]$ exists for $Re\left(\frac{1}{v}\right) > \varphi$ and satisfies

$$|f(t)| \leq Re^{\varphi t},$$

where R is positive constant.Then, Kamal transform integral converges absolutely for $Re\left(\frac{1}{v}\right) > \varphi.$

Proof of Theorem 2.1. Using the definition of Kamal transform, we get

$$|G(v)| = \left|\int_0^\infty e^{-\frac{t}{v}}[f(t)]\,dt\right| \leq \int_0^\infty e^{-\frac{t}{v}}|f(t)|\,dt \leq R\int_0^\infty e^{-\left(\frac{1}{v}-\varphi\right)t}dt = \frac{vR}{1-v\varphi}$$

Thus, Kamal transform integral converges absolutely for $Re\left(\frac{1}{v}\right) > \varphi$.

In the following arguments, we present some properties of Kamal transform.

Assume that $G(v) = K[f(t)]$ and $H(v) = K[h(t)]$ and $a, b \in \mathcal{R}$.Then, we have the following properties:

- $K[a\,f(t) + b\,h(t)] = a\,K[f(t)] + b\,K[h(t)].$
- $K^{-1}[a\,G(v) + b\,H(v)] = a\,K^{-1}[G(v)] + b\,K^{-1}[H(v)].$
- $K[t^n] = \Gamma(n+1)v^{n+1}, n \geq 0,$ **and** Γ is the regular gamma function.

- $K[e^{at}] = \frac{v}{1-av}, a \in \mathcal{R}.$
- $K[\sin at] = \frac{av^2}{1+a^2v^2}, a \in \mathcal{R}.$
- $K[\cos at] = \frac{v}{1+a^2v^2}, a \in \mathcal{R}.$
- $K[\sinh at] = \frac{av^2}{1-a^2v^2}, a \in \mathcal{R}.$
- $K[\cosh at] = \frac{v}{1-a^2v^2}, a \in \mathcal{R}.$
- $K[f'(t)] = \frac{1}{v}G(v) - f(0).$
- $K\left[f^{(n)}(t)\right] = v^{(-n)}G(v) - \sum_{k=0}^{n-1} v^{k-n+1}f^{(k)}(0)$

The above results can be obtained from the definition of Kamal transform with simple calculations (See [22])

3. Basic Concepts of Triple Kamal Transform

This section concerned with presentation of the new triple Kama lintegral transform in three-dimensional spaces termed. We present basic properties concerning the existence conditions, linearity and the inverse of this transform. Moreover, some essential properties and results are used to compute the TKT for some basic functions. We introduce the triple convolution theorem and the derivatives properties of the new transform

Definition 3.1. Let$w(x,y,t)$becontinuous function of three variables$x,y,t > 0$.Then the TKT of $w(x,y,t)$ denoted by $K_3[w(x,y,t)]$ and is defined as

$$K_3[w(x,y,t),(s,u,v)] = G(s,u,v) = K_x\left[K_y[K_t[w(x,y,t);t \to v]y \to u]x \to s\right], s,u,v > 0,$$

$$= \int_0^\infty e^{-\frac{x}{s}}\left(\int_0^\infty e^{-\frac{y}{u}}\left(\int_0^\infty e^{-\frac{t}{v}}[w(x,y,t)]\,dt\right)dy\right)dx$$

$$= \int_0^\infty\int_0^\infty\int_0^\infty e^{-\frac{x}{s}-\frac{y}{u}-\frac{t}{v}}[w(x,y,t)]dxdy\,dt.$$

where s,u and v are transform functions for $x,y$and$t$,respectively.

Thus, the inverse TKT is defined by

$$K_3^{-1}[G(s,u,v)] = K_s^{-1}\left[K_u^{-1}\left[K_v^{-1}[G(s,u,v)]\right]\right] = w(x,y,t) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{\frac{x}{s}}ds\,\frac{1}{2\pi i}\int_{b-i\infty}^{b+i\infty} e^{\frac{y}{u}}du\,\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} e^{\frac{t}{v}}G(s,u,v)dv. \quad (3)$$

where $a, b$and c are real constants.

3.1. Some Properties and Theorems of Triple Kamal Transform

In this section, we proceed to prove some basic properties and theorems such as existence, triple convolution.

Property 3.1. (*Linearity*) If $K_3[w(x,y,t)] = G(s,u,v)$ and $K_3[h(x,y,t)] = H(s,u,v)$, then for any constants A and B , we have

$$K_3[A\,w(x,y,t) + B\,h(x,y,t)] = A\,K_3[w(x,y,t)] + B\,K_3[h(x,y,t)]. \quad (4)$$

Proof of Property 3.1

$$K_3[A\,w(x,y,t)+B\,h(x,y,t)]=\int_0^\infty\int_0^\infty\int_0^\infty e^{-\frac{x}{s}-\frac{y}{u}-\frac{t}{v}}[A\,w(x,y,t)+B\,h(x,y,t)]dxdy\,dt.$$

$$=A\int_0^\infty\int_0^\infty\int_0^\infty e^{-\frac{x}{s}-\frac{y}{u}-\frac{t}{v}}[w(x,y,t)]dxdy\,dt+\int_0^\infty\int_0^\infty\int_0^\infty e^{-\frac{x}{s}-\frac{y}{u}-\frac{t}{v}}[h(x,y,t)]dxdy\,dt.$$

$$=A\,K_3[w(x,y,t)]+B\,K_3[h(x,y,t)].$$

Thus, TKT is linear integral transformations. Similarly, we can show the inverse TKT is also linear.

Property 3.2. Let $w(x,y,t)=f(x)h(y)g(t)\ ,x>0,y>0,t>0$. Then

$$K[w(x,y,t)]=K_x[f(x)]K_y[h(y)]K_t[g(t)]. \tag{5}$$

where K_x,$K_y$and $K_t$are Kamal integral transform for $f(x)$,$h(y)$and $g(t)$respectively.

Proof of Property 3.2

$$K_3[w(x,y,t)]=\int_0^\infty\int_0^\infty\int_0^\infty e^{-\frac{x}{s}-\frac{y}{u}-\frac{t}{v}}[f(x)h(y)g(t)]dxdy\,dt$$

$$=\int_0^\infty e^{-\frac{x}{s}}[f(x)]\,dx\int_0^\infty e^{-\frac{y}{u}}[h(y)]\,dy\int_0^\infty e^{-\frac{t}{v}}[g(t)]\,dt$$

$$=K_x[f(x)]K_y[h(y)]K_t[g(t)].$$

Definition 3.2. If $w(x,y,t)$ defined on $[0,X]\times[0,Y]\times[0,T]$, and satisfies the condition

$$|w(x,y,t)|\le R\,e^{\alpha x+\delta y+\lambda t},\qquad \exists R>0,\qquad \forall x>X,\qquad \forall y>Y\ and\ \ t>T$$

Then,$w(x,y,t)$ is called a function of exponential orders α ,δ and λ *as* $x,y,t\to\infty$.

Theorem 3.1. The existence condition of triple Kamal transform of the continuous function$w(x,y,t)$defined on$[0,X]\times[0,Y]\times[0,T]$ is to be of exponential orders α ,$\delta$and λ , for $\mathrm{Re}\left(\frac{1}{s}\right)>\alpha$, $\mathrm{Re}\left(\frac{1}{u}\right)>\delta$and $\mathrm{Re}\left(\frac{1}{v}\right)>\lambda$.

Proof of Theorem 3.1. Using the definition of TKT,we get

$$|G(s,u,v)|=\left|\int_0^\infty\int_0^\infty\int_0^\infty e^{-\frac{x}{s}-\frac{y}{u}-\frac{t}{v}}[w(x,y,t)]dxdy\,dt\right|\le\int_0^\infty\int_0^\infty\int_0^\infty e^{-\frac{x}{s}-\frac{y}{u}-\frac{t}{v}}|w(x,y,t)|dxdy\,dt$$

$$\le R\int_0^\infty e^{-\left(\frac{1}{s}-\alpha\right)x}dx\int_0^\infty e^{-\left(\frac{1}{u}-\delta\right)y}dy\int_0^\infty e^{-\left(\frac{1}{u}-\lambda\right)t}dt=\frac{R\,suv}{(1-\alpha s)(1-\delta u)(1-\lambda v)}.$$

where $\mathrm{Re}\left(\frac{1}{s}\right)>\alpha$, $\mathrm{Re}\left(\frac{1}{u}\right)>\delta$ **and**$\mathrm{Re}\left(\frac{1}{v}\right)>\lambda$.

Definition 3.3. The convolution of $w(x,y,t)$ and $\psi(x,y,t)$ is denoted by $(w***\psi)(x,y,t)$ and defined by

$$(w***\psi)(x,y,t)=\int_0^x\int_0^y\int_0^t w(x-\alpha,y-\delta,t-\lambda)\psi(\alpha,\delta,\lambda)\,d\alpha d\delta d\lambda \tag{6}$$

Theorem 3.2. Let $K_3[w(x,y,t)]=G(s,u,v)$. Then,

$$K_3[w(x-\alpha,y-\delta,t-\lambda)H(x-\alpha,y-\delta,t-\lambda)]=e^{-\frac{1}{s}\alpha-\frac{1}{u}\delta-\frac{1}{v}\lambda}G(s,u,v) \tag{7}$$

where $H(x,y,t)$ denotes the unit step function defined by

$$H(x-\alpha,y-\delta,t-\lambda)=\begin{cases}1, x>\alpha,y>\delta,t>\lambda\\ 0, otherwise.\end{cases}$$

Proof of Theorem 3.2. From the definition of TKT, we have

$$K_3[w(x-\alpha,y-\delta,t-\lambda)H(x-\alpha,y-\delta,t-\lambda)]=\int_0^\infty\int_0^\infty\int_0^\infty e^{-\frac{x}{s}-\frac{y}{u}-\frac{t}{v}}[w(x-\alpha,y-\delta,t-\lambda)H(x-\alpha,y-\delta,t-\lambda)]\,dxdydt=\int_0^\infty\int_0^\infty\int_0^\infty e^{-\frac{x}{s}-\frac{y}{u}-\frac{t}{v}}[w(x-\alpha,y-\delta,t-\lambda)]\,dxdydt \tag{8}$$

Putting $-\alpha=\rho$, $y-\delta=\beta$ and $t-\lambda=\mu$ in Eq.(8), we obtain

$$K_3[w(x-\alpha,y-\delta,t-\lambda)H(x-\alpha,y-\delta,t-\lambda)]=\int_0^\infty\int_0^\infty\int_0^\infty e^{-\frac{1}{s}(\rho+\alpha)-\frac{1}{u}(\beta+\delta)-\frac{1}{v}(\lambda+\mu)}[w(\rho,\beta,\mu)]\,d\rho d\beta d\mu. \tag{9}$$

Thus, Eq.(9) can be simplified into

$$K_3[w(x-\alpha,y-\delta,t-\lambda)H(x-\alpha,y-\delta,t-\lambda)] = e^{-\frac{1}{s}\alpha-\frac{1}{u}\delta-\frac{1}{v}\lambda}\left(\int_0^\infty\int_0^\infty\int_0^\infty e^{e^{-\frac{1}{s}\rho-\frac{1}{u}\beta-\frac{1}{v}\mu}}[w(\rho,\beta,\lambda)]d\rho d\beta d\mu\right)$$

$$= e^{-\frac{1}{s}\alpha-\frac{1}{u}\delta-\frac{1}{v}\lambda}G(s,u,v).$$

Theorem 3.3. (*Triple Convolution Theorem*) If $K_3[w(x,y,t)] = G(s,u,v)$ and $K_3[\psi(x,y,t)] = \Psi(s,u,v)$, then

$$K_3[(w *** \psi)(x,y,t)] = G(s,u,v)\Psi(s,u,v). \tag{10}$$

Proof of Theorem 3.3. From the definition of TKT, we have

$$K_3[(w *** \psi)(x,y,t)] = \int_0^\infty\int_0^\infty\int_0^\infty e^{-\frac{x}{s}-\frac{y}{u}-\frac{t}{v}}\left[\int_0^x\int_0^y\int_0^t w(x-\alpha,y-\delta,t-\lambda)\psi(\alpha,\delta,\lambda)\,d\alpha d\delta d\lambda\right]dxdydt. \tag{11}$$

The definition of unit step,Eq. (11) can be written as

$$K_3[(w *** \psi)(x,y,t)] =$$

$$\int_0^\infty\int_0^\infty\int_0^\infty e^{-\frac{x}{s}-\frac{y}{u}-\frac{t}{v}}\left[\int_0^\infty\int_0^\infty\int_0^\infty w(x-\alpha,y-\delta,t-\lambda)H(x-\alpha,y-\delta,t-\lambda)\psi(\alpha,\delta,\lambda)\,d\alpha d\delta d\lambda\right]dxdydt. \tag{12}$$

Thus,Eq.(12) can be written as

$$K_3[(w *** \psi)(x,y,t)]$$

$$= \int_0^\infty\int_0^\infty\int_0^\infty \psi(\alpha,\delta,\lambda)d\alpha d\delta d\lambda\left[\int_0^\infty\int_0^\infty\int_0^\infty e^{-\frac{x}{s}-\frac{y}{u}-\frac{t}{v}}w(x-\alpha,y-\delta,t-\lambda)H(x-\alpha,y-\delta,t-\lambda)\right]dxdydt.$$

Using **Theorem 3.2**, we have

$$K_3[(w *** \psi)(x,y,t)]\int_0^\infty\int_0^\infty\int_0^\infty \psi(\alpha,\delta,\lambda)d\alpha d\delta d\lambda\, e^{-\frac{1}{s}\alpha-\frac{1}{u}\delta-\frac{1}{v}\lambda}\,\mathrm{G}(s,u,v)$$

$$= \mathrm{G}(s,u,v)\int_0^\infty\int_0^\infty\int_0^\infty e^{-\frac{1}{s}\alpha-\frac{1}{u}\delta-\frac{1}{v}\lambda}\psi(\alpha,\delta,\lambda)d\alpha d\delta d\lambda = G(s,u,v)\Psi(s,u,v).$$

2.3.Triple Kamal Transform of Some Basic Functions

In this section, we introduce the TKT for some elementary functions.

i. Let $w(x,y,t) = 1$.Then

$$K_3[1] = \int_0^\infty\int_0^\infty\int_0^\infty e^{-\frac{x}{s}-\frac{y}{u}-\frac{t}{v}}dxdy\,dt = \int_0^\infty e^{-\frac{x}{s}}dx\int_0^\infty e^{-\frac{y}{u}}dy\int_0^\infty e^{-\frac{t}{v}}dt$$

Using Property 2.1 and single Kamal transform,we have

$$K_3[1] = K_x[1]K_y[1]K_t[1] = suv.$$

ii. Let $w(x,y,t) = x\,y\,t \quad ,x>0,y>0$.Then

$$K_3[x\,y\,t] = \int_0^\infty\int_0^\infty\int_0^\infty e^{-\frac{x}{s}-\frac{y}{u}-\frac{t}{v}}[xyt]dx\,dy\,dt = \int_0^\infty e^{-\frac{x}{s}}[x]\,dx\int_0^\infty e^{-\frac{y}{u}}[y]dy\int_0^\infty e^{-\frac{t}{v}}[t]\,dt$$

Using Property 2.1 and integrating by parts, we obtain:

$$K_3[xyt] = K_x[x]K_y[y]K_t[t] = s^2u^2v^2$$

Thus, by induction, we prove

$$K_3[(xyt)^n] = s^{n+1}u^{n+1}v^{n+1}(\Gamma(n+1))^3, n\in\mathcal{R}.$$

iii. Let $w(x,y,t) = e^{ax+by+ct} \quad ,x>0,y>0$ and $t>0$ and a,b and c are constants. Then

$$K_3\left[e^{ax+by+ct}\right] = \int_0^\infty\int_0^\infty\int_0^\infty e^{-\frac{x}{s}-\frac{y}{u}-\frac{t}{v}}\left[e^{ax+by+ct}\right]dx\,dy\,dt = \int_0^\infty e^{-\left(\frac{1}{s}-a\right)x}d\,x\int_0^\infty e^{-\left(\frac{1}{u}-b\right)y}dy\int_0^\infty e^{-\left(\frac{1}{v}-c\right)t}dt$$

Using Property 2.1 and single Kamal transform, we have

$$K_3\left[e^{ax+by+ct}\right] = K_x[e^{ax}]K_y\left[e^{by}\right]K_t[e^{ct}] = \frac{s}{(1-as)}\frac{u}{(1-bu)}\frac{v}{(1-cv)}$$

Similarly,

$$K_3\left[e^{i(ax+by+ct)}\right]=\frac{usv}{(1-ias)(1-ibu)(1-icv)}.$$

Thus, one can obtain

$$K_3\left[e^{i(ax+by+ct)}\right]=\frac{[1-absu-bcuv-acsv]}{(1+a^2s^2)(1+b^2u^2)(1+c^2v^2)}+\frac{[bu+as+cv-abcsuv]}{(1+a^2s^2)(1+b^2u^2)(1+c^2v^2)}i$$

Using Euler's formulas:

$$\sin(ax+by+ct)=\frac{e^{i(ax+by+ct)}-e^{-i(ax+by+ct)}}{2i}$$

$$\cos(ax+by+ct)=\frac{e^{i(ax+by+ct)}+e^{-i(ax+by+ct)}}{2}.$$

And the formulas:

$$\sinh(ax+by+ct)=\frac{e^{ax+by+ct}-e^{-(ax+by+ct)}}{2}$$

$$\cosh(ax+by+ct)=\frac{e^{ax+by+ct)}+e^{-(ax+by+ct)}}{2}.$$

Thus, we find the GTT of the following functions:

$$K_3[\cos(ax+by+ct)]=\frac{[1-absu-bcuv-acsv]}{(1+a^2s^2)(1+b^2u^2)(1+c^2v^2)},$$

$$K_3[\sin(ax+by+ct)]=\frac{[bu+as+cv-abcsuv]}{(1+a^2s^2)(1+b^2u^2)(1+c^2v^2)},$$

$$K_3[\cosh(ax+by+ct)]=\frac{[1-absu-bcuv-acsv]}{(1-a^2s^2)(1-b^2u^2)(1-c^2v^2)},$$

$$K_3[\sinh(ax+by+ct)]=\frac{[bu+as+cv+abcsuv]}{(1-a^2s^2)(1-b^2u^2)(1-c^2v^2)}.$$

4.Triple Kamal Transform for Partial Differential Derivatives

In this section, we present some results related to the new triple Kamal integral transform of partial derivatives, we begin by obtaining partial derivatives with respect to x, y and t.

Theorem 4.1. Let $G(s,u,v)$ is triple Kamal transform of $w(x,y,t)$, then

a) $K_3\left[\frac{\partial w(x,y,t)}{\partial x}\right]=\frac{1}{s}G(s,u,v)-K_yK_t[w(0,y,t)].$

b) $K_3\left[\frac{\partial^2 w(x,y,t)}{\partial x^2}\right]=\frac{1}{s^2}G(s,u,v)-\frac{1}{s}K_yK_t[w(0,y,t)]-K_yK_t\left[\frac{\partial w(0,y,t)}{\partial x}\right].$

c) $K_3\left[\frac{\partial^n w(x,y,t)}{\partial x^n}\right]=\frac{1}{s^n}G(s,u,v)-\sum_{i=0}^{n-1}s^{i-n+1}K_yK_t\left[\frac{\partial^i w(0,y,t)}{\partial x^i}\right].$

d) $K_3\left[\frac{\partial w(x,y,t)}{\partial y}\right]=\frac{1}{u}G(s,u,v)-K_xK_t[w(x,0,t)].$

e) $K_3\left[\frac{\partial^2 w(x,y,t)}{\partial y^2}\right]=\frac{1}{u^2}G(s,u,v)-\frac{1}{u}K_xK_t[w(x,0,t)]-K_xK_t\left[\frac{\partial w(x,0,t)}{\partial y}\right].$

f) $K_3\left[\frac{\partial^n w(x,y,t)}{\partial y^n}\right]=\frac{1}{u^n}G(s,u,v)-\sum_{i=0}^{n-1}u^{i-n+1}K_xK_t\left[\frac{\partial^i w(x,0,t)}{\partial y^i}\right].$

g) $K_3\left[\frac{\partial w(x,y,t)}{\partial t}\right]=\frac{1}{v}G(s,u,v)-K_xK_y[w(x,y,0)].$

h) $K_3\left[\frac{\partial^2 w(x,y,t)}{\partial t^2}\right]=\frac{1}{v^2}G(s,u,v)-\frac{1}{v}K_xK_y[w(x,y,0)]-K_xK_y\left[\frac{\partial w(x,y,0)}{\partial t}\right].$

i) $K_3\left[\frac{\partial^n w(x,y,t)}{\partial t^n}\right]=\frac{1}{v^n}G(s,u,v)-\sum_{i=0}^{n-1}v^{i-n+1}K_xK_y\left[\frac{\partial^i w(x,y,0)}{\partial t^i}\right].$

Proof of Theorem 4.1. From the definition of TKT, we have

a) $K_3\left[\frac{\partial w(x,y,t)}{\partial x}\right]=\int_0^\infty\int_0^\infty\int_0^\infty e^{-\frac{x}{s}-\frac{y}{u}-\frac{t}{v}}\left[\frac{\partial w(x,y,t)}{\partial x}\right]dxdy\,dt=\int_0^\infty\int_0^\infty e^{-\frac{y}{u}-\frac{t}{v}}dydt\int_0^\infty e^{-\frac{x}{s}}\left[\frac{\partial w(x,y,t)}{\partial x}\right]dx$

Using integrating by parts, we obtain:

Let $u=e^{-\frac{x}{s}}\Rightarrow du=-\frac{1}{s}e^{-\frac{x}{s}}dx$ and $dv=\frac{\partial w(x,y,t)}{\partial x}dx \Rightarrow v=w(x,y,t)$.Thus

$$\int_0^\infty e^{-\frac{x}{s}}\left[\frac{\partial w(x,y,t)}{\partial x}\right]dx = \left(-w(0,y,t) + \frac{1}{s}\int_0^\infty e^{-\frac{x}{s}}[w(x,y,t)]dx\right)$$

$$\therefore \quad K_3\left[\frac{\partial w(x,y,t)}{\partial x}\right] = \frac{1}{s}G(s,u,v) - K_yK_t[w(0,y,t)]. \tag{13}$$

b) $K_3\left[\frac{\partial^2 w(x,y,t)}{\partial x^2}\right] = \int_0^\infty\int_0^\infty\int_0^\infty e^{-\frac{x}{s}-\frac{y}{u}-\frac{t}{v}}\left[\frac{\partial^2 w(x,y,t)}{\partial x^2}\right]dxdy\,dt.$

$$= \int_0^\infty\int_0^\infty e^{-\frac{y}{u}-\frac{t}{v}}\,dydt\int_0^\infty e^{-\frac{x}{s}}\left[\frac{\partial^2 w(x,y,t)}{\partial x^2}\right]dx$$

Using integrating by parts, we obtain:

Let $u = e^{-\frac{x}{s}} \implies du = -\frac{1}{s}e^{-\frac{x}{s}}dx$ and $dv = \frac{\partial^2 w(x,y,t)}{\partial x^2}dx \implies v = \frac{\partial w(x,y,t)}{\partial x}$. Thus

$$\int_0^\infty e^{-\frac{x}{s}}\left[\frac{\partial^2 w(x,y,t)}{\partial x^2}\right]dx = \left(-\frac{\partial w(0,y,t)}{\partial x} + \frac{1}{s}\int_0^\infty e^{-\frac{x}{s}}\left[\frac{\partial w(x,y,t)}{\partial x}\right]dx\right)$$

Using Eq.(13), we get

$$K_3\left[\frac{\partial^2 w(x,y,t)}{\partial x^2}\right] = \frac{1}{s^2}G(s,u,v) - \frac{1}{s}K_yK_t[w(0,y,t)] - K_yK_t\left[\frac{\partial w(0,y,t)}{\partial x}\right]. \tag{14}$$

c) Using induction, we can easily prove c.

Similarly, we can prove that:

$$K_3\left[\frac{\partial w(x,y,t)}{\partial y}\right] = \frac{1}{u}G(s,u,v) - K_xK_t[w(x,0,t)].$$

$$K_3\left[\frac{\partial^2 w(x,y,t)}{\partial y^2}\right] = \frac{1}{u^2}G(s,u,v) - \frac{1}{u}K_xK_t[w(x,0,t)] - K_xK_t\left[\frac{\partial w(x,0,t)}{\partial y}\right].$$

$$K_3\left[\frac{\partial^n w(x,y,t)}{\partial y^n}\right] = \frac{1}{u^n}G(s,u,v) - \sum_{i=0}^{n-1}u^{i-n+1}K_xK_t\left[\frac{\partial^i w(x,0,t)}{\partial y^i}\right].$$

$$K_3\left[\frac{\partial w(x,y,t)}{\partial t}\right] = \frac{1}{v}G(s,u,v) - K_xK_y[w(x,y,0)].$$

$$K_3\left[\frac{\partial^2 w(x,y,t)}{\partial t^2}\right] = \frac{1}{v^2}G(s,u,v) - \frac{1}{v}K_xK_y[w(x,y,0)] - K_xK_y\left[\frac{\partial w(x,y,0)}{\partial t}\right].$$

$$K_3\left[\frac{\partial^n w(x,y,t)}{\partial t^n}\right] = \frac{1}{v^n}G(s,u,v) - \sum_{i=0}^{n-1}v^{i-n+1}K_xK_y\left[\frac{\partial^i w(x,y,0)}{\partial t^i}\right].$$

Theorem 4.2. (*Derivative properties with respect to* x, y *and* t). Let $G(s,u,v)$ is Kamal triple transform of $w(x,y,t)$,then

$$K_3\left[\frac{\partial^3 w(x,y,t)}{\partial x\partial y\partial t}\right] = \frac{1}{suv}G(s,u,v) - \frac{1}{su}K_xK_y[w(x,y,0)] - \frac{1}{sv}K_xK_t[w(x,0,t)] - \frac{1}{uv}K_yK_t[w(0,y,t)] + \frac{1}{s}K_x[w(x,0,0)] + \frac{1}{u}K_y[w(0,y,0)] + \frac{1}{v}K_t[w(0,0,t)] - w(0,0,0)$$

Proof of Theorem 4.2. From the dentition of TKT, we have

$$K_xK_yK_t\left[\frac{\partial^3 w(x,y,t)}{\partial x\partial y\partial t}\right] = \int_0^\infty\int_0^\infty e^{-\frac{x}{s}-\frac{y}{u}}\left(\int_0^\infty e^{-\frac{t}{v}}\left[\frac{\partial^3 w(x,y,t)}{\partial x\partial y\partial t}\right]dt\right)dxdy.$$

Using integrating by parts

Let $u = e^{-\frac{t}{v}} \implies du = -\frac{1}{v}e^{-\frac{t}{v}}dt$ and $dv = \frac{\partial^3 w(x,y,t)}{\partial x\partial y\partial t}dt \implies v = \frac{\partial^2 w(x,y,t)}{\partial x\partial y}$.

Then, we have

$$K_xK_yK_t\left[\frac{\partial^3 w(x,y,t)}{\partial x\partial y\partial t}\right] = \int_0^\infty\int_0^\infty e^{-\frac{x}{s}-\frac{y}{u}}\left(\left(-\frac{\partial^2 w(x,y,0)}{\partial x\partial y} + \frac{1}{v}\int_0^\infty e^{-\frac{t}{v}}\left[\frac{\partial^2 w(x,y,t)}{\partial x\partial y}\right]dt\right)\right)dxdy.$$

Therefore,

$$K_xK_yK_t\left[\frac{\partial^3 w(x,y,t)}{\partial x\partial y\partial t}\right] = -K_xK_y\left[\frac{\partial^2 w(x,y,0)}{\partial x\partial y}\right] + \frac{1}{v}K_xK_yK_t\left[\frac{\partial^2 w(x,y,t)}{\partial x\partial y}\right]. \tag{15}$$

We take the first term:

$$-K_xK_y\left[\frac{\partial^2 w(x,y,0)}{\partial x\partial y}\right]=-\int_0^\infty e^{-\frac{x}{s}}\left(\int_0^\infty e^{-\frac{y}{u}}\left[\frac{\partial^2 w(x,y,0)}{\partial x\partial y}\right]dy\right)dx.$$

Using integrating by parts

Let $u=e^{-\frac{y}{u}} \implies du=-\frac{1}{u}e^{-\frac{y}{u}}dy$ and $dv=\frac{\partial^2 w(x,y,0)}{\partial x\partial y}dy \implies v=\frac{\partial w(x,y,0)}{\partial x}$. Then, we have

$$-K_xK_y\left[\frac{\partial^2 w(x,y,0)}{\partial x\partial y}\right]=-\int_0^\infty e^{-\frac{x}{s}}\left(-\frac{\partial w(x,0,0)}{\partial x}+\frac{1}{u}\int_0^\infty e^{-\frac{y}{u}}\left[\frac{\partial w(x,y,0)}{\partial x}\right]dy\right)dx.$$

$$=K_x\left[\frac{\partial w(x,0,0)}{\partial x}\right]-\frac{1}{u}K_xK_y\left[\frac{\partial w(x,y,0)}{\partial x}\right]$$

Applying single Kamal transform for $\frac{\partial w(x,0,0)}{\partial x}$ and double Kamal to $\frac{\partial w(x,y,0)}{\partial x}$, we have

$$K_x\left[\frac{\partial w(x,0,0)}{\partial x}\right]-\frac{1}{u}K_xK_y\left[\frac{\partial w(x,y,0)}{\partial x}\right]$$

$$=-w(0,0,0)+\frac{1}{s}L_x[w(x,0,0)]+\frac{1}{u}K_y[w(0,y,0)]-\frac{1}{su}K_xK_y[w(x,y,0)]. \tag{16}$$

We take the second:

$$\frac{1}{v}K_xK_yK_t\left[\frac{\partial^2 w(x,y,t)}{\partial x\partial y}\right]=\frac{1}{v}\int_0^\infty\int_0^\infty e^{-\frac{x}{s}-\frac{t}{v}}\left(\int_0^\infty e^{-\frac{y}{u}}\left[\frac{\partial^2 w(x,y,t)}{\partial x\partial y}\right]dy\right)dxdt$$

Using integrating by parts

$$\frac{1}{v}K_xK_yK_t\left[\frac{\partial^2 w(x,y,t)}{\partial x\partial y}\right]=\frac{1}{v}\int_0^\infty\int_0^\infty e^{-\frac{x}{s}-\frac{t}{v}}\left(-\frac{\partial w(x,0,t)}{\partial x}+\frac{1}{u}\int_0^\infty e^{-\frac{y}{u}}\left[\frac{\partial w(x,y,t)}{\partial x}\right]dy\right)dxdt$$

Therefore,

$$\frac{1}{v}K_xK_yK_t\left[\frac{\partial^2 w(x,y,t)}{\partial x\partial y}\right]=-\frac{1}{v}K_xK_t\left[\frac{\partial w(x,0,t)}{\partial x}\right]+\frac{1}{vu}K_xK_yK_t\left[\frac{\partial w(x,y,t)}{\partial x}\right]$$

Using theorem and applying double Kamal transform to $\frac{\partial w(x,0,t)}{\partial x}$, we have

$$\frac{1}{v}K_xK_yK_t\left[\frac{\partial^2 w(x,y,t)}{\partial x\partial y}\right]=\frac{1}{v}K_t[w(0,0,t)]-\frac{1}{vs}K_xK_t[w(x,0,t)]+\frac{1}{suv}G(s,u,v)-\frac{1}{vu}K_yK_t[w(0,y,t)]. \tag{17}$$

Substituting Eq. (16) and Eq. (17) into Eq.(15), we get

$$K_3\left[\frac{\partial^3 w(x,y,t)}{\partial x\partial y\partial t}\right]=\frac{1}{suv}G(s,u,v)-\frac{1}{su}K_xK_y[w(x,y,0)]-\frac{1}{sv}K_xK_t[w(x,0,t)]-\frac{1}{uv}K_yK_t[w(0,y,t)]+\frac{1}{s}K_x[w(x,0,0)]$$
$$+\frac{1}{u}K_y[w(0,y,0)]+\frac{1}{v}K_t[w(0,0,t)]-w(0,0,0).$$

4.Applications

In this section, we apply the properties associated with triple Kamal transform established above to solvethree-dimensional Mboctara partial differential equations.

Example 4.1

Consider the following homogeneous three-dimensional Mboctara partial differential equation

$$w_{xyt}(x,y,t)+w(x,y,t)=0 \tag{18}$$

Subject to the boundary and initial conditions

$$\begin{cases} w(x,y,0)=e^{x+y} & w(x,0,0)=e^{x} \\ w(x,0,t)=e^{x-t}, & w(0,y,0)=e^{y} \\ w(0,y,t)=e^{y-t} & w(0,0,t)=e^{-t} \end{cases} \tag{19}$$

Applying TKT on both sides of Eq. (18), we have

$$K_3\left[w_{xyt}(x,y,t)+w(x,y,t)\right]=0 \tag{20}$$

By linearity property and partial derivative properties of TKT, we get

$$\frac{1}{suv}G(s,u,v)-\frac{1}{su}K_xK_y[w(x,y,0)]-\frac{1}{sv}K_xK_t[w(x,0,t)]-\frac{1}{uv}K_yK_t[w(0,y,t)]+\frac{1}{s}K_x[w(x,0,0)]+\frac{1}{u}K_y[w(0,y,0)]+\frac{1}{v}K_t[w(0,0,t)]-w(0,0,0)+G(s,u,v)=0 \tag{21}$$

Substituting

$$K_xK_y[w(x,y,0)]=\frac{su}{(1-s)(1-u)},\quad K_x[w(x,0,0)]=\frac{s}{1-s},$$
$$K_xK_t[w(x,0,t)]=\frac{sv}{(1-s)(1+v)},\quad K_y[w(0,y,0)]=\frac{u}{1-u},$$
$$K_yK_t[w(0,y,t)]=\frac{uv}{(1-u)(1+v)},\quad K_t[w(0,0,t)]=\frac{v}{1+v},$$
$$w(0,0,0)=1.$$

in Eq. (21) and simplifying, we obtain

$$G(s,u,v)=\frac{suv}{(1-s)(1-u)(1+v)} \tag{22}$$

Taking inverse TKT for Eq. (22), we get

$$w(x,y,t)={K_3}^{-1}[G(s,u,v)]={K_3}^{-1}\left[\frac{suv}{(1-s)(1-u)(1+v)}\right]=e^{x+y-t}.$$

Example 4.2

Consider the following nonhomogeneous third-order Mboctara partial differential equation

$$w_{xyt}(x,y,t)+w(x,y,t)=3e^{-x-2y+t} \tag{23}$$

Subject to the boundary and initial conditions

$$\begin{cases} w(x,y,0)=e^{-x-2y} & w(x,0,0)=e^{-x} \\ w(x,0,t)=e^{-x+t}, & w(0,y,0)=e^{-2y} \\ w(0,y,t)=e^{-2y+t} & w(0,0,t)=e^{t} \end{cases} \tag{24}$$

Applying TKT on both sides of Eq. (23), we have

$$K_3\left[w_{xyt}(x,y,t)+w(x,y,t)\right]=K_3[3e^{-x-2y+t}] \tag{25}$$

By linearity property and partial derivative properties of TKT, we get

$$\frac{1}{suv}G(s,u,v)-\frac{1}{su}K_xK_y[w(x,y,0)]-\frac{1}{sv}K_xK_t[w(x,0,t)]-\frac{1}{uv}K_yK_t[w(0,y,t)]+\frac{1}{s}K_x[w(x,0,0)]+\frac{1}{u}K_y[w(0,y,0)]+\frac{1}{v}K_t[w(0,0,t)]-w(0,0,0)+G(s,u,v)=\frac{3suv}{(1+s)(1+2u)(1-v)} \tag{26}$$

Substituting

$$K_xK_y[w(x,y,0)]=\frac{su}{(1+s)(1+2u)},\quad K_x[w(x,0,0)]=\frac{s}{1+s},$$
$$K_xK_t[w(x,0,t)]=\frac{sv}{(1+s)(1-v)},\quad K_y[w(0,y,0)]=\frac{u}{1+2u},$$
$$K_yK_t[w(0,y,t)]=\frac{uv}{(1+2u)(1-v)},\quad K_t[w(0,0,t)]=\frac{v}{1-v}$$
$$w(0,0,0)=1$$

in Eq. (26) and simplifying, we obtain

$$G(s,u,v)=\frac{suv}{(1+s)(1+2u)(1-v)} \tag{27}$$

Taking inverse TKT for Eq. (27), we get

$$w(x,y,t)={K_3}^{-1}[G(s,u,v)]={K_3}^{-1}\left[\frac{suv}{(1+s)(1+2u)(1-v)}\right]=e^{-x-2y+t}.$$

Example 4.3

Consider the following nonhomogeneous third-order Mboctara partial differential equation

$$w_{xyt}(x,y,t)+w(x,y,t)=\cos x\cos y\cos t-\sin x\sin y\sin t \tag{28}$$

Subject to the boundary and initial conditions

$$\begin{cases} w(x,y,0) = \cos x \cos y \quad w(x,0,0) = \cos x \\ w(x,0,t) = \cos x \cos t\ ,\ w(0,y,0) = \cos y \\ w(0,y,t) = \cos y \cos t \quad w(0,0,t) = \cos t \end{cases} \tag{29}$$

Applying TKT on both sides of Eq. (28), we have

$$\mathrm{K}_3\left[w_{xyt}(x,y,t) + w(x,y,t)\right] = \mathrm{K}_3[\cos x \cos y \cos t - \sin x \sin y \sin \mathrm{t}] \tag{30}$$

By linearity property and partial derivative properties of TKT, we get

$$\frac{1}{suv}G(s,u,v) - \frac{1}{su}K_xK_y[w(x,y,0)] - \frac{1}{sv}K_xK_t[w(x,0,t)] - \frac{1}{uv}K_yK_t[w(0,y,t)] + \frac{1}{s}K_x[w(x,0,0)]$$
$$+\frac{1}{u}K_y[w(0,y,0)] + \frac{1}{v}K_t[w(0,0,t)] - w(0,0,0) + G(s,u,v) = \frac{suv - s^2u^2v^2}{(1+\mathrm{s}^2)(1+u^2)(1+v^2)} \tag{31}$$

Substituting

$$K_xK_y[w(x,y,0)] = \frac{su}{(1+\mathrm{s}^2)(1+u^2)}, \quad K_x[w(x,0,0)] = \frac{s}{(1+\mathrm{s}^2)},$$
$$K_xK_t[w(x,0,t)] = \frac{sv}{(1+\mathrm{s}^2)(1+v^2)}, \quad K_y[w(0,y,0)] = \frac{u}{(1+u^2)},$$
$$K_yK_t[w(0,y,t)] = \frac{uv}{(1+u^2)(1+v^2)}, \qquad K_t[w(0,0,t)] = \frac{v}{(1+v^2)},$$
$$w(0,0,0) = 1.$$

in Eq. (31) and simplifying, we obtain

$$G(s,u,v) = \frac{suv}{(1+\mathrm{s}^2)(1+u^2)(1+v^2)} \tag{32}$$

Taking inverse TKT for Eq. (32), we get

$$w(x,y,t) = {K_3}^{-1}[G(s,u,v)] = {K_3}^{-1}\left[\frac{suv}{(1+s^2)(1+u^2)(1+v^2)}\right] = \cos x \cos y \cos t.$$

5. Conclusion

Taking clue from work in single integral transforms in one - dimensional spaces and double integral transform in tow - dimensional spaces, we introduce a novel transform known as triple Kamal integral transform. This new transform refines and implies the original triple Laplace transform in positive quadrant planes. We proved some main properties related to suggested triple Kamal transform such as triple convolution theorem and other properties. To evaluate the effectiveness of this transform, we apply to solve a range of PDEs subject to standard conditions. It is recommended that researchers under take studies concerning the applicability of using this transform to solve integral deferential equations, nonlinear PDEs and fractional deferential equations.

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