



Solution of Three - Dimensional Mboctara Equation via Triple Kamal Transform

Abdelilah Kamal.H.Sedeeg

Department of Mathematics, Faculty of Education, University of the Holy Quran and Islamic Sciences, Sudan.

ARTICLE INFO

Article history:

Received: 23 May 2023;

Received in revised form:

10 August 2023;

Accepted: 25 August 2023;

Keywords

Laplace Transform,
Single Kamal Transforms,
Triple Kamal Transform,
Triple convolution Theorem,
Partial Derivatives,
Mboctara Equation.

ABSTRACT

The goal of this study is to suggest a new triple Kamal integral transform. We outline its essential properties and proved some important results, including existence theorem, triple convolution theorem and derivatives properties. Moreover, the proposed new transform is utilized to solve Mboctara partial differential equations.

© 2023 Elixir All rights reserved.

1. Introduction

Integral transform methods have been proved as one of the most effective means for solving partial differential equations. Their applications extend to many phenomena in mathematical physics, engineering and many other scientific fields. These sciences can be modeled in the form of mathematical equations expressed in terms of PDEs [1–11]. There by, it is feasible to transform PDEs in terms of algebraic equations and thus obtain exact solutions of PDEs. Many researchers and academics have exerted great efforts to develop and perfect these methods and apply them to deal with a wide range of problems in Mathematics. Well known examples of these methods are the Fourier and Laplace transforms among many other methods [12–22].

Recently, triple Laplace transform has widely been applied to solve PDEs with unknown functions of three variables to obtain more satisfactory results compared with numerical methods [27–29]. In addition, researchers have contributed extensions the original triple Laplace transform such as triple Sumudu transform [30], triple Elzaki transform [31], triple Shehu transform [32], triple Aboodh transform [33] and triple Natural transform [34] extensions to the original triple Laplace transform.

Abdelilah [22–26] is introduced a novel integral transform known as the Kamal transform is defined as

$$K[f(t)] = G(v) = \int_0^{\infty} e^{-\frac{t}{v}} [f(t)] dt \quad t \geq 0 \quad (1)$$

where v in this transform is used to factor of the variable t in the argument of the function f .

In the present study, we introduce a new notion of triple transform of functions with three variables, known as triple Kamal transform (TKT). First, the definition of the new TKT for functions of three positive variables is provided. Next, we proceed to prove some basic theorems such as existence, triple convolution and other properties. Also, we obtain the TKT of some basic functions. It is established that the novel TKT implies the original triple Laplace transform. Finally, the new TKT is effectively utilized to solve Mboctara partial differential equations.

2. Basic Definitions and Theorems for Single Kamal Transform [22]

In this section, we introduce the basic properties of Kamal transform.

Definition 2.1. Let $f(t)$ be a continuous function of t specified for $t > 0$. Then, Kamal transform of $f(t)$, denoted by $K[f(t)]$, and is defined by

$$K[f(t)] = G(v) = \int_0^{\infty} e^{-\frac{t}{v}} [f(t)] dt \quad t \geq 0$$

The inverse of Kamal transform is given by

$$K^{-1}[G(v)] = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{\frac{t}{v}} G(v) dv = f(t) \quad , \quad t > 0 \quad (2)$$

where c is real number.

Theorem 2.1. (Existence conditions). If $f(t)$ is a piecewise continuous function on the interval $[0, \infty)$ and is of exponential order φ . Then $K[f(t)]$ exists for $\operatorname{Re}\left(\frac{1}{v}\right) > \varphi$ and satisfies

$$|f(t)| \leq R e^{\varphi t},$$

where R is positive constant. Then, Kamal transform integral converges absolutely for $\operatorname{Re}\left(\frac{1}{v}\right) > \varphi$.

Proof of Theorem 2.1. Using the definition of Kamal transform, we get

$$\begin{aligned} |G(v)| &= \left| \int_0^\infty e^{-\frac{t}{v}} [f(t)] dt \right| \leq \int_0^\infty e^{-\frac{t}{v}} |f(t)| dt \leq R \int_0^\infty e^{-(\frac{1}{v}-\varphi)t} dt \\ &= \frac{vR}{1-v\varphi} \end{aligned}$$

Thus, Kamal transform integral converges absolutely for $\operatorname{Re}\left(\frac{1}{v}\right) > \varphi$.

In the following arguments, we present some properties of Kamal transform.

Assume that $G(v) = K[f(t)]$ and $H(v) = K[h(t)]$ and $a, b \in \mathcal{R}$. Then, we have the following properties:

- $K[af(t) + bh(t)] = aK[f(t)] + bK[h(t)]$.
- $K^{-1}[aG(v) + bH(v)] = aK^{-1}[G(v)] + bK^{-1}[H(v)]$.
- $K[t^n] = \Gamma(n+1)v^{n+1}$, $n \geq 0$, and Γ is the regular gamma function.

- $K[e^{at}] = \frac{v}{1-av}$, $a \in \mathcal{R}$.

- $K[\sin at] = \frac{av^2}{1+a^2v^2}$, $a \in \mathcal{R}$.

- $K[\cos at] = \frac{v}{1+a^2v^2}$, $a \in \mathcal{R}$.

- $K[\sinh at] = \frac{av^2}{1-a^2v^2}$, $a \in \mathcal{R}$.

- $K[\cosh at] = \frac{v}{1-a^2v^2}$, $a \in \mathcal{R}$.

- $K[f'(t)] = \frac{1}{v}G(v) - f(0)$.

- $K[f^{(n)}(t)] = v^{(-n)}G(v) - \sum_{k=0}^{n-1} v^{k-n+1} f^{(k)}(0)$

The above results can be obtained from the definition of Kamal transform with simple calculations (See [22])

3. Basic Concepts of Triple Kamal Transform

This section concerned with presentation of the new triple Kama lintegral transform in three-dimensional spaces termed. We present basic properties concerning the existence conditions, linearity and the inverse of this transform. Moreover, some essential properties and results are used to compute the TKT for some basic functions. We introduce the triple convolution theorem and the derivatives properties of the new transform

Definition 3.1. Let $w(x, y, t)$ be continuous function of three variables $x, y, t > 0$. Then the TKT of $w(x, y, t)$ denoted by $K_3[w(x, y, t)]$ and is defined as

$$K_3[w(x, y, t), (s, u, v)] = G(s, u, v) = K_x[K_y[K_t[w(x, y, t); t \rightarrow v]y \rightarrow u]x \rightarrow s], s, u, v > 0,$$

$$\begin{aligned} &= \int_0^\infty e^{-\frac{x}{s}} \left(\int_0^\infty e^{-\frac{y}{u}} \left(\int_0^\infty e^{-\frac{t}{v}} [w(x, y, t)] dt \right) dy \right) dx \\ &= \int_0^\infty \int_0^\infty \int_0^\infty e^{-\frac{x}{s} - \frac{y}{u} - \frac{t}{v}} [w(x, y, t)] dx dy dt. \end{aligned}$$

where s, u and v are transform functions for x, y and t , respectively.

Thus, the inverse TKT is defined by

$$K_3^{-1}[G(s, u, v)] = K_s^{-1} \left[K_u^{-1} \left[K_v^{-1}[G(s, u, v)] \right] \right] = w(x, y, t) = \frac{1}{2\pi i} \int_{a-i\infty}^{a+i\infty} e^{\frac{x}{s}} ds \frac{1}{2\pi i} \int_{b-i\infty}^{b+i\infty} e^{\frac{y}{u}} du \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{\frac{t}{v}} G(s, u, v) dv. \quad (3)$$

where a, b and c are real constants.

3.1. Some Properties and Theorems of Triple Kamal Transform

In this section, we proceed to prove some basic properties and theorems such as existence, triple convolution.

Property 3.1. (Linearity) If $K_3[w(x, y, t)] = G(s, u, v)$ and $K_3[h(x, y, t)] = H(s, u, v)$, then for any constants A and B , we have

$$K_3[A w(x, y, t) + B h(x, y, t)] = A K_3[w(x, y, t)] + B K_3[h(x, y, t)]. \quad (4)$$

Proof of Property 3.1

$$\begin{aligned}
K_3[A w(x, y, t) + B h(x, y, t)] &= \int_0^\infty \int_0^\infty \int_0^\infty e^{-\frac{x}{s} - \frac{y}{u} - \frac{t}{v}} [A w(x, y, t) + B h(x, y, t)] dx dy dt. \\
&= A \int_0^\infty \int_0^\infty \int_0^\infty e^{-\frac{x}{s} - \frac{y}{u} - \frac{t}{v}} [w(x, y, t)] dx dy dt + \int_0^\infty \int_0^\infty \int_0^\infty e^{-\frac{x}{s} - \frac{y}{u} - \frac{t}{v}} [h(x, y, t)] dx dy dt. \\
&= A K_3[w(x, y, t)] + B K_3[h(x, y, t)].
\end{aligned}$$

Thus, TKT is linear integral transformations. Similarly, we can show the inverse TKT is also linear.

Property 3.2. Let $w(x, y, t) = f(x)h(y)g(t)$, $x > 0, y > 0, t > 0$. Then

$$K[w(x, y, t)] = K_x[f(x)]K_y[h(y)]K_t[g(t)]. \quad (5)$$

where K_x, K_y and K_t are Kamal integral transform for $f(x), h(y)$ and $g(t)$ respectively.

Proof of Property 3.2

$$\begin{aligned}
K_3[w(x, y, t)] &= \int_0^\infty \int_0^\infty \int_0^\infty e^{-\frac{x}{s} - \frac{y}{u} - \frac{t}{v}} [f(x)h(y)g(t)] dx dy dt \\
&= \int_0^\infty e^{-\frac{x}{s}} [f(x)] dx \int_0^\infty e^{-\frac{y}{u}} [h(y)] dy \int_0^\infty e^{-\frac{t}{v}} [g(t)] dt \\
&= K_x[f(x)]K_y[h(y)]K_t[g(t)].
\end{aligned}$$

Definition 3.2. If $w(x, y, t)$ defined on $[0, X] \times [0, Y] \times [0, T]$, and satisfies the condition

$$|w(x, y, t)| \leq R e^{\alpha x + \delta y + \lambda t}, \quad \exists R > 0, \quad \forall x > X, \quad \forall y > Y \text{ and } t > T$$

Then, $w(x, y, t)$ is called a function of exponential orders α, δ and λ as $x, y, t \rightarrow \infty$.

Theorem 3.1. The existence condition of triple Kamal transform of the continuous function $w(x, y, t)$ defined on $[0, X] \times [0, Y] \times [0, T]$ is to be of exponential orders α, δ and λ , for $\operatorname{Re}\left(\frac{1}{s}\right) > \alpha$, $\operatorname{Re}\left(\frac{1}{u}\right) > \delta$ and $\operatorname{Re}\left(\frac{1}{v}\right) > \lambda$.

Proof of Theorem 3.1. Using the definition of TKT, we get

$$\begin{aligned}
|G(s, u, v)| &= \left| \int_0^\infty \int_0^\infty \int_0^\infty e^{-\frac{x}{s} - \frac{y}{u} - \frac{t}{v}} [w(x, y, t)] dx dy dt \right| \leq \int_0^\infty \int_0^\infty \int_0^\infty e^{-\frac{x}{s} - \frac{y}{u} - \frac{t}{v}} |w(x, y, t)| dx dy dt \\
&\leq R \int_0^\infty e^{-(\frac{1}{s}-\alpha)x} dx \int_0^\infty e^{-(\frac{1}{u}-\delta)y} dy \int_0^\infty e^{-(\frac{1}{v}-\lambda)t} dt = \frac{R s u v}{(1-\alpha s)(1-\delta u)(1-\lambda v)}.
\end{aligned}$$

where $\operatorname{Re}\left(\frac{1}{s}\right) > \alpha$, $\operatorname{Re}\left(\frac{1}{u}\right) > \delta$ and $\operatorname{Re}\left(\frac{1}{v}\right) > \lambda$.

Definition 3.3. The convolution of $w(x, y, t)$ and $\psi(x, y, t)$ is denoted by $(w *** \psi)(x, y, t)$ and defined by

$$(w *** \psi)(x, y, t) = \int_0^x \int_0^y \int_0^t w(x-\alpha, y-\delta, t-\lambda) \psi(\alpha, \delta, \lambda) d\alpha d\delta d\lambda \quad (6)$$

Theorem 3.2. Let $K_3[w(x, y, t)] = G(s, u, v)$. Then,

$$K_3[w(x-\alpha, y-\delta, t-\lambda)H(x-\alpha, y-\delta, t-\lambda)] = e^{-\frac{1}{s}\alpha - \frac{1}{u}\delta - \frac{1}{v}\lambda} G(s, u, v) \quad (7)$$

where $H(x, y, t)$ denotes the unit step function defined by

$$H(x-\alpha, y-\delta, t-\lambda) = \begin{cases} 1, & x > \alpha, y > \delta, t > \lambda \\ 0, & \text{otherwise.} \end{cases}$$

Proof of Theorem 3.2. From the definition of TKT, we have

$$\begin{aligned}
K_3[w(x-\alpha, y-\delta, t-\lambda)H(x-\alpha, y-\delta, t-\lambda)] &= \int_0^\infty \int_0^\infty \int_0^\infty e^{-\frac{x}{s} - \frac{y}{u} - \frac{t}{v}} [w(x-\alpha, y-\delta, t-\lambda)H(x-\alpha, y-\delta, t-\lambda)] dx dy dt \\
&= \int_0^\infty \int_0^\infty \int_0^\infty e^{-\frac{x}{s} - \frac{y}{u} - \frac{t}{v}} [w(x-\alpha, y-\delta, t-\lambda)] dx dy dt
\end{aligned} \quad (8)$$

Putting $-\alpha = \rho$, $y - \delta = \beta$ and $t - \lambda = \mu$ in Eq.(8), we obtain

$$K_3[w(x-\alpha, y-\delta, t-\lambda)H(x-\alpha, y-\delta, t-\lambda)] = \int_0^\infty \int_0^\infty \int_0^\infty e^{-\frac{1}{s}(\rho+\alpha) - \frac{1}{u}(\beta+\delta) - \frac{1}{v}(\lambda+\mu)} [w(\rho, \beta, \mu)] d\rho d\beta d\mu. \quad (9)$$

Thus, Eq.(9) can be simplified into

$$K_3[w(x-\alpha, y-\delta, t-\lambda)H(x-\alpha, y-\delta, t-\lambda)] = e^{-\frac{1}{s}\alpha - \frac{1}{u}\delta - \frac{1}{v}\lambda} \left(\int_0^\infty \int_0^\infty \int_0^\infty e^{e^{-\frac{1}{s}\rho - \frac{1}{u}\beta - \frac{1}{v}\mu}} [w(\rho, \beta, \lambda)] d\rho d\beta d\mu \right) \\ = e^{-\frac{1}{s}\alpha - \frac{1}{u}\delta - \frac{1}{v}\lambda} G(s, u, v).$$

Theorem 3.3. (Triple Convolution Theorem) If $K_3[w(x, y, t)] = G(s, u, v)$ and $K_3[\psi(x, y, t)] = \Psi(s, u, v)$, then

$$K_3[(w *** \psi)(x, y, t)] = G(s, u, v)\Psi(s, u, v). \quad (10)$$

Proof of Theorem 3.3. From the definition of TKT, we have

$$K_3[(w *** \psi)(x, y, t)] = \int_0^\infty \int_0^\infty \int_0^\infty e^{-\frac{x}{s} - \frac{y}{u} - \frac{t}{v}} \left[\int_0^x \int_0^y \int_0^t w(x-\alpha, y-\delta, t-\lambda) \psi(\alpha, \delta, \lambda) d\alpha d\delta d\lambda \right] dx dy dt. \quad (11)$$

The definition of unit step, Eq. (11) can be written as

$$K_3[(w *** \psi)(x, y, t)] = \int_0^\infty \int_0^\infty \int_0^\infty e^{-\frac{x}{s} - \frac{y}{u} - \frac{t}{v}} \left[\int_0^\infty \int_0^\infty \int_0^\infty w(x-\alpha, y-\delta, t-\lambda) H(x-\alpha, y-\delta, t-\lambda) \psi(\alpha, \delta, \lambda) d\alpha d\delta d\lambda \right] dx dy dt. \quad (12)$$

Thus, Eq. (12) can be written as

$$K_3[(w *** \psi)(x, y, t)] = \int_0^\infty \int_0^\infty \int_0^\infty \psi(\alpha, \delta, \lambda) d\alpha d\delta d\lambda \left[\int_0^\infty \int_0^\infty \int_0^\infty e^{-\frac{x}{s} - \frac{y}{u} - \frac{t}{v}} w(x-\alpha, y-\delta, t-\lambda) H(x-\alpha, y-\delta, t-\lambda) \right] dx dy dt.$$

Using Theorem 3.2, we have

$$K_3[(w *** \psi)(x, y, t)] \int_0^\infty \int_0^\infty \int_0^\infty \psi(\alpha, \delta, \lambda) d\alpha d\delta d\lambda e^{-\frac{1}{s}\alpha - \frac{1}{u}\delta - \frac{1}{v}\lambda} G(s, u, v) \\ = G(s, u, v) \int_0^\infty \int_0^\infty \int_0^\infty e^{-\frac{1}{s}\alpha - \frac{1}{u}\delta - \frac{1}{v}\lambda} \psi(\alpha, \delta, \lambda) d\alpha d\delta d\lambda = G(s, u, v)\Psi(s, u, v).$$

2.3. Triple Kamal Transform of Some Basic Functions

In this section, we introduce the TKT for some elementary functions.

i. Let $w(x, y, t) = 1$. Then

$$K_3[1] = \int_0^\infty \int_0^\infty \int_0^\infty e^{-\frac{x}{s} - \frac{y}{u} - \frac{t}{v}} dx dy dt = \int_0^\infty e^{-\frac{x}{s}} dx \int_0^\infty e^{-\frac{y}{u}} dy \int_0^\infty e^{-\frac{t}{v}} dt$$

Using Property 2.1 and single Kamal transform, we have

$$K_3[1] = K_x[1]K_y[1]K_t[1] = suv.$$

ii. Let $w(x, y, t) = x y t$, $x > 0, y > 0$. Then

$$K_3[x y t] = \int_0^\infty \int_0^\infty \int_0^\infty e^{-\frac{x}{s} - \frac{y}{u} - \frac{t}{v}} [x y t] dx dy dt = \int_0^\infty e^{-\frac{x}{s}} [x] dx \int_0^\infty e^{-\frac{y}{u}} [y] dy \int_0^\infty e^{-\frac{t}{v}} [t] dt$$

Using Property 2.1 and integrating by parts, we obtain:

$$K_3[x y t] = K_x[x]K_y[y]K_t[t] = s^2 u^2 v^2$$

Thus, by induction, we prove

$$K_3[(x y t)^n] = s^{n+1} u^{n+1} v^{n+1} (\Gamma(n+1))^3, n \in \mathcal{R}.$$

iii. Let $w(x, y, t) = e^{ax+by+ct}$, $x > 0, y > 0$ and $t > 0$ and a, b and c are constants. Then

$$K_3[e^{ax+by+ct}] = \int_0^\infty \int_0^\infty \int_0^\infty e^{-\frac{x}{s} - \frac{y}{u} - \frac{t}{v}} [e^{ax+by+ct}] dx dy dt = \int_0^\infty e^{-(\frac{1}{s}-a)x} dx \int_0^\infty e^{-(\frac{1}{u}-b)y} dy \int_0^\infty e^{-(\frac{1}{v}-c)t} dt$$

Using Property 2.1 and single Kamal transform, we have

$$K_3[e^{ax+by+ct}] = K_x[e^{ax}]K_y[e^{by}]K_t[e^{ct}] = \frac{s}{(1-as)} \frac{u}{(1-bu)} \frac{v}{(1-cv)}$$

Similarly,

$$K_3[e^{i(ax+by+ct)}] = \frac{usv}{(1-ias)(1-ibu)(1-icv)}.$$

Thus, one can obtain

$$K_3[e^{i(ax+by+ct)}] = \frac{[1-absu-bcuv-acsv]}{(1+a^2s^2)(1+b^2u^2)(1+c^2v^2)} + \frac{[bu+as+cv-abcsuv]}{(1+a^2s^2)(1+b^2u^2)(1+c^2v^2)} i$$

Using Euler's formulas:

$$\sin(ax+by+ct) = \frac{e^{i(ax+by+ct)} - e^{-i(ax+by+ct)}}{2i}$$

$$\cos(ax+by+ct) = \frac{e^{i(ax+by+ct)} + e^{-i(ax+by+ct)}}{2}.$$

And the formulas:

$$\sinh(ax+by+ct) = \frac{e^{ax+by+ct} - e^{-(ax+by+ct)}}{2}$$

$$\cosh(ax+by+ct) = \frac{e^{ax+by+ct} + e^{-(ax+by+ct)}}{2}.$$

Thus, we find the GTT of the following functions:

$$K_3[\cos(ax+by+ct)] = \frac{[1-absu-bcuv-acsv]}{(1+a^2s^2)(1+b^2u^2)(1+c^2v^2)},$$

$$K_3[\sin(ax+by+ct)] = \frac{[bu+as+cv-abcsuv]}{(1+a^2s^2)(1+b^2u^2)(1+c^2v^2)},$$

$$K_3[\cosh(ax+by+ct)] = \frac{[1-absu-bcuv-acsv]}{(1-a^2s^2)(1-b^2u^2)(1-c^2v^2)},$$

$$K_3[\sinh(ax+by+ct)] = \frac{[bu+as+cv+abcsuv]}{(1-a^2s^2)(1-b^2u^2)(1-c^2v^2)}.$$

4. Triple Kamal Transform for Partial Differential Derivatives

In this section, we present some results related to the new triple Kamal integral transform of partial derivatives, we begin by obtaining partial derivatives with respect to x , y and t .

Theorem 4.1. Let $G(s, u, v)$ is triple Kamal transform of $w(x, y, t)$, then

$$\begin{aligned} \text{a) } K_3 \left[\frac{\partial w(x, y, t)}{\partial x} \right] &= \frac{1}{s} G(s, u, v) - K_y K_t [w(0, y, t)]. \\ \text{b) } K_3 \left[\frac{\partial^2 w(x, y, t)}{\partial x^2} \right] &= \frac{1}{s^2} G(s, u, v) - \frac{1}{s} K_y K_t [w(0, y, t)] - K_y K_t \left[\frac{\partial w(0, y, t)}{\partial x} \right]. \\ \text{c) } K_3 \left[\frac{\partial^n w(x, y, t)}{\partial x^n} \right] &= \frac{1}{s^n} G(s, u, v) - \sum_{i=0}^{n-1} s^{i-n+1} K_y K_t \left[\frac{\partial^i w(0, y, t)}{\partial x^i} \right]. \\ \text{d) } K_3 \left[\frac{\partial w(x, y, t)}{\partial y} \right] &= \frac{1}{u} G(s, u, v) - K_x K_t [w(x, 0, t)]. \\ \text{e) } K_3 \left[\frac{\partial^2 w(x, y, t)}{\partial y^2} \right] &= \frac{1}{u^2} G(s, u, v) - \frac{1}{u} K_x K_t [w(x, 0, t)] - K_x K_t \left[\frac{\partial w(x, 0, t)}{\partial y} \right]. \\ \text{f) } K_3 \left[\frac{\partial^n w(x, y, t)}{\partial y^n} \right] &= \frac{1}{u^n} G(s, u, v) - \sum_{i=0}^{n-1} u^{i-n+1} K_x K_t \left[\frac{\partial^i w(x, 0, t)}{\partial y^i} \right]. \\ \text{g) } K_3 \left[\frac{\partial w(x, y, t)}{\partial t} \right] &= \frac{1}{v} G(s, u, v) - K_x K_y [w(x, y, 0)]. \\ \text{h) } K_3 \left[\frac{\partial^2 w(x, y, t)}{\partial t^2} \right] &= \frac{1}{v^2} G(s, u, v) - \frac{1}{v} K_x K_y [w(x, y, 0)] - K_x K_y \left[\frac{\partial w(x, y, 0)}{\partial t} \right]. \\ \text{i) } K_3 \left[\frac{\partial^n w(x, y, t)}{\partial t^n} \right] &= \frac{1}{v^n} G(s, u, v) - \sum_{i=0}^{n-1} v^{i-n+1} K_x K_y \left[\frac{\partial^i w(x, y, 0)}{\partial t^i} \right]. \end{aligned}$$

Proof of Theorem 4.1. From the definition of TKT, we have

$$\text{a) } K_3 \left[\frac{\partial w(x, y, t)}{\partial x} \right] = \int_0^\infty \int_0^\infty \int_0^\infty e^{-\frac{x}{s} - \frac{y}{u} - \frac{t}{v}} \left[\frac{\partial w(x, y, t)}{\partial x} \right] dx dy dt = \int_0^\infty \int_0^\infty e^{-\frac{y}{u} - \frac{t}{v}} dy dt \int_0^\infty e^{-\frac{x}{s}} \left[\frac{\partial w(x, y, t)}{\partial x} \right] dx$$

Using integrating by parts, we obtain:

$$\text{Let } u = e^{-\frac{x}{s}} \Rightarrow du = -\frac{1}{s} e^{-\frac{x}{s}} dx \text{ and } dv = \frac{\partial w(x, y, t)}{\partial x} dx \Rightarrow v = w(x, y, t). \text{ Thus}$$

$$\int_0^\infty e^{-\frac{x}{s}} \left[\frac{\partial w(x, y, t)}{\partial x} \right] dx = \left(-w(0, y, t) + \frac{1}{s} \int_0^\infty e^{-\frac{x}{s}} [w(x, y, t)] dx \right)$$

$$\therefore K_3 \left[\frac{\partial w(x, y, t)}{\partial x} \right] = \frac{1}{s} G(s, u, v) - K_y K_t [w(0, y, t)]. \quad (13)$$

$$\begin{aligned} \text{b) } K_3 \left[\frac{\partial^2 w(x, y, t)}{\partial x^2} \right] &= \int_0^\infty \int_0^\infty \int_0^\infty e^{-\frac{x}{s} - \frac{y}{u} - \frac{t}{v}} \left[\frac{\partial^2 w(x, y, t)}{\partial x^2} \right] dx dy dt \\ &= \int_0^\infty \int_0^\infty e^{-\frac{y}{u} - \frac{t}{v}} dy dt \int_0^\infty e^{-\frac{x}{s}} \left[\frac{\partial^2 w(x, y, t)}{\partial x^2} \right] dx \end{aligned}$$

Using integrating by parts, we obtain:

$$\text{Let } u = e^{-\frac{x}{s}} \Rightarrow du = -\frac{1}{s} e^{-\frac{x}{s}} dx \text{ and } dv = \frac{\partial^2 w(x, y, t)}{\partial x^2} dx \Rightarrow v = \frac{\partial w(x, y, t)}{\partial x}. \text{ Thus}$$

$$\int_0^\infty e^{-\frac{x}{s}} \left[\frac{\partial^2 w(x, y, t)}{\partial x^2} \right] dx = \left(-\frac{\partial w(0, y, t)}{\partial x} + \frac{1}{s} \int_0^\infty e^{-\frac{x}{s}} \left[\frac{\partial w(x, y, t)}{\partial x} \right] dx \right)$$

Using Eq.(13), we get

$$K_3 \left[\frac{\partial^2 w(x, y, t)}{\partial x^2} \right] = \frac{1}{s^2} G(s, u, v) - \frac{1}{s} K_y K_t [w(0, y, t)] - K_y K_t \left[\frac{\partial w(0, y, t)}{\partial x} \right]. \quad (14)$$

c) Using induction, we can easily prove **c**.

Similarly, we can prove that:

$$K_3 \left[\frac{\partial w(x, y, t)}{\partial y} \right] = \frac{1}{u} G(s, u, v) - K_x K_t [w(x, 0, t)].$$

$$K_3 \left[\frac{\partial^2 w(x, y, t)}{\partial y^2} \right] = \frac{1}{u^2} G(s, u, v) - \frac{1}{u} K_x K_t [w(x, 0, t)] - K_x K_t \left[\frac{\partial w(x, 0, t)}{\partial y} \right].$$

$$K_3 \left[\frac{\partial^n w(x, y, t)}{\partial y^n} \right] = \frac{1}{u^n} G(s, u, v) - \sum_{i=0}^{n-1} u^{i-n+1} K_x K_t \left[\frac{\partial^i w(x, 0, t)}{\partial y^i} \right].$$

$$K_3 \left[\frac{\partial w(x, y, t)}{\partial t} \right] = \frac{1}{v} G(s, u, v) - K_x K_y [w(x, y, 0)].$$

$$K_3 \left[\frac{\partial^2 w(x, y, t)}{\partial t^2} \right] = \frac{1}{v^2} G(s, u, v) - \frac{1}{v} K_x K_y [w(x, y, 0)] - K_x K_y \left[\frac{\partial w(x, y, 0)}{\partial t} \right].$$

$$K_3 \left[\frac{\partial^n w(x, y, t)}{\partial t^n} \right] = \frac{1}{v^n} G(s, u, v) - \sum_{i=0}^{n-1} v^{i-n+1} K_x K_y \left[\frac{\partial^i w(x, y, 0)}{\partial t^i} \right].$$

Theorem 4.2. (Derivative properties with respect to **x, y and t**). Let $G(s, u, v)$ is Kamal triple transform of $w(x, y, t)$, then

$$\begin{aligned} K_3 \left[\frac{\partial^3 w(x, y, t)}{\partial x \partial y \partial t} \right] &= \frac{1}{suv} G(s, u, v) - \frac{1}{su} K_x K_y [w(x, y, 0)] - \frac{1}{sv} K_x K_t [w(x, 0, t)] - \frac{1}{uv} K_y K_t [w(0, y, t)] + \frac{1}{s} K_x [w(x, 0, 0)] \\ &\quad + \frac{1}{u} K_y [w(0, y, 0)] + \frac{1}{v} K_t [w(0, 0, t)] - w(0, 0, 0) \end{aligned}$$

Proof of Theorem 4.2. From the definition of TKT, we have

$$K_x K_y K_t \left[\frac{\partial^3 w(x, y, t)}{\partial x \partial y \partial t} \right] = \int_0^\infty \int_0^\infty e^{-\frac{x}{s} - \frac{y}{u}} \left(\int_0^\infty e^{-\frac{t}{v}} \left[\frac{\partial^3 w(x, y, t)}{\partial x \partial y \partial t} \right] dt \right) dx dy.$$

Using integrating by parts

$$\text{Let } u = e^{-\frac{t}{v}} \Rightarrow du = -\frac{1}{v} e^{-\frac{t}{v}} dt \text{ and } dv = \frac{\partial^3 w(x, y, t)}{\partial x \partial y \partial t} dt \Rightarrow v = \frac{\partial^2 w(x, y, t)}{\partial x \partial y}.$$

Then, we have

$$K_x K_y K_t \left[\frac{\partial^3 w(x, y, t)}{\partial x \partial y \partial t} \right] = \int_0^\infty \int_0^\infty e^{-\frac{x}{s} - \frac{y}{u}} \left(\left(-\frac{\partial^2 w(x, y, 0)}{\partial x \partial y} + \frac{1}{v} \int_0^\infty e^{-\frac{t}{v}} \left[\frac{\partial^2 w(x, y, t)}{\partial x \partial y} \right] dt \right) \right) dx dy.$$

Therefore,

$$K_x K_y K_t \left[\frac{\partial^3 w(x, y, t)}{\partial x \partial y \partial t} \right] = -K_x K_y \left[\frac{\partial^2 w(x, y, 0)}{\partial x \partial y} \right] + \frac{1}{v} K_x K_y K_t \left[\frac{\partial^2 w(x, y, t)}{\partial x \partial y} \right]. \quad (15)$$

We take the first term:

$$-K_x K_y \left[\frac{\partial^2 w(x, y, 0)}{\partial x \partial y} \right] = - \int_0^\infty e^{-\frac{x}{s}} \left(\int_0^\infty e^{-\frac{y}{u}} \left[\frac{\partial^2 w(x, y, 0)}{\partial x \partial y} \right] dy \right) dx.$$

Using integrating by parts

Let $u = e^{-\frac{y}{u}} \Rightarrow du = -\frac{1}{u} e^{-\frac{y}{u}} dy$ and $dv = \frac{\partial^2 w(x, y, 0)}{\partial x \partial y} dy \Rightarrow v = \frac{\partial w(x, y, 0)}{\partial x}$. Then, we have

$$\begin{aligned} -K_x K_y \left[\frac{\partial^2 w(x, y, 0)}{\partial x \partial y} \right] &= - \int_0^\infty e^{-\frac{x}{s}} \left(-\frac{\partial w(x, 0, 0)}{\partial x} + \frac{1}{u} \int_0^\infty e^{-\frac{y}{u}} \left[\frac{\partial w(x, y, 0)}{\partial x} \right] dy \right) dx. \\ &= K_x \left[\frac{\partial w(x, 0, 0)}{\partial x} \right] - \frac{1}{u} K_x K_y \left[\frac{\partial w(x, y, 0)}{\partial x} \right] \end{aligned}$$

Applying single Kamal transform for $\frac{\partial w(x, 0, 0)}{\partial x}$ and double Kamal to $\frac{\partial w(x, y, 0)}{\partial x}$, we have

$$\begin{aligned} K_x \left[\frac{\partial w(x, 0, 0)}{\partial x} \right] - \frac{1}{u} K_x K_y \left[\frac{\partial w(x, y, 0)}{\partial x} \right] \\ = -w(0, 0, 0) + \frac{1}{s} L_x[w(x, 0, 0)] + \frac{1}{u} K_y[w(0, y, 0)] - \frac{1}{su} K_x K_y[w(x, y, 0)]. \end{aligned} \quad (16)$$

We take the second:

$$\frac{1}{v} K_x K_y K_t \left[\frac{\partial^2 w(x, y, t)}{\partial x \partial y} \right] = \frac{1}{v} \int_0^\infty \int_0^\infty e^{-\frac{x}{s} - \frac{t}{v}} \left(\int_0^\infty e^{-\frac{y}{u}} \left[\frac{\partial^2 w(x, y, t)}{\partial x \partial y} \right] dy \right) dx dt$$

Using integrating by parts

$$\frac{1}{v} K_x K_y K_t \left[\frac{\partial^2 w(x, y, t)}{\partial x \partial y} \right] = \frac{1}{v} \int_0^\infty \int_0^\infty e^{-\frac{x}{s} - \frac{t}{v}} \left(-\frac{\partial w(x, 0, t)}{\partial x} + \frac{1}{u} \int_0^\infty e^{-\frac{y}{u}} \left[\frac{\partial w(x, y, t)}{\partial x} \right] dy \right) dx dt$$

Therefore,

$$\frac{1}{v} K_x K_y K_t \left[\frac{\partial^2 w(x, y, t)}{\partial x \partial y} \right] = -\frac{1}{v} K_x K_t \left[\frac{\partial w(x, 0, t)}{\partial x} \right] + \frac{1}{vu} K_x K_y K_t \left[\frac{\partial w(x, y, t)}{\partial x} \right]$$

Using theorem and applying double Kamal transform to $\frac{\partial w(x, 0, t)}{\partial x}$, we have

$$\frac{1}{v} K_x K_y K_t \left[\frac{\partial^2 w(x, y, t)}{\partial x \partial y} \right] = \frac{1}{v} K_t[w(0, 0, t)] - \frac{1}{vs} K_x K_t[w(x, 0, t)] + \frac{1}{suv} G(s, u, v) - \frac{1}{vu} K_y K_t[w(0, y, t)]. \quad (17)$$

Substituting Eq. (16) and Eq. (17) into Eq.(15), we get

$$\begin{aligned} K_3 \left[\frac{\partial^3 w(x, y, t)}{\partial x \partial y \partial t} \right] &= \frac{1}{suv} G(s, u, v) - \frac{1}{su} K_x K_y[w(x, y, 0)] - \frac{1}{sv} K_x K_t[w(x, 0, t)] - \frac{1}{uv} K_y K_t[w(0, y, t)] + \frac{1}{s} K_x[w(x, 0, 0)] \\ &\quad + \frac{1}{u} K_y[w(0, y, 0)] + \frac{1}{v} K_t[w(0, 0, t)] - w(0, 0, 0). \end{aligned}$$

4.Applications

In this section, we apply the properties associated with triple Kamal transform established above to solve three-dimensional Mboctara partial differential equations.

Example 4.1

Consider the following homogeneous three-dimensional Mboctara partial differential equation

$$w_{xyt}(x, y, t) + w(x, y, t) = 0 \quad (18)$$

Subject to the boundary and initial conditions

$$\begin{cases} w(x, y, 0) = e^{x+y} & w(x, 0, 0) = e^x \\ w(x, 0, t) = e^{x-t}, & w(0, y, 0) = e^y \\ w(0, y, t) = e^{y-t} & w(0, 0, t) = e^{-t} \end{cases} \quad (19)$$

Applying TKT on both sides of Eq. (18), we have

$$K_3[w_{xyt}(x, y, t) + w(x, y, t)] = 0 \quad (20)$$

By linearity property and partial derivative properties of TKT, we get

$$\begin{aligned} & \frac{1}{suv} G(s, u, v) - \frac{1}{su} K_x K_y [w(x, y, 0)] - \frac{1}{sv} K_x K_t [w(x, 0, t)] - \frac{1}{uv} K_y K_t [w(0, y, t)] + \frac{1}{s} K_x [w(x, 0, 0)] + \frac{1}{u} K_y [w(0, y, 0)] + \\ & \frac{1}{v} K_t [w(0, 0, t)] - w(0, 0, 0) + G(s, u, v) = 0 \end{aligned} \quad (21)$$

Substituting

$$\begin{aligned} K_x K_y [w(x, y, 0)] &= \frac{su}{(1-s)(1-u)}, & K_x [w(x, 0, 0)] &= \frac{s}{1-s}, \\ K_x K_t [w(x, 0, t)] &= \frac{sv}{(1-s)(1+v)}, & K_y [w(0, y, 0)] &= \frac{u}{1-u}, \\ K_y K_t [w(0, y, t)] &= \frac{uv}{(1-u)(1+v)}, & K_t [w(0, 0, t)] &= \frac{v}{1+v}, \\ w(0, 0, 0) &= 1. \end{aligned}$$

in Eq. (21) and simplifying, we obtain

$$G(s, u, v) = \frac{suv}{(1-s)(1-u)(1+v)} \quad (22)$$

Taking inverse TKT for Eq. (22), we get

$$w(x, y, t) = K_3^{-1} [G(s, u, v)] = K_3^{-1} \left[\frac{suv}{(1-s)(1-u)(1+v)} \right] = e^{x+y-t}.$$

Example 4.2

Consider the following nonhomogeneous third-order Mboctara partial differential equation

$$w_{xyt}(x, y, t) + w(x, y, t) = 3e^{-x-2y+t} \quad (23)$$

Subject to the boundary and initial conditions

$$\begin{cases} w(x, y, 0) = e^{-x-2y} & w(x, 0, 0) = e^{-x} \\ w(x, 0, t) = e^{-x+t} & w(0, y, 0) = e^{-2y} \\ w(0, y, t) = e^{-2y+t} & w(0, 0, t) = e^t \end{cases} \quad (24)$$

Applying TKT on both sides of Eq. (23), we have

$$K_3 [w_{xyt}(x, y, t) + w(x, y, t)] = K_3 [3e^{-x-2y+t}] \quad (25)$$

By linearity property and partial derivative properties of TKT, we get

$$\begin{aligned} & \frac{1}{suv} G(s, u, v) - \frac{1}{su} K_x K_y [w(x, y, 0)] - \frac{1}{sv} K_x K_t [w(x, 0, t)] - \frac{1}{uv} K_y K_t [w(0, y, t)] + \frac{1}{s} K_x [w(x, 0, 0)] + \frac{1}{u} K_y [w(0, y, 0)] + \\ & \frac{1}{v} K_t [w(0, 0, t)] - w(0, 0, 0) + G(s, u, v) = \frac{3suv}{(1+s)(1+2u)(1-v)} \end{aligned} \quad (26)$$

Substituting

$$\begin{aligned} K_x K_y [w(x, y, 0)] &= \frac{su}{(1+s)(1+2u)}, & K_x [w(x, 0, 0)] &= \frac{s}{1+s}, \\ K_x K_t [w(x, 0, t)] &= \frac{sv}{(1+s)(1-v)}, & K_y [w(0, y, 0)] &= \frac{u}{1+2u}, \\ K_y K_t [w(0, y, t)] &= \frac{uv}{(1+2u)(1-v)}, & K_t [w(0, 0, t)] &= \frac{v}{1-v}, \\ w(0, 0, 0) &= 1 \end{aligned}$$

in Eq. (26) and simplifying, we obtain

$$G(s, u, v) = \frac{suv}{(1+s)(1+2u)(1-v)} \quad (27)$$

Taking inverse TKT for Eq. (27), we get

$$w(x, y, t) = K_3^{-1} [G(s, u, v)] = K_3^{-1} \left[\frac{suv}{(1+s)(1+2u)(1-v)} \right] = e^{-x-2y+t}.$$

Example 4.3

Consider the following nonhomogeneous third-order Mboctara partial differential equation

$$w_{xyt}(x, y, t) + w(x, y, t) = \cos x \cos y \cos t - \sin x \sin y \sin t \quad (28)$$

Subject to the boundary and initial conditions

$$\begin{cases} w(x, y, 0) = \cos x \cos y & w(x, 0, 0) = \cos x \\ w(x, 0, t) = \cos x \cos t, & w(0, y, 0) = \cos y \\ w(0, y, t) = \cos y \cos t & w(0, 0, t) = \cos t \end{cases} \quad (29)$$

Applying TKT on both sides of Eq. (28), we have

$$K_3[w_{xyt}(x, y, t) + w(x, y, t)] = K_3[\cos x \cos y \cos t - \sin x \sin y \sin t] \quad (30)$$

By linearity property and partial derivative properties of TKT, we get

$$\begin{aligned} \frac{1}{suv} G(s, u, v) - \frac{1}{su} K_x K_y[w(x, y, 0)] - \frac{1}{sv} K_x K_t[w(x, 0, t)] - \frac{1}{uv} K_y K_t[w(0, y, t)] + \frac{1}{s} K_x[w(x, 0, 0)] \\ + \frac{1}{u} K_y[w(0, y, 0)] + \frac{1}{v} K_t[w(0, 0, t)] - w(0, 0, 0) + G(s, u, v) = \frac{su v - s^2 u^2 v^2}{(1+s^2)(1+u^2)(1+v^2)} \end{aligned} \quad (31)$$

Substituting

$$\begin{aligned} K_x K_y[w(x, y, 0)] &= \frac{su}{(1+s^2)(1+u^2)}, & K_x[w(x, 0, 0)] &= \frac{s}{(1+s^2)}, \\ K_x K_t[w(x, 0, t)] &= \frac{sv}{(1+s^2)(1+v^2)}, & K_y[w(0, y, 0)] &= \frac{u}{(1+u^2)}, \\ K_y K_t[w(0, y, t)] &= \frac{uv}{(1+u^2)(1+v^2)}, & K_t[w(0, 0, t)] &= \frac{v}{(1+v^2)}, \\ w(0, 0, 0) &= 1. \end{aligned}$$

in Eq. (31) and simplifying, we obtain

$$G(s, u, v) = \frac{su v}{(1+s^2)(1+u^2)(1+v^2)} \quad (32)$$

Taking inverse TKT for Eq. (32), we get

$$w(x, y, t) = K_3^{-1}[G(s, u, v)] = K_3^{-1}\left[\frac{su v}{(1+s^2)(1+u^2)(1+v^2)}\right] = \cos x \cos y \cos t.$$

5. Conclusion

Taking clue from work in single integral transforms in one - dimensional spaces and double integral transform in tow - dimensional spaces, we introduce a novel transform known as triple Kamal integral transform. This new transform refines and implies the original triple Laplace transform in positive quadrant planes. We proved some main properties related to suggested triple Kamal transform such as triple convolution theorem and other properties. To evaluate the effectiveness of this transform, we apply to solve a range of PDEs subject to standard conditions. It is recommended that researchers under take studies concerning the applicability of using this transform to solve integral deferential equations, nonlinear PDEs and fractional deferential equations.

Acknowledgments

I acknowledge my colleagues Holy Quran and Islamic Sciences University for their technical advice. I am also grateful to my student Muneerah Al-Ghamdi for her support, both morally and financially.

References

1. Ahmad.L ;Khan .M. Numerical simulation for MHD flow of Sisko nanofluid over a moving curved surface: A revised model. *Microsystem Technologies*.2019, 25 (6), 2411 – 2428 .
2. Constanda, C. *Solution Techniques for Elementary Partial Differential Equations*; Chapman and Hall/CRC: New York, NY, USA, 2002.
3. Debnath,L.The double Laplace transforms and their properties with applications to functional, integral and partial differential equations. *Int. J. Appl. Comput. Math.* 2016, 2, 223–241.
4. Muatjetjeja, B. Group classification and conservation laws of the generalized Klein–Gordon–Fock equation. *Int. J. Mod. Phys. B* 2016, 30, 1640023.
5. A.M.Wazwaz, *Partial Differential Equations and Solitary Waves Theory*. Springer Dordrecht Heidelberg .London New York , 2009.
6. Eddine, N.C.; Ragusa, M.A. Generalized critical Kirchhoff-type potential systems with Neumann boundary conditions. *Appl. Anal.* 2022. <https://doi.org/10.1080/00036811.2022.2057305>.
7. Rashid, S.; Ashraf, R.; Bonyah,E.On analytical solution of time-fractional biological population model by means of generalized integral transform with their uniqueness and convergence analysis. *J. Funct. Spaces* 2022, 2022, 7021288.
8. Zid, S.; Menkad, S. The lambda-Aluthge transform and its applications to some classes of operators. *Filomat* 2022, 36, 289.
9. Debnath, L. *Nonlinear Partial Differential Equations for Scientists and Engineers*; Birkh User: Boston, MA, USA, 1997.
10. Qazza, A.; Hatamleh, R.; Alodat, N. About the solution stability of Volterra integral equation with random kernel. *Far East J. Math. Sci.* 2016, 100, 671–680.
11. Gharib, G.; Saadeh, R. Reduction of the self-dual yang-mills equations to sinh-poisson equation and exact solutions. *WSEAS Interact. Math.* 2021, 20, 540–554.
12. Widder, V. *The Laplace Transform*; PrincetonUniversity Press: Princeton, NJ, USA, 1941.
13. Bochner, S.; Chandrasekharan, K. *Fourier Transforms*; Princeton University Press: London, UK, 1949.

14. Atangana, A.; Kiliçman, A. A novel integral operator transform and its application to some FODE and FPDE with some kind of singularities. *Math. Probl. Eng.* 2013, 2013, 531984.
15. Srivastava, H.; Luo, M.; Raina, R. A new integral transform and its applications. *Acta Math. Sci.* 2015, 35B, 1386–1400.
16. Watugula, G.K. Sumudu transform: A new integral transform to solve differential equations and control engineering problems. *Int. J. Math. Educ. Sci. Technol.* 1993, 24, 35–43.
17. Khan, Z.H.; Khan, W.A. Natural transform-properties and applications. *NUST J. Eng. Sci.* 2008, 1, 127–133.
18. .Elzaki, T.M. The new integral transform “Elzaki transform”. *Glob. J. Pure Appl. Math.* 2011, 7, 57–64.
19. Saadeh, R.; Qazza, A.; Burqan, A. A new integral transform: ARA transform and its properties and applications. *Symmetry* 2020, 12, 925.
20. Zafar, Z. ZZ transform method. *Int. J. Adv. Eng. Glob. Technol.* 2016, 4, 1605–1611.
21. Mahgoub MMA (2019) The new integral transform “Sawi Transform”, *Advances in Theoretical and Applied Mathematics*, 14(1):81–87.
22. Sedeeg, A .H. The New Integral Transform “ Kamal Transform ” , *Advances in Theoretical and Applied Mathematics* , 2016, 11, 451-458.
23. Abdelilah Kamal .H. Sedeeg and Zahra.I. Adam Mahamoud , The Use of Kamal Transform for Solving Partial Differential Equations , *Advances in Theoretical and Applied Mathematics* , Vol.12, No.1, 2017, pp.7-13.
24. Abdelilah Kamal .H. Sedeeg and Zahra.I. Adam Mahamoud, Application of New Transform " Kamal Transform" to Ordinary Differential Equations with Non-Constants Coefficients .*ElixirInternational Journal ,Applied Mathematics*.103 (2017) pp .45727-45730 .
25. Abdelilah Kamal .H. Sedeeg and Zahra.I. Adam Mahamoud , Kamal Transform Integrated with Adomian Decomposition Method for Solving Predator-Prey Systems. *International Journal of Current Research* Vol. 9, Issue, 02, February, 2017, pp.46080-46086
26. Aboodh, K.; Abdullahi, I.; Nuruddeen, R. On the Aboodh transform connections with some famous integral transforms. *Int. J. Eng. Inform. Syst.* 2017, 1, 143–151.
27. Thakur, A. K .and Panda, S. “Some Properties of Triple Laplace Transform”, *Journal of Mathematics and Computer Applications Research (JMCAR)*, 2250-2408, 2015.
28. Atangana, A. A note on the Triple Laplace Transform and Its Applications to Some Kind of Third-Order Differential Equation. *Abstract and Applied Analysis*, Vol. 2013, Article ID 769102, Pages 1-10.
29. A.Khan, T.Khan, G.Zaman, Extension of triple Laplace transform for solving fractional differential equations, *J.Discrete Contin. Dyn. Syst.* 13 (3) 755–768 (2020).
30. Mechee, M. S. and Naemah, A. J. (2020). A Study of Triple Sumudu Transform for Solving Partial Differential Equations with Some Applications. *Multidisciplinary European Academic Journal*, 2020, Vol.2, No.2.
31. Elzaki, T., Adil. M. On the convergence of triple Elzaki transform, *SN Applied Sciences*, 2019 1:275. doi:10.1007/s42452-019-0257-2.
32. Alfaqeh, S. and Ozis, T. Note on Triple Aboodh Transform and Its Application. *International Journal of Engineering and Information Systems (IJEAIS)*, ISSN: 2000-000X, Vol. 3, Issue 3, March 2019, Pages: 41-50.
33. Alkaleeli, S., Mtawal, A., and Hmad, M. (2021). Triple Shehu transform and its properties with applications. *African Journal of Mathematics and Computer*, 2021, 14(1), 4-12.
34. Sinha, A., Kumar Three dimensional natural transform and its applications. *National Conference on Advanced Materials and Applications*, 2019. doi:10.1088/1757-899X/798/1/012037