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The Study of Evolution of Linearized Perturbations in a Thermally Stratified Magnetohydrodynamic Bounded Couette Flow

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ABSTRACT

Using Fourier transforms, the evolution of linearized perturbations in a thermally stratified magnetohydrodynamic shear flow is solved as an initial value problem. The resulting equation in terms of the Fourier amplitudes is solved for the case of bounded couette flow with a point source of the field of transverse velocity, density and temperature. Solutions are obtained for small values of Alfve′n velocity and Brunt Vaisala frequency. The velocity plots are drawn for different values of Alfve′n velocity and Brunt frequency.

1. Introduction

The stability of electrically conducting shear flows with thermal stratification is of importance to geophysicists and astrophysicists. Magnetic field sometimes exerts constraints which prevent or inhibit certain types of motion and can be stabilizing. As a result, flow in the presence of magnetic field may remain laminar even at high Reynolds number. In addition to magnetic field if destabilizing agent such as shear is present then the result may be amplification of wavelike motions and thermal stratification also act as destabilizing agent.

Many researchers have considered the effect of shear on the stability of conducting fluid. Lerner and. Knobloch [3] using the method of separation of variables studied the stability of dissipative magnetohydrodynamic shear flow in a parallel magnetic field for unbounded plane Couette flow and found that the finite conductivity and molecular viscosity were stabilising. Vijayalakshmi and Balagondar [6] studied the evolution of general three-dimensional perturbations in a thermally stratified couette flow and found graphically that the behaviour of the total energy and the sum of first five components of energy are qualitatively similar for different values of Brunt Vaisala frequency. Vijayalakshmi and Balagondar [7] studied the evolution of general three-dimensional perturbations in a magnetohydrodynamiccouette flow as an initial value problem and found that the behaviour of the total energy and the sum of first five components of energy which are qualitatively similar for different values of Alfven velocity.

Venkatachalappa and Soward [5] have shown that the addition of small diffusivity, dissipation is strongly stabilising and causes eventual collapse of all the modes. Damien Biau and France Alessandro Bottaro [2] studied the effect of buoyancy on shear flow stability with a positive thermal gradient. A linear stability analysis was carried out, using Normal mode analysis focusing on both exponential and transient growth. In both cases, positive thermal stratification was found to stabilize the disturbances**.** Martin Withalm and Hoffmann [4]studied the influence of thermal stratification on the stability of Ekman-Couette-flow and found stable stratification is suppressing the emergence of stationary as well as shear-instabilities, while unstable stratification is supporting them**.**

In the present paper, we have extended the work of Criminale andDrazin[1] for the case of thermally stratifiedmagnetohydrodynamic bounded couette flow with unit pulse of velocity, magnetic field and temperature as initial conditions. The complete general solution to the linearized equations of motion are obtained as function of all space variables and time. The disturbances are resolved into rotational and irrotational components. The rotational solution is the solution for the hypothetical initial-value problem for which the mean flow is unbounded but coincides with the actual flow in the layer. The irrotational solution in each layer is specified uniquely by satisfying the interfacial and boundary conditions.

2. Mathematical formulation

We consider an electrically conducting fluid of density ρ, moving with velocity $\vec{q}$ in the presence of a magnetic field $\vec{H}$ under the influence of gravity $\vec{g}$. Small density changes are caused by variations in the temperature T. We assume that the fluid is Boussinesq, for which motion is governed by the equations

$$\nabla.\mathbf{q} = 0, \qquad (2.1)$$

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$$\nabla.\mathbf{H} = 0, \tag{2.2}$$

$$\rho\left(\frac{\partial \mathbf{q}}{\partial t}+(\mathbf{q}.\nabla)\mathbf{q}\right) = -\nabla P+\rho\mathbf{g}+\mu_m(\mathbf{H}.\nabla)\ \mathbf{H}, \tag{2.3}$$

$$\frac{\partial \mathbf{H}}{\partial t}+(\mathbf{q}.\nabla)\mathbf{H} = (\mathbf{H}.\nabla)\mathbf{q}, \tag{2.4}$$

$$\frac{\partial T}{\partial t}+(\mathbf{q}.\nabla)T = 0, \tag{2.5}$$

$$\rho=\rho_e\left[1-\alpha_1\left(T-T_e\right)\right], \tag{2.6}$$

where $P = p+\frac{\mu_m H^2}{2}$ is the total pressure, $\boldsymbol{\alpha_1}$ is the coefficient of expansion, $\boldsymbol{\mu_m}$ is the magnetic permeability. The constant density ρ_e corresponds to some reference temperature T_e.

In the linear stability theory we superimpose a small wave like perturbation upon the mean flow i,e.,

$$\mathbf{q} = \mathbf{q}_0+\mathbf{q}', \mathbf{H}=\mathbf{H}'_0+\mathbf{H}', P=P_0+P', T=T_0+T' \tag{2.7}$$

Where

$$\mathbf{q}_0 = (U(y)=\sigma y,0,0), \mathbf{H}_0=(H_0,0,0), P = P_0(y), T_0=-\beta_1\ y \tag{2.8}$$

are the ambient velocity, magnetic field, pressure and temperature respectively. The shear σ, magnetic field H_0 and temperature gradient β_1 are all constants. $\mathbf{q}', \mathbf{H}', P', T'=-\beta_1\theta'$ are the perturbed quantities of velocity, By (i) employing moving co-ordinates transformation,

$$T = t,\ \xi = x-\sigma y\, t,\ \eta=y,\ \zeta=z \tag{2.9}$$

(ii) Using three – dimensional Fourier transformation given by

$$\hat{u}(\alpha;\beta;\gamma;T) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u(\xi;\eta;\zeta;T)\ e^{i(\alpha\xi+\beta\eta+\gamma\zeta)}\, d\xi\, d\eta\, d\zeta \tag{2.10}$$

with similar expressions for $\hat{v}, \hat{w}$, $\hat{H}_x, \hat{H}_y, \hat{H}_z$ and $\hat{P}$ and (iii) changing the quantities in the αγ plane (i,e.,ξζ plane in real space) to polar variables ($\bar{\alpha}$, φ) using Squire transformation by defining

$$\bar{H}_x=\frac{\alpha\hat{H}_x+\gamma\hat{H}_z}{\bar{\alpha}}, \bar{H}_z=\frac{-\gamma\hat{H}_x+\alpha\hat{H}_z}{\bar{\alpha}}\ \bar{u}=\frac{\alpha\hat{u}+\gamma\hat{w}}{\bar{\alpha}}, \bar{w}=\frac{-\gamma\hat{u}+\alpha\hat{w}}{\bar{\alpha}}, \tag{2.11}$$

andby eliminating $\hat{P}$ the linearized equations of motion with Boussinesq approximation and omitting the primes reduces to

$$\frac{d}{dT}\left(K^2\hat{v}\right)-N^2\bar{\alpha}^2\hat{\theta}+i\alpha V_A^2\ K^2\hat{H}_y=0, \tag{2.12}$$

$$\frac{d\hat{H}_y}{dT} = -i\alpha\hat{v}, \tag{2.13}$$

$$\frac{d\hat{\theta}}{dT} = \beta_1\hat{v}\ . \tag{2.14}$$

where V_A is the Alfve'n velocity, N is the Brunt Vaisala frequency, ρ_0 is the equilibrium density. $V_A^2=\frac{\mu_m H_0^2}{\rho_0}$,

$$N^2=-\frac{g}{\rho_0}\frac{d\rho_0}{dy}=-\alpha_1\beta_1 g,\ \bar{\alpha}^2=\left(\alpha^2+\gamma^2\right),\ K^2=\bar{\alpha}^2+(\beta-\sigma\alpha T)^2.$$

Once equations (2.12) - (2.14) are solved with appropriate initial conditions for $\hat{v}$, $\hat{\theta}$ and $\hat{H}_y$, other velocity and magnetic field components $\hat{u}$, $\hat{w}$, $\hat{H}_x$ and $\hat{H}_z$ can be obtained by inverting the relations (2.11). The pressure amplitude $\hat{P}$ is obtained by taking the divergence of the momentum equations and is found to be

$$\hat{P} = \frac{-i}{K^2}\left(2\sigma\alpha\,\hat{v} + N^2\left((\beta - \sigma\alpha T)\hat{\theta}\right)\right) \text{if } K^2 \neq 0. \tag{2.15}$$

Two sets of solutions exist for equation (2.12) for $\hat{v}$. First, for $K^2 \neq 0$, the disturbance is rotational. Second, for $K^2 = 0$, corresponding to irrotational disturbances, since $K^2\hat{v} = 0$ corresponds to $\nabla^2\hat{v} = 0$ in real space. But for $\hat{H}_y$ only one solution exists for $K^2 \neq 0$, since for $K^2 = 0$, i,e., $K^2\hat{H}_y = 0$ corresponds to $\nabla^2\hat{H}_y = 0$ which implies that $\hat{H}_y$ is force free magnetic field i,e., there is no magnetic field. Hence $\nabla^2\hat{H}_y = 0$ which corresponds to irrotational solution is not taken into consideration.

Now considering the case $K^2 \neq 0$, we assume

$$\hat{v}_R(\alpha,\beta,\gamma,T) = \hat{v}_0(\alpha,\beta,\gamma,T) + N^2\hat{v}_1(\alpha,\beta,\gamma,T) + V_A^2\,\hat{v}_2(\alpha,\beta,\gamma,T) + \left(N^2\right)^2\hat{v}_3(\alpha,\beta,\gamma,T) + \left(V_A^2\right)^2\hat{v}_4(\alpha,\beta,\gamma,T) + N^2V_A^2\,\hat{v}_5(\alpha,\beta,\gamma,T) + \ldots \tag{2.16}$$

with similar expressions for $\hat{H}_y(\alpha,\beta,\gamma,T)$ and $\hat{\theta}(\alpha,\beta,\gamma,T)$.
At the zeroth, first and second order we have,

$$\hat{v}_0 = \frac{\hat{\Omega}_0(\alpha,\beta,\gamma)}{\bar{\alpha}^2 + (\beta - \sigma\alpha T)^2}, \tag{2.17}$$

$$\hat{\theta}_0 = \hat{\Omega}_1(\alpha,\beta,\gamma), \tag{2.18}$$

$$\hat{H}_{y_0} = \frac{i}{\sigma\bar{\alpha}}\hat{\Omega}_0(\alpha,\beta,\gamma)\tan^{-1}\left(\frac{\beta - \sigma\alpha T}{\bar{\alpha}}\right) + \hat{\Omega}_2(\alpha,\beta,\gamma) \tag{2.19}$$

$$\hat{v}_1 = \frac{1}{\bar{\alpha}^2 + (\beta - \sigma\alpha T)^2}\left(\bar{\alpha}^2 T\left(\hat{\Omega}_0 + \hat{\Omega}_2\right) - \frac{\hat{\Omega}_2}{3\sigma\alpha}\left(\frac{\beta - \sigma\alpha T}{\bar{\alpha}}\right)^3\right) \tag{2.20}$$

$$\hat{\theta}_1 = \frac{\hat{\Omega}_0}{\sigma\alpha\bar{\alpha}\alpha_1 g}\tan^{-1}\left(\frac{\beta - \sigma\alpha T}{\bar{\alpha}}\right), \tag{2.21}$$

$$\hat{H}_{y_1} = \frac{i\bar{\alpha}}{\sigma^2\alpha}\left(\hat{\Omega}_0 + \hat{\Omega}_2\right)\left[\beta\tan^{-1}\left(\frac{\beta - \sigma\alpha T}{\bar{\alpha}}\right) - \bar{\alpha}\log\left(\frac{\bar{\alpha}^2 + (\beta - \sigma\alpha T)^2}{\bar{\alpha}^2}\right)\right] - \hat{\Omega}_2\left(\frac{\beta - \sigma\alpha T}{\bar{\alpha}}\right)^2 \tag{2.22}$$

$$\hat{v}_2 = \frac{1}{\bar{\alpha}^2 + (\beta - \sigma\alpha T)^2}\left\{-\frac{\hat{\Omega}_0\bar{\alpha}^3}{\sigma}\left[\left(\frac{\beta - \sigma\alpha T}{\bar{\alpha}}\right)\tan^{-1}\left(\frac{\beta - \sigma\alpha T}{\bar{\alpha}}\right) + \frac{1}{3}\left(\frac{\beta - \sigma\alpha T}{\bar{\alpha}}\right)^3\tan^{-1}\left(\frac{\beta - \sigma\alpha T}{\bar{\alpha}}\right) - \frac{1}{3}\log\left(\frac{\bar{\alpha}^2 + (\beta - \sigma\alpha T)^2}{\bar{\alpha}^2}\right) - \frac{1}{6}\left(\tan^{-1}\left(\frac{\beta - \sigma\alpha T}{\bar{\alpha}}\right)\right)^2\right] + \frac{i\bar{\alpha}^3\hat{\Omega}_2}{\sigma}\left(\left(\frac{\beta - \sigma\alpha T}{\bar{\alpha}}\right) + \frac{1}{3}\left(\frac{\beta - \sigma\alpha T}{\bar{\alpha}}\right)^3\right)\right\} \tag{2.23}$$

$$\hat{\theta}_2 = 0, \tag{2.24}$$

$$\hat{H}_{y_2} = -\frac{i\hat{\Omega}_0\bar{\alpha}^2}{\sigma^2\alpha}\left\{\frac{1}{2}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)^2\tan^{-1}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)-\frac{2}{3}\left(\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)-\tan^{-1}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)\right)\right.$$
$$\left.-\frac{2}{3}\tan^{-1}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)\cos\left(\tan^{-1}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)\right)-\sin\left(\tan^{-1}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)\right)\right\}\Bigg]+\frac{\alpha\bar{\alpha}^2\hat{\Omega}_2}{\sigma}$$
$$\left[\frac{2}{3}\log\left(\frac{\bar{\alpha}^2+(\beta-\sigma\alpha T)^2}{\bar{\alpha}^2}\right)+\frac{1}{6}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)^2\right] \tag{2.25}$$

The solution for $K^2 = 0$ is found by considering the perturbation equations where two – dimensional Fourier transform is used instead of the full three – dimensional decomposition. Using moving co-ordinate transformation (2.9), $K^2\hat{v}=0$ corresponds to

$$\frac{\partial^2\breve{v}_I}{\partial\eta^2}+2i\sigma\alpha T\frac{\partial\breve{v}_I}{\partial\eta}-\left(\bar{\alpha}^2+\sigma^2\alpha^2T^2\right)\breve{v}_I = 0, \tag{2.26}$$

$$\breve{v}_I = \breve{v}_I(\alpha,\eta,\gamma;T) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} v_I(\xi,\eta,\varsigma,T)\, e^{i(\alpha\xi+\gamma\zeta)}\, d\xi\, d\zeta, \tag{2.27}$$

is the irrotational part of v. The solution of equation (2.26) is found to be

$$\breve{v}_I = A(T)\,e^{-\bar{\alpha}\eta-i\sigma\alpha T\eta} + B(T)\,e^{\bar{\alpha}\eta-i\sigma\alpha T\eta} \tag{2.28}$$

where A(T) and B(T) are constants of integration.

In order to combine $\hat{v}_R$ and $\breve{v}_I$ to obtain the complete solution and satisfy the matching condition $\hat{v}_R$ must be inverted once to obtain $\breve{v}_R(\alpha,\eta,\gamma;T)$ i,e.,

$$\breve{v}_R(\alpha,\eta,\gamma;T) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\hat{v}_R(\alpha,\beta,\gamma;T)\, e^{i\beta\eta}\, d\beta \tag{2.29}$$

With initial velocity, initial magnetic field and initial temperature given by

$$v(x,y,z,0) = V_0\,\delta(x-x_0)\,\delta(y-y_0)\,\delta(z-z_0) \tag{2.30}$$

$$\theta(x,y,z,0) = \tilde{\theta}_0\,\delta(x-x_0)\delta(y-y_0)\delta(z-z_0) \tag{2.31}$$

$$H_y(x,y,z,0) = H_0\,\delta(x-x_0)\delta(y-y_0)\delta(z-z_0). \tag{2.32}$$

In terms of moving co-ordinates and three-dimensional Fourier transform is

$$\breve{v}_0(\alpha,\beta,\gamma) = \Omega_0(\alpha,\beta,\gamma) = V_0 e^{i(\alpha x_0+\beta y_0+\gamma z_0)} \tag{2.33}$$

$$\tilde{\theta}_0(\alpha,\beta,\gamma) = \Omega_1(\alpha,\beta,\gamma) = \tilde{\theta}_0 e^{i(\alpha x_0+\beta y_0+\gamma z_0)}, \tag{2.34}$$

$$\breve{H}_{y_0}(\alpha,\beta,\gamma) = \Omega_2(\alpha,\beta,\gamma) = \tilde{H}_0 e^{i(\alpha x_0+\beta y_0+\gamma z_0)}. \tag{2.35}$$

$\breve{v}_R$ is found to be

$$\breve{v}_R = e^{i\left(\alpha x_0+\gamma z_0-\sigma\alpha T\bar{\eta}\right)}\Bigg\{\left(V_0+N^2\bar{\alpha}^2 T\left(\tilde{\theta}_0+\tilde{H}_0\right)\right)e^{-\bar{\alpha}|\bar{\eta}|}+\left(\frac{2N^4\bar{\alpha}^2 V_0 V_A^2}{\sigma^2\alpha_1\alpha^2 g}+\frac{14V_0V_A^4}{45\sigma^3\alpha}-\right.$$

$$\left.\frac{2i\alpha\bar{\alpha}^3V_A^4\tilde{\theta}_0}{3\sigma}-\frac{\bar{\alpha}^4N^2V_A^4\tilde{H}_0}{\sigma^3\alpha}\left(\frac{2}{3}+\frac{\bar{\alpha}}{2}\right)\right)\left(\int_{-\infty}^{\infty}\frac{\eta' e^{-\bar{\alpha}|\bar{\eta}-\eta'|-\bar{\alpha}|\bar{\eta}|}}{(\bar{\eta}-\eta')}d\eta'\right)+\left(V_A^4\left(\frac{V_0\bar{\alpha}^3}{3\sigma\alpha}-\frac{7V_0}{45\sigma^3\alpha}\right.\right.$$

$$\left.\left.+\frac{i\alpha\bar{\alpha}^3\tilde{\theta}_0}{36\sigma}\right)+\frac{N^4\bar{\alpha}^2V_0}{2\sigma^2\alpha_1\alpha^2 g}+\frac{3N^2V_A^2\bar{\alpha}^4V_0}{2\sigma^3\alpha}\right)\left(\int_{-\infty}^{\infty}\frac{e^{-\bar{\alpha}|\bar{\eta}-\eta'|-\bar{\alpha}|\bar{\eta}|}}{\eta'}d\eta'\right)\left(\frac{V_A^2\bar{\alpha}^3V_0}{6\sigma\alpha}+V_A^4\left(\frac{43V_0}{90}+\frac{i\alpha\bar{\alpha}^3\tilde{\theta}_0}{36\sigma}\right)\right.$$

$$\left.-\frac{N^2V_A^2\bar{\alpha}^4\tilde{\theta}_0}{6\sigma^3\alpha}\right)e^{-\bar{\alpha}|\bar{\eta}|}+\left(\frac{2V_A^2 i\bar{\alpha}\tilde{H}_0}{3\sigma}+V_A^4\left(\frac{V_0}{20\sigma^3\alpha}+\frac{32i\alpha\bar{\alpha}^3\tilde{\theta}_0}{270\sigma^2}\right)-\frac{N^2V_A^2\bar{\alpha}^4\tilde{H}_0}{6\sigma^3\alpha}\right)\left(\frac{-i\bar{\eta}e^{-\bar{\alpha}|\bar{\eta}|}}{2}\right)+$$

$$\left(-\frac{5iV_A^4\alpha\bar{\alpha}^3\tilde{\theta}_0}{6\sigma}+\frac{N^2V_A^2\bar{\alpha}^5\tilde{\theta}_0}{2\sigma^3\alpha}\right)\left(\int_{-\infty}^{\infty}-i\eta'\frac{e^{-\bar{\alpha}|\bar{\eta}-\eta'|-\bar{\alpha}|\eta'|}}{2(\bar{\eta}-\eta')}d\eta'\right)\Bigg\}. \qquad (2.36)$$

Then the total solution will be

$$\breve{v} = \breve{v}_R + \breve{v}_I \qquad (2.37)$$

3. Thermally Stratified Magnetohydrodynamic Bounded Plane Couette Flow

In this case, a plane magnetohydrodynamicCouette flow with thermal stratification which is bounded at $y = \pm H$ is considered (Fig.1). Here velocity $\breve{v}$ vanishes at $\eta = \pm H$, hence we have

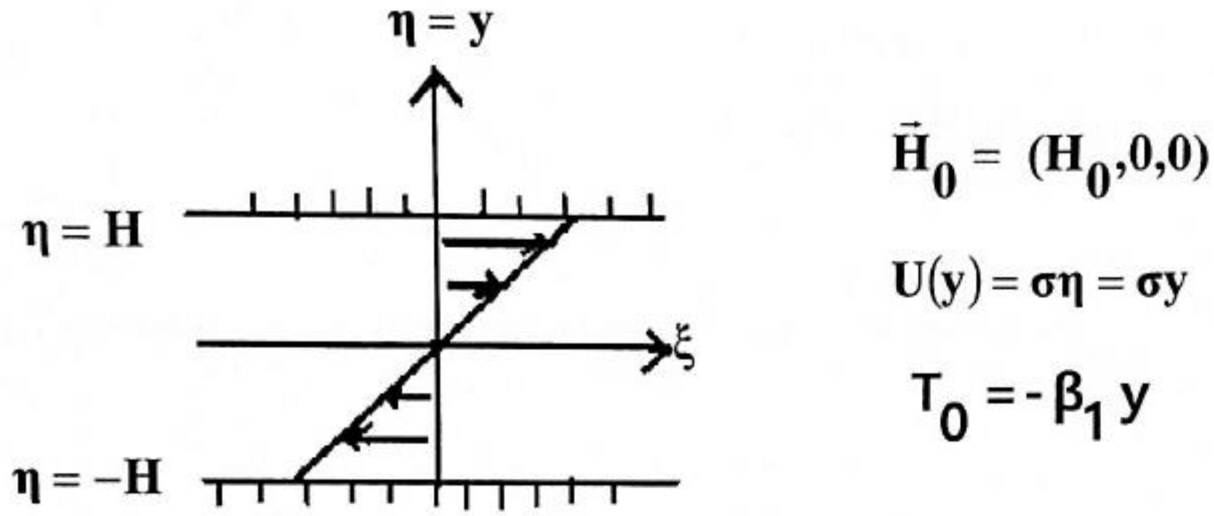

Fig 1.Sketch of bounded Magnetohydrodynamic Bounded Couette Flow with Thermal Stratification

$$e^{-\bar{\alpha}H-i\sigma\alpha TH}A+e^{\bar{\alpha}H-i\sigma\alpha TH}B=-\left[\breve{v}_R\right]_{\eta=+H} \qquad (3.1)$$

$$e^{\bar{\alpha}H+i\sigma\alpha TH}A+e^{-\bar{\alpha}H+i\sigma\alpha TH}B=-\left[\breve{v}_R\right]_{\eta=-H} \qquad (3.2)$$

From equations (3.1) and (3.2), A and B are found to be are found to be

$$A=\frac{1}{2\sinh(2\bar{\alpha}H)}\left[\breve{v}_R(+H)e^{-\bar{\alpha}H+i\sigma\alpha TH}-\breve{v}_R(+H)e^{\bar{\alpha}H-i\sigma\alpha TH}\right] \qquad (3.3)$$

$$B=\frac{1}{2\sinh\left(2\bar{\alpha}H\right)}\left[\breve{v}_R\left(-H\right)e^{-\bar{\alpha}H-i\sigma\alpha TH}-\breve{v}_R\left(-H\right)e^{\bar{\alpha}H+i\sigma\alpha TH}\right] \quad (3.4)$$

where $\breve{v}_R\left(\pm H\right)=-\left\lfloor\breve{v}_R\right\rfloor_{\eta=\pm H}$ It is found that

$$\breve{v}_R\left(+H\right)=\left(A_1T+B_1\right)e^{i\left(\alpha x_0+\gamma z_0-\sigma\alpha T\left(H-y_0\right)\right)}. \quad (3.5)$$

$$\breve{v}_R\left(-H\right)=\left(A_2T+B_2\right)e^{i\left(\alpha x_0+\gamma z_0-\sigma\alpha T\left(H-y_0\right)\right)} \quad (3.6)$$

where

$$A_1=N^2\bar{\alpha}^2\left(\tilde{\theta}_0+\tilde{H}_0\right)e^{-\bar{\alpha}\left|H-y_0\right|} \quad (3.7)$$

$$B_1=V_0e^{-\bar{\alpha}\left|H-y_0\right|}+\left(\frac{2N^4\bar{\alpha}^2V_0V_A^2}{\sigma^2\alpha_1\alpha^2g}+-\frac{2i\alpha\bar{\alpha}^3V_A^4\tilde{\theta}_0}{3\sigma}-\frac{\bar{\alpha}^4N^2V_A^4\tilde{H}_0}{\sigma^3\alpha}\left(\frac{2}{3}+\frac{\bar{\alpha}}{2}\right)+\frac{14V_0V_A^4}{45\sigma^3\alpha}\right.$$

$$\int_{-\infty}^{\infty}\frac{\eta' e^{-\bar{\alpha}\left|\eta-\eta'\right|-\bar{\alpha}\left|\eta\right|}}{\left(\eta-\eta'\right)}d\eta'+\left(V_A^4\left(\frac{V_0\bar{\alpha}^3}{3\sigma\alpha}-\frac{7V_0}{45\sigma^3\alpha}+\frac{i\alpha\bar{\alpha}^3\tilde{\theta}_0}{36\sigma}\right)+\frac{N^4\bar{\alpha}^2V_0}{2\sigma^2\alpha_1\alpha^2g}+\frac{3N^2V_A^2\bar{\alpha}^4V_0}{2\sigma^3\alpha}\right)$$

$$\left(\int_{-\infty}^{\infty}\frac{e^{-\bar{\alpha}\left|\eta-\eta'\right|-\bar{\alpha}\left|\eta\right|}}{\eta'}d\eta'\right)-\left(\frac{V_A^2\bar{\alpha}^3V_0}{6\sigma\alpha}+V_A^4\left(\frac{43V_0}{90}+\frac{i\alpha\bar{\alpha}^3\tilde{\theta}_0}{36\sigma}\right)\right)-\left(\frac{V_A^2\bar{\alpha}^3V_0}{6\sigma\alpha}+V_A^4\left(\frac{43V_0}{90}+\frac{i\alpha\bar{\alpha}^3\tilde{\theta}_0}{36\sigma}\right)\right)$$

$$-\left(\frac{V_A^2\bar{\alpha}^3V_0}{6\sigma\alpha}+V_A^4\left(\frac{43V_0}{90}+\frac{i\alpha\bar{\alpha}^3\tilde{\theta}_0}{36\sigma}\right)-\frac{N^2V_A^2\bar{\alpha}^4\tilde{\theta}_0}{6\sigma^3\alpha}\right)e^{-\bar{\alpha}\left|H-y_0\right|}-i\left(\frac{5iV_A^4\alpha\bar{\alpha}^3\tilde{\theta}_0}{6\sigma}-\frac{N^2V_A^2\bar{\alpha}^5\tilde{\theta}_0}{2\sigma^3\alpha}\right)$$

$$\left.\int_{-\infty}^{\infty}\eta'\frac{e^{-\bar{\alpha}\left|\eta-\eta'\right|-\bar{\alpha}\left|\eta'\right|}}{2\left(H-y_0-\eta'\right)}d\eta'\right\} \quad (3.8)$$

By replacing H by –H in A_1 and B_1 we obtain A_2 and B_2.

4. Results and discussion

In this problem, we have studied the lnearized perturbations of a basic flow of an inviscidmagnetohydrodynamic bounded couette flow with thermal stratification using piecewise linear velocity profiles. We have used unit pulse of velocity, magnetic field and temperature as initial distributions.

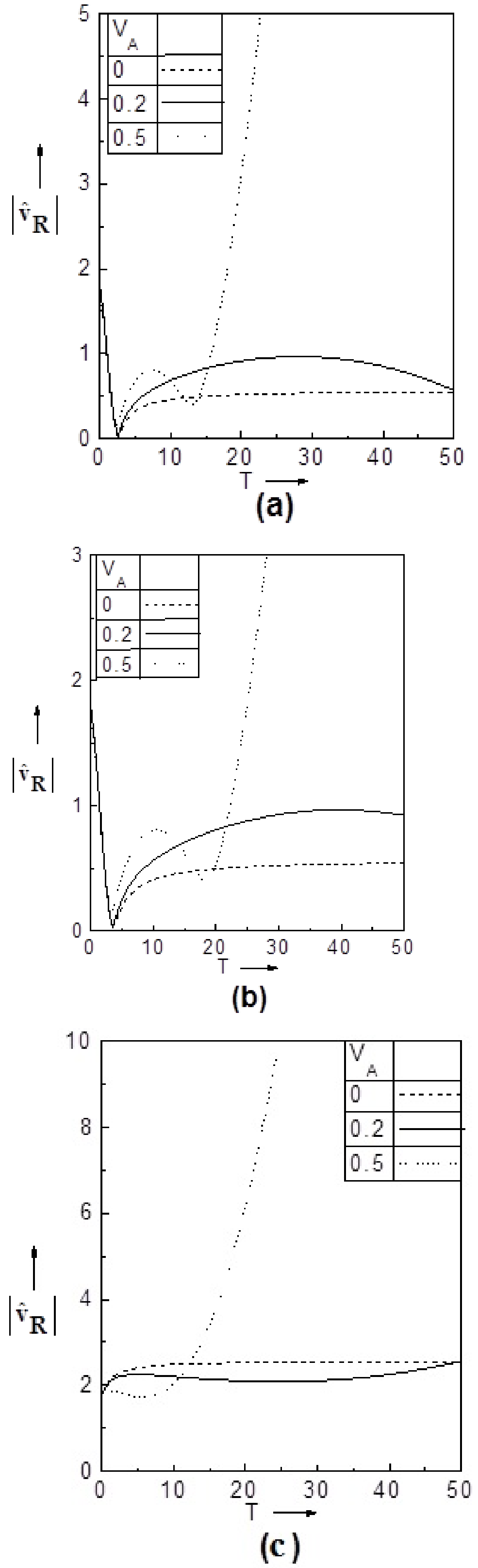

Fig 2. Curves of $|\hat{v}_R|$ versus T for (a) $\varphi = 0^o$, (b) $\varphi = 45^o$ and (c) $\varphi = 180^o$ for different values of V_A and N= 0

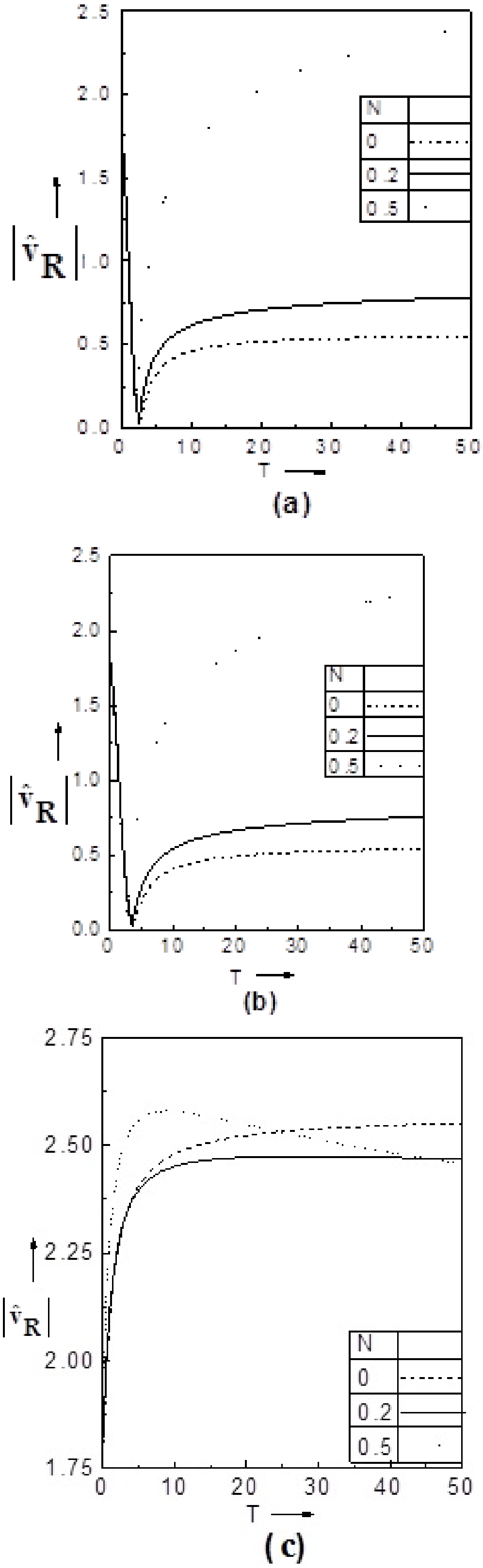

Fig 3.Curves of $|\hat{v}_R|$ versus T for (a) $\varphi = 0^o$, (b) $\varphi = 45^o$ and (c) $\varphi = 180^o$ for different values of N and $V_A = 0$.

We have resolved the perturbations into rotational and irrotational components. Plots are drawn to observe the variation of amplitude of rotational velocity $|\hat{v}_R|$ with time.

Figs. (2) and (3) are plots of $\left|\hat{v}_R\right|$ Vs T. These plots are drawn for different values of N and V_A (N $= V_A$ = 0, 0.2, 0.5) and for different values of φ ($\varphi = 0^0, 45^0, 180^0$). We see that for V_A = 0 as N increases, there is decay in $\left|\hat{v}_R\right|$, For N = 0 as V_A increases for all values of φ there is growth in $\left|\hat{v}_R\right|$.

5. Conclusions

Graphically it is found that as time elapses, the amplitude of rotational velocity disturbances decay as thermal stratification increases and due to the increase in magnetic field there is growth in the amplitude of rotational velocity disturbances. In the absence of thermal stratification and magnetic field, the results obtained here obtained coincides with Criminale and Drazin [1].

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