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The Evolution of Linearized Disturbances in a Stratified Bounded Shear Flow with Rotation

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ABSTRACT

Using the initial value problem approach, the evolution of linearized perturbations in a stratified shear flow with rotation is studied. Here the resulting equation in time posed by using Fourier transformation and Square transformation is solved for the Fourier amplitudes for bounded couette flow with a point source of the field of transverse velocity and density as the initial distributions. Perturbation solutions are obtained and the velocity and density plots are drawn to find the effect of density variation and rotation.

1. Introduction

The study of stability of stratified shear flows forms an important part of fluid dynamics due to its relevance to meteorology and oceanography.

There are two fundamental methods which have been used in the theory of linear stability of shear flows. The first is the method of normal modes in which the shear flow under consideration is superimposed with disturbances where growth behaviour determines the instability. But a complete transient and asymptotic behaviour could not be traced. In the other method, temporal evolution of the disturbance is obtained by using transform methods and deem the flow unstable if any disturbance grows without bound after a long enough time. This method does not require any eigen value calculations and it does not involve critical levels. More number of research papers are available related to normal mode analysis compared to papers on initial-value problem approach.

The dynamics of fluids in rotating system has developed rapidly in recent years due to interest in geophysical flow problems. Earth's atmosphere, oceans and core of stars and galaxies all exhibit this phenomenon.

Peterson et al. [5] studied the theoretical aspects of modeling stratified turbulent flows subjected to rotation and found that the most commonly used linear models are ill-posed when the combined effect of system rotation and stratification is imposed; the models do not exhibit a steady state solution. Cambon [2] studied theoretically, experimentally and numerically linear and nonlinear structuring effects caused by coriolis force and/or buoyancy force/density stratification. Salhi and Cambon [6] studied the stability problem of unbounded shear flow, subjected to a uniform vertical density stratification, with Brunt-Väisälä frequency and rotating about an axis aligned with the span wise direction and obtained stability diagrams. Benoit Cushman et al. [1] studied mechanism by which stability may occur in rotating stratified flows. They are motion of individual particles (inertial instability) and organized motions across the flow (baroclinic instability).

Chen Wang et al [3] studied ageostrophic instability in rotating, stratified interior vertical shear flows. Two types of baroclinic, ageostrophic instability were found for intermediate Rossby number which resulted in unbalanced instabilities. Wang et al. [9] studied the instability associated baroclinic critical layers in rotating stratified shear flow using normal mode analysis and showed that the resulting coupling between the Kelvin and gravity waves leads to an over reflectional instability. Vijayalakshmi and Balagondar [7] studied the stability of linearized disturbances in a two-layered stratified shear flow with rotation using initial value problem approach and concluded that for large rotation the system is destabilized. Vijayalakshmi [8] studied the stability of linearized disturbances in a stratified bounded couette flow using initial value problem approach.

In this paper we have considered the evolution of linearized perturbations in a stratified shear flow with rotation for bounded couette flow with unit pulse of velocity and density as initial conditions which is due to the work of Criminale and Drazin [4]. The distributions are resolved into two components, rotational and irrotational. The solution for the hypothetical initial value problem for which the basic flow is unbounded but coincides with the actual flow in the layer is the rotational solution. The irrotational solution in each layer is specified uniquely by satisfying the interfacial conditions and boundary conditions.

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2. Mathematical Formulation

Consider a cartesian co-ordinate system OXYZ such that OY is vertical with an incompressible, heterogeneous, inviscid fluid which rotates with an uniform angular velocity $\boldsymbol{\Omega}$ about the y–axis i,e., $\boldsymbol{\Omega} = (0, \Omega, 0)$. The equations of motion of an incompressible, inviscid, stratified fluid in the presence of Coriolis force are

$$\nabla.\mathbf{q} = 0, \tag{2.1}$$

$$\rho\left(\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q} - 2\vec{\Omega}\ x\ \vec{q}\right) = -\nabla p + \rho\vec{g}, \tag{2.2}$$

$$\frac{\partial \rho}{\partial t} + (\vec{q}.\nabla)\rho = 0. \tag{2.3}$$

Here $\vec{q}$ is the velocity vector, p, ρ , $\vec{g}$ respectively denotes the pressure, density at a point and the acceleration due to gravity with component –g in the y–direction.

The basic unperturbed state considered here is $\mathbf{q}_0 = (U(y) = \sigma y, 0, 0), \rho = \rho_0(y), p = p_0(y)$, balance $\frac{\partial p_0}{\partial y} = -\rho_0 g$ and for geostrophic balance $\frac{\partial p_0}{\partial z} = -2\rho_0 \Omega U$ are obtained.

To study the evolution of linearized disturbances in a stratified shear flow with rotation, we linearize equations (2.1) - (2.3) about the basic state and non dimensionalise the equations using the quantities $\left(u^*, v^*, w^*\right) = \left(\frac{u}{V}, \frac{v}{V}, \frac{w}{V}\right)$, $\left(x^*, y^*, z^*\right) = \left(\frac{x}{L}, \frac{y}{L}, \frac{z}{L}\right), p^* = \frac{p}{\rho V^2}$

$t^* = \frac{tV}{L}$, with V and L as the characteristic velocity and length, the above set of equations reduces to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{2.4}$$

$$\frac{\partial u}{\partial t} + \sigma y \frac{\partial u}{\partial x} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + \frac{2}{Ro}w \tag{2.5}$$

$$\frac{\partial v}{\partial t} + \sigma y \frac{\partial v}{\partial x} = -\frac{1}{\rho_0}\frac{\partial p}{\partial y} - \frac{\rho g}{\rho_0}u \tag{2.6}$$

$$\frac{\partial w}{\partial t} + \sigma y \frac{\partial w}{\partial x} = -\frac{1}{\rho_0}\frac{\partial p}{\partial z} - \frac{2}{Ro}u \tag{2.7}$$

$$\frac{\partial \rho}{\partial t} + \sigma y \frac{\partial \rho}{\partial x} + v\frac{d\rho_0}{dy} = 0 \tag{2.8}$$

Using (i) moving coordinates transformation by defining $T = t, \xi = x - \sigma y\, t, \eta = y, \zeta = z$
(ii) three - dimensional Fourier transformation given by
$\hat{u}(\alpha;\beta;\gamma;T) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u(\xi;\eta;\zeta;T)\, e^{i(\alpha\xi + \beta\eta + \gamma\zeta)}\, d\xi\, d\eta\, d\zeta$, with similar expressions

for $\hat{v}, \hat{w}$, $\hat{p}$ and $\hat{\rho}$ and (iii) squire transformation by using $\bar{\alpha} = \left(\alpha^2 + \gamma^2\right)^{1/2}$ and $\varphi = \arctan\left(\gamma/\alpha\right)$ as the polar variables in wave – number space and defining the velocity components in the $\bar{\alpha}$ and φ directions as $\bar{u} = \dfrac{\alpha\hat{u} + \gamma\hat{w}}{\bar{\alpha}}$, $\bar{w} = \dfrac{-\gamma\hat{u} + \alpha\hat{w}}{\bar{\alpha}}$, the above set of equations will be reduced to

$$\frac{d^2}{dT^2}\left(K^2\hat{v}\right) + \left(\frac{2\sigma\alpha}{Ro}(\beta - \sigma\alpha T) + \frac{4}{Ro^2}(\beta - \sigma\alpha T)^2 + N^2\bar{\alpha}^2\right)\hat{v} = \frac{2\sigma\alpha\bar{\alpha}}{Ro}\bar{w}, \tag{2.9}$$

$$\frac{d\bar{w}}{dT} = \frac{1}{\bar{\alpha}}\left(\sigma\gamma + \frac{2}{Ro}(\beta - \sigma\alpha T)\right)\hat{v}. \tag{2.10}$$

Where $K^2 = \bar{\alpha}^2 + (\beta - \sigma\alpha T)^2$, $\bar{\alpha}^2 = \left(\alpha^2 + \gamma^2\right)$, $Ro = \dfrac{V}{\Omega L}$ is Rossby number.

$N^2 = -\dfrac{g}{\rho_0}\dfrac{d\rho_0}{dy}$, ρ_0 is the equilibrium density, N is the Brunt Vaisala frequency

Two solutions exist for (2.9). For $K^2 \neq 0$, the disturbance is rotational and for $K^2 = 0$, the disturbance is irrotational, since $K^2\hat{v} = 0$ results in Laplace equation $\nabla^2\hat{v} = 0$ in real space.

Now considering the case $K^2 \neq 0$, we assume the perturbation expression of the form

$$\hat{v}_R(\alpha,\beta,\gamma,T) = \hat{v}_0(\alpha,\beta,\gamma,T) + N^2\hat{v}_1(\alpha,\beta,\gamma,T) + \frac{1}{Ro}\hat{v}_2(\alpha,\beta,\gamma,T) + \left(N^2\right)^2\hat{v}_3(\alpha,\beta,\gamma,T)$$
$$+\frac{1}{Ro^2}\hat{v}_4(\alpha,\beta,\gamma,T) + \frac{N^2}{Ro}\hat{v}_5(\alpha,\beta,\gamma,T) + \ldots \tag{2.11}$$

$$\bar{w}(\alpha,\beta,\gamma,T) = \bar{w}_0(\alpha,\beta,\gamma,T) + N^2\bar{w}_1(\alpha,\beta,\gamma,T) + \frac{1}{Ro}\bar{w}_2(\alpha,\beta,\gamma,T) + \left(N^2\right)^2\bar{w}_3(\alpha,\beta,\gamma,T)$$
$$+\frac{1}{Ro^2}\bar{w}_4(\alpha,\beta,\gamma,T) + \frac{N^2}{Ro}\bar{w}_5(\alpha,\beta,\gamma,T) + \ldots \tag{2.12}$$

where $\hat{v}_R$ is the velocity for rotational disturbances.

Hence,

$$\hat{v}_R = \frac{T\hat{\Omega}_0(\alpha,\beta,\gamma) + \hat{\Omega}_1(\alpha,\beta,\gamma)}{\bar{\alpha}^2 + (\beta - \sigma\alpha T)^2} + \frac{N^2}{\bar{\alpha}^2 + (\beta - \sigma\alpha T)^2}\left(-\frac{\bar{\alpha}}{\sigma^2\alpha^2}\right)\left[\left(\frac{\beta\hat{\Omega}_0}{\sigma\alpha} + \hat{\Omega}_1\right)\right.$$
$$\left[(\beta - \sigma\alpha T)\tan^{-1}\left(\frac{\beta - \sigma\alpha T}{\bar{\alpha}}\right) - \frac{1}{2}\log\left(\frac{\bar{\alpha}^2 + (\beta - \sigma\alpha T)^2}{\bar{\alpha}^2}\right)\right] - \frac{\hat{\Omega}_0\bar{\alpha}}{2\sigma\alpha}\left(\left(\frac{\beta - \sigma\alpha T}{\bar{\alpha}}\right)\log\left(\frac{\bar{\alpha}^2 + (\beta - \sigma\alpha T)^2}{\bar{\alpha}^2}\right)\right.$$
$$\left.\left.-2\left(\left(\frac{\beta - \sigma\alpha T}{\bar{\alpha}}\right) - \tan^{-1}\left(\frac{\beta - \sigma\alpha T}{\bar{\alpha}}\right)\right)\right)\right] + \frac{1}{Ro}\left\{\left(-\frac{\gamma\bar{\alpha}}{\sigma\alpha^2}\left(\beta\hat{\Omega}_0 + \hat{\Omega}_1\right) + \right.\right.$$
$$+\frac{\gamma}{\alpha\bar{\alpha}}\left(\frac{\beta\hat{\Omega}_0}{\sigma\alpha} + \hat{\Omega}_1\right)\left(-\frac{\bar{\alpha}^3}{\sigma\alpha}\right)\left[\left(\frac{\beta - \sigma\alpha T}{\bar{\alpha}}\right)\log\left(\frac{\bar{\alpha}^2 + (\beta - \sigma\alpha T)^2}{\bar{\alpha}^2}\right) - 2\left(\left(\frac{\beta - \sigma\alpha T}{\bar{\alpha}}\right) - \tan^{-1}\left(\frac{\beta - \sigma\alpha T}{\bar{\alpha}}\right)\right]\right. +$$

$$\left(\frac{\gamma\bar{\alpha}^2\hat{\Omega}_0}{\sigma\alpha^2}+\frac{\gamma\hat{\Omega}_0}{2\sigma\alpha^2}\right)\left[\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)^2-2\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)\tan^{-1}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)+\log\left(\frac{\bar{\alpha}^2+(\beta-\sigma\alpha T)^2}{\bar{\alpha}^2}\right)\right]$$

$$-\frac{\gamma\bar{\alpha}^2}{\sigma\alpha^2}\left(\frac{\beta\hat{\Omega}_0}{\sigma\alpha}+\hat{\Omega}_1\right)\left[\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)^2\tan^{-1}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)-\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)+\tan^{-1}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)\right]-\frac{\gamma\hat{\Omega}_0}{2\sigma\alpha^2}$$

$$\Bigg[\log\left(\frac{\bar{\alpha}^2+(\beta-\sigma\alpha T)^2}{\bar{\alpha}^2}\right)\left(\frac{1}{2}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)^2+1\right)-\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)^2\Bigg]-\frac{2\bar{\alpha}^2}{\sigma^2\alpha}\hat{\Omega}_2\Bigg[\Bigg[\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)\tan^{-1}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)$$

$$-\frac{1}{2}\log\left(\frac{\bar{\alpha}^2+(\beta-\sigma\alpha T)^2}{\bar{\alpha}^2}\right)\Bigg]\Bigg]\Bigg\}\frac{1}{\bar{\alpha}^2+(\beta-\sigma\alpha T)^2}, \qquad (2.13)$$

$$\bar{w}=-\frac{\gamma}{\alpha\bar{\alpha}}\left[\left(\frac{\beta\hat{\Omega}_0}{\sigma\alpha}+\hat{\Omega}_1\right)\tan^{-1}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)-\frac{\bar{\alpha}\hat{\Omega}_0}{2\sigma\alpha}\log\left(\frac{\bar{\alpha}^2+(\beta-\sigma\alpha T)^2}{\bar{\alpha}^2}\right)+\hat{\Omega}_2\right]+$$

$$N^2\Bigg\{\frac{1}{\sigma^2\alpha^2\bar{\alpha}}\Bigg[\left(\frac{\beta\hat{\Omega}_0}{\sigma\alpha}+\hat{\Omega}_1\right)\Bigg[\frac{1}{2}\tan^{-1}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)\log\left(\frac{\bar{\alpha}^2+(\beta-\sigma\alpha T)^2}{\bar{\alpha}^2}\right)+\cos\left(\tan^{-1}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)\right)\Bigg]\Bigg]$$

$$-\frac{\hat{\Omega}_0}{2\sigma\alpha}\Bigg[\frac{1}{2}\left(\log\left(\frac{\bar{\alpha}^2+(\beta-\sigma\alpha T)^2}{\bar{\alpha}^2}\right)\right)^2-\log\left(\frac{\bar{\alpha}^2+(\beta-\sigma\alpha T)^2}{\bar{\alpha}^2}\right)+2\left(\tan^{-1}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)\right)^2\Bigg]\Bigg]\Bigg\}+$$

$$\frac{1}{Ro}\Bigg\{-\frac{\gamma}{\alpha\bar{\alpha}^2}\Bigg\{\Bigg(-\frac{\gamma\bar{\alpha}}{\sigma\alpha^2}\left(\beta\hat{\Omega}_0+\hat{\Omega}_1\right)-\frac{\bar{\alpha}^2\gamma}{\sigma\alpha^2}\left(\frac{\beta\hat{\Omega}_0}{\sigma\alpha}+\hat{\Omega}_1-\frac{2\bar{\alpha}^2}{\sigma^2\alpha}\hat{\Omega}_2\right)\Bigg)\Bigg[-\frac{1}{2}\Bigg(\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)^2\log\left(\frac{\bar{\alpha}^2+(\beta-\sigma\alpha T)^2}{\bar{\alpha}^2}\right)$$

$$\left(1+\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)\right)\Bigg)\Bigg)-\left(\log\left(\frac{\bar{\alpha}^2+(\beta-\sigma\alpha T)^2}{\bar{\alpha}^2}\right)\right)^2\Bigg]-\Bigg[\Bigg(-\frac{\gamma\bar{\alpha}}{\sigma\alpha^2}\left(\beta\hat{\Omega}_0+\hat{\Omega}_1\right)+\frac{\gamma\bar{\alpha}^2}{\sigma\alpha^2}\left(\frac{\beta\hat{\Omega}_0}{\sigma\alpha}+\hat{\Omega}_1\right)\Bigg]$$

$$-\left(\tan^{-1}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)\right)^2\Bigg]+\left(\frac{\gamma\bar{\alpha}^2\hat{\Omega}_0}{\sigma\alpha^2}+\frac{4\gamma\hat{\Omega}_0}{2\sigma\alpha^2}\right)\Bigg[\Bigg[\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)-\tan^{-1}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)+\frac{1}{2}\tan^{-1}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)$$

$$\log\left(\frac{\bar{\alpha}^2+(\beta-\sigma\alpha T)^2}{\bar{\alpha}^2}\right)-\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)\left(\tan^{-1}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)\right)^2+\frac{1}{3}\left(\tan^{-1}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)\right)^3\Bigg]\Bigg]\Bigg\}-\frac{\gamma\bar{\alpha}^2}{\sigma\alpha^2}\left(\frac{\beta\hat{\Omega}_0}{\sigma\alpha}+\hat{\Omega}_1\right)$$

$$2\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)\tan^{-1}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)-\frac{2}{\bar{\alpha}}\left(\frac{\beta\hat{\Omega}_0}{\sigma^2\alpha^2}+\frac{\hat{\Omega}_1}{\sigma\alpha\bar{\alpha}}\right)\tan^{-1}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)+\frac{\hat{\Omega}_0}{\sigma^2\alpha^2}\log\left(\frac{\bar{\alpha}^2+(\beta-\sigma\alpha T)^2}{\bar{\alpha}^2}\right)\Bigg)\Bigg\} \qquad (2.14)$$

$$\hat{\rho} = -\frac{\bar{\rho}}{\bar{\alpha}^2 g}\left\{\frac{d}{dT}\left(k^2\hat{v}\right) + \frac{2}{Ro}\bar{\alpha}\left(\beta - \sigma\alpha T\right)\bar{w}\right\}, \tag{2.15}$$

The solution for $K^2 = 0$ is found by considering the perturbation equations by using two – dimensional Fourier transform. Using moving co-ordinate transformation, $K^2\hat{v} = 0$ simplifies to

$$\frac{\partial^2 \breve{v}_I}{\partial\eta^2} + 2i\sigma\,\alpha T\frac{\partial\breve{v}_I}{\partial\eta} - \left(\bar{\alpha}^2 + \sigma^2\alpha^2T^2\right)\breve{v}_I = 0 \tag{2.16}$$

with

$$\breve{v}_I = \breve{v}_I(\alpha,\eta,\gamma;T) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} v_I(\xi,\eta,\varsigma,T)\,e^{i(\alpha\xi+\gamma\zeta)}\,d\xi\,d\zeta \tag{2.17}$$

as the irrotational part of $\breve{v}$ and its solution is given by

$$\breve{v}_I = A(T)e^{-\bar{\alpha}\eta - i\sigma\,\alpha T\eta} + B(T)e^{\bar{\alpha}\eta - i\sigma\,\alpha T\eta}, \tag{2.18}$$

where A(T) and B(T) are the constants of integration .

In order to combine $\hat{v}_R$ and $\breve{v}_I$ to obtain the complete solution and satisfy the matching condition $\hat{v}_R$must be inverted once to obtain $\breve{v}_R(\alpha,\eta,\gamma;T)$i,e.,

$$\breve{v}_R(\alpha,\eta,\gamma;T) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\hat{v}_R(\alpha,\beta,\gamma;T)\,e^{-i\beta\eta}\,d\beta. \tag{2.19}$$

With initial velocity and initial density as unit pulse, the initial conditions are given by

$$v(x,y,z,0) = V_0\,\delta(x - x_0)\delta(y - y_0)\delta(z - z_0),$$

$$\rho(x,y,z,0) = \tilde{\rho}_0\,\delta(x - x_0)\delta(y - y_0)\delta(z - z_0),$$

$$\bar{w}_0(x,y,z,0) = \bar{W}_0\,\delta(x - x_0)\delta(y - y_0)\delta(z - z_0) \tag{2.20}$$

In terms of moving co-ordinates and three-dimensional Fourier transform,

$$\breve{v}_0(\alpha,\beta,\gamma) = \Omega_0(\alpha,\beta,\gamma) = V_0 e^{i(\alpha x_0+\beta y_0+\gamma z_0)},$$

$$\tilde{\rho}_0(\alpha,\beta,\gamma) = \Omega_1(\alpha,\beta,\gamma) = \tilde{\rho}_0 e^{i(\alpha x_0+\beta y_0+\gamma z_0)}$$

$$\bar{w}_0(\xi,\eta,\zeta,0) = \Omega_2(\alpha,\beta,\gamma) = \bar{W}_0 e^{i(\alpha x_0+\beta y_0+\gamma z_0)} \tag{2.21}$$

$\breve{v}_R$ is found to be

$$\breve{v}_R = e^{i(\alpha x_0+\gamma z_0-\sigma\alpha T\bar{\eta})}\left\{\left(TV_0 + \tilde{\rho}_0\right)e^{-\bar{\alpha}|\bar{\eta}|} + N^2\left\{\frac{V_0}{\sigma\alpha}\left(\int_{-\infty}^{\infty}\frac{e^{-\bar{\alpha}|\bar{\eta}-\eta'|-\bar{\alpha}|\eta'|}}{\eta'}d\eta' -\right.\right.\right.$$

$$\left.-\frac{i}{2}\int_{-\infty}^{\infty}(\bar{\eta}-\eta')\frac{e^{-\bar{\alpha}|\bar{\eta}-\eta'|-\bar{\alpha}|\eta'|}}{\eta'}d\eta'\right) + \tilde{\rho}_0\left(-\frac{1}{2}\int_{-\infty}^{\infty}(\bar{\eta}-\eta')\frac{e^{-\bar{\alpha}|\bar{\eta}-\eta'|-\bar{\alpha}|\eta'|}}{\eta'}d\eta'\right.$$

$$\left.\left.\left.-\frac{1}{2}\frac{e^{-\bar{\alpha}|\bar{\eta}|}}{\bar{\eta}}\right) - \frac{V_0\bar{\alpha}}{2\sigma\alpha}\left(-\frac{i}{2}\int_{-\infty}^{\infty}\frac{(\bar{\eta}-\eta')e^{-\bar{\alpha}|\bar{\eta}-\eta'|-\bar{\alpha}|\eta'|}}{\eta'}d\eta' + i\,\bar{\eta}e^{-\bar{\alpha}|\bar{\eta}|} - \frac{2ie^{-\bar{\alpha}|\bar{\eta}|}}{\bar{\eta}}\right)\right\}\right\}\left(-\frac{1}{\sigma^3\alpha^3}\right)$$

$$\begin{aligned}
&+\frac{1}{Ro}\left\{\left(-\frac{\gamma\bar{\alpha}}{\sigma\alpha^{2}}-\frac{\gamma\bar{\alpha}^{3}}{\sigma^{2}\alpha^{3}}\right)V_{0}\left[\int_{-\infty}^{\infty}\frac{e^{-\bar{\alpha}|\bar{\eta}-\eta'|-\bar{\alpha}|\eta'|}}{\eta'}d\eta'-2\int_{-\infty}^{\infty}\frac{(\bar{\eta}-\eta')e^{-\bar{\alpha}|\bar{\eta}-\eta'|-\bar{\alpha}|\eta'|}}{\eta'}d\eta'\right.\right.\\
&\quad\left.+\eta e^{-\bar{\alpha}|\bar{\eta}|}\right)-\left(-\frac{\gamma\bar{\alpha}}{\sigma\alpha^{2}}-\frac{\gamma\bar{\alpha}^{3}}{\sigma^{2}\alpha^{3}}\right)\tilde{\rho}_{0}\left(i\int_{-\infty}^{\infty}\frac{(\bar{\eta}-\eta')e^{-\bar{\alpha}|\bar{\eta}-\eta'|-\bar{\alpha}|\eta'|}}{\eta'}d\eta'-\frac{2ie^{-\bar{\alpha}|\bar{\eta}-\eta'|}}{(\bar{\eta}-\eta')}\right.\\
&\quad\left.+i(\bar{\eta}-\eta')e^{-\bar{\alpha}|\bar{\eta}-\eta'|}\right)+\left(\frac{\gamma\bar{\alpha}}{\sigma\alpha^{2}}+-\frac{\gamma\bar{\alpha}^{3}}{\sigma^{2}\alpha^{3}}\right)V_{0}\left(e^{-\bar{\alpha}|\bar{\eta}|}+\int_{-\infty}^{\infty}\frac{(\bar{\eta}-\eta')e^{-\bar{\alpha}|\bar{\eta}-\eta'|-\bar{\alpha}|\eta'|}}{2\eta'}d\eta'\right.\\
&\quad\left.-\frac{e^{-\bar{\alpha}|\bar{\eta}|}}{\bar{\eta}}\right)-\frac{\gamma\bar{\alpha}^{2}}{\sigma\alpha^{2}}\tilde{\rho}_{0}\left(-i\int_{-\infty}^{\infty}\frac{(\bar{\eta}-\eta')e^{-\bar{\alpha}|\bar{\eta}-\eta'|-\bar{\alpha}|\eta'|}}{\eta'}d\eta'-\frac{i\bar{\eta}e^{-\bar{\alpha}|\bar{\eta}|}}{2}-\frac{ie^{-\bar{\alpha}|\bar{\eta}|}}{\bar{\eta}}\right)-\frac{\gamma V_{0}}{2\sigma\alpha^{2}}\\
&\quad\left(\frac{1}{2}\int_{-\infty}^{\infty}\frac{e^{-\bar{\alpha}|\bar{\eta}-\eta'|-\bar{\alpha}|\eta'|}}{\eta'}d\eta'+\frac{e^{-\bar{\alpha}|\bar{\eta}|}}{\bar{\eta}}-e^{-\bar{\alpha}|\bar{\eta}|}\right)-\frac{2\bar{\alpha}^{2}\bar{W}_{0}}{\sigma^{2}\alpha}\left(\int_{-\infty}^{\infty}\frac{e^{-\bar{\alpha}|\bar{\eta}-\eta'|-\bar{\alpha}|\eta'|}}{\eta'}d\eta'\right.\\
&\quad\left.\left.\left.-\int_{-\infty}^{\infty}e^{-\bar{\alpha}|\bar{\eta}-\eta'|-\bar{\alpha}|\eta'|}d\eta'\right)\right\}\right\} \qquad (2.22)
\end{aligned}$$

Here $\bar{\eta}=\eta-y_0$. Now the complete solution will be

$$\breve{v}=\breve{v}_R+\breve{v}_I. \qquad (2.23)$$

$\breve{v}_R$ and $\breve{v}_I$ given by equations (2.22) and (2.18).

3. Stratified Bounded Couette Flow with Rotation

In this case, a stratified plane Couette flow bounded at $y=\pm H$ is considered (Fig. 1). Here velocity $\breve{v}$ vanishes at $\eta=\pm H$, hence we have

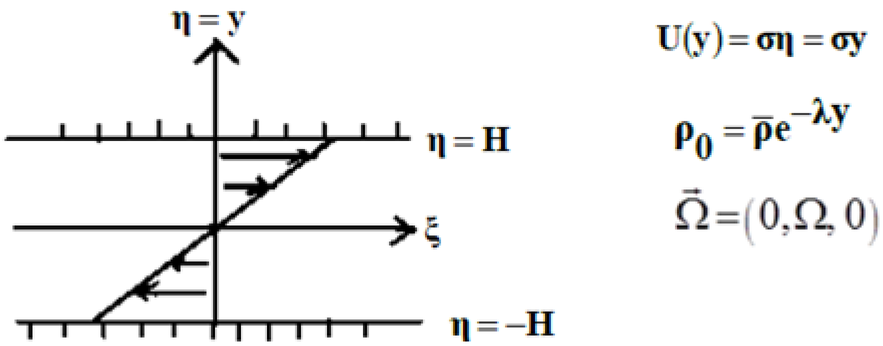


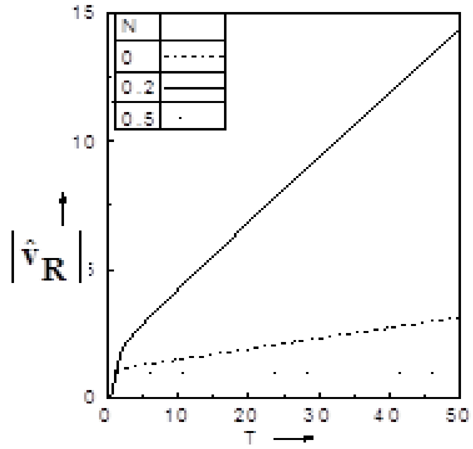
Fig 1. Sketch of Stratified Bounded Couette Flow with Rotation

$$e^{-\bar{\alpha}H-i\sigma\alpha TH}A+e^{\bar{\alpha}H-i\sigma\alpha TH}B=-\left[\breve{v}_R\right]_{\eta=+H}, \qquad (3.1)$$

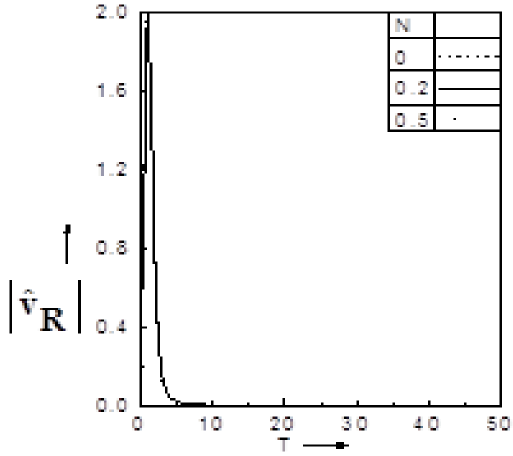
$$e^{\bar{\alpha}H+i\sigma\alpha TH}A+e^{-\bar{\alpha}H+i\sigma\alpha TH}B=-\left[\breve{v}_R\right]_{\eta=-H}, \qquad (3.2)$$

From equations (3.1) and (3.2), A and B are found to be

$$A=\frac{\breve{v}_R(+H)e^{-\bar{\alpha}H+i\sigma\alpha TH}-\breve{v}_R(+H)e^{\bar{\alpha}H-i\sigma\alpha TH}}{2\sinh(2\bar{\alpha}H)}. \qquad (3.3)$$



(a)



(a)

$$B = \frac{\breve{v}_R(-H)e^{-\bar{\alpha}H - i\sigma\alpha TH} - \breve{v}_R(-H)e^{\bar{\alpha}H + i\sigma\alpha TH}}{2\sinh(2\bar{\alpha}H)}. \quad (3.4)$$

where $\breve{v}_R(\pm H) = -\lfloor \breve{v}_R \rfloor_{\eta = \pm H}$.

It is found that

$$\breve{v}_R(+H) = (A_1 T + B_1)e^{i(\alpha x_0 + \gamma z_0 - \sigma\alpha T(H - y_0))}. \quad (3.5)$$

$$\breve{v}_R(-H) = (A_2 T + B_2)e^{i(\alpha x_0 + \gamma z_0 + \sigma\alpha T(H + y_0))}. \quad (3.6)$$

$$A_1 = V_0 e^{-\bar{\alpha}|H - y_0|}, \; B_1 = \left(\breve{v}_R - e^{i(\alpha x_0 + \gamma z_0 - \sigma\alpha T\bar{\eta})} T V_0 e^{-\bar{\alpha}|\bar{\eta}|} \right)\Bigg|_{\bar{\eta} = H - y_0}$$

By replacing H by –H in A_1 and B_1 we obtain A_2 and B_2.

4. Results and Discussions

In this paper, we have considered the linear stability of a basic flow of an inviscid stratified shear flow with rotation using piecewise linear velocity profiles with unit pulse for velocity and density as initial distributions. Here we have concentrated on bounded couette flow. In these piecewise linear profiles, the disturbance in each layer is resolved into the sum of two components, the rotational and irrotational solution. We have drawn plots for the variation of rotational velocity $\hat{v}_R$ (Fig.2 & 3) with time.

Figs. 2(a)–(c) and 3(a)–(c) are plots of $\hat{v}_R$ versus T for different values of Brunt Vaisala frequency N (N = 0, 0.2, 0.5) and $\varphi(\varphi = 0^o, 45^o, 90^o)$ for $\frac{1}{R} = 0$ and 2 respectively.

We see that there is decay in $\hat{v}_R$ for $\varphi = 0^0, 450^0$ but for $\varphi = 90^0$ there is increase in $\hat{v}_R$ for large time for all values of N and Ro.

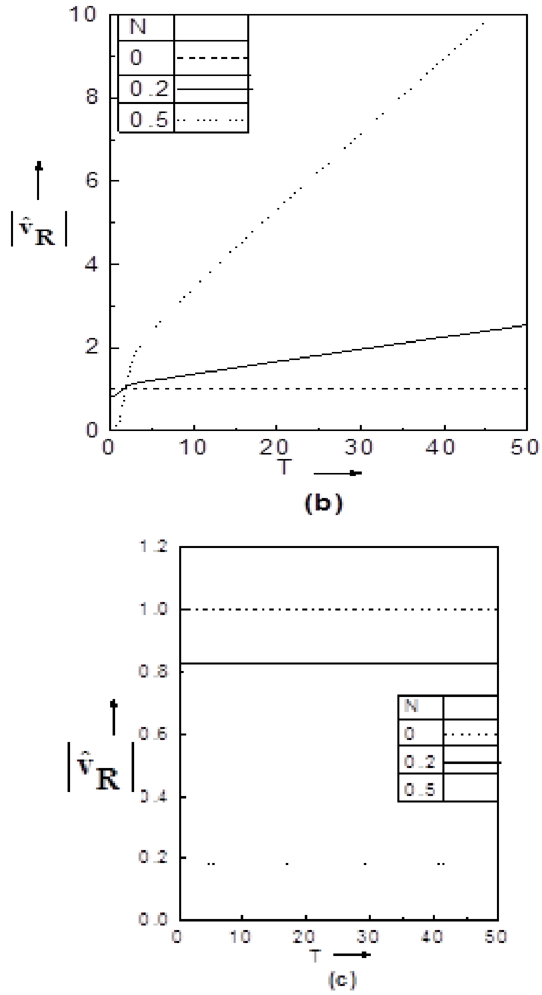


Fig 2. Curves of $\left|\hat{v}_R\right|$ versus T for (a) $\varphi = 0^o$, (b) $\varphi = 45^o$, (c) $\varphi = 90^o$ for different values of N and $\frac{1}{Ro}$ = 0 .

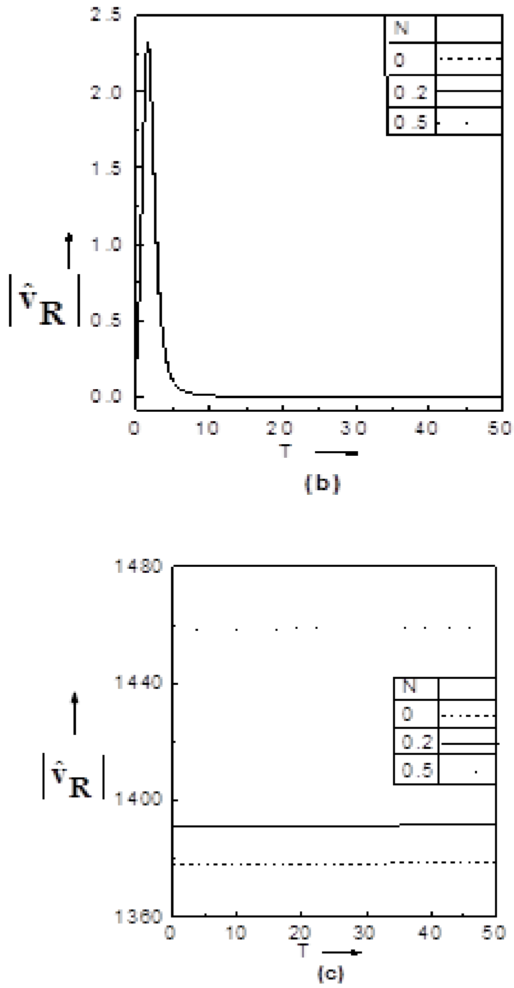


Fig 3. Curves of $\left|\hat{v}_R\right|$ versus T for (a) $\varphi = 0^o$, (b) $\varphi = 45^o$, (c) $\varphi = 90^o$ for different values of N and $\frac{1}{Ro}$ = 2 .

5. Conclusions

Graphically, we can conclude that rotation highly destabilizes the system. For $\varphi = 90^0$ there is increase in the amplitude of rotational velocity disturbances for large time for all values of N and Ro. $\varphi = 90^0$ implies that the boundaries are placed far away from each other. When $\frac{1}{Ro} = 0$ i,e., for large rotation the disturbances grows with time making the system unstable. In the absence of stratification and rotation the results obtained here coincides with Criminale and Drazin [4] and in the absence of rotation, the results obtained coincides with Vijayalakshmi [8].

6. References

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