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One-dimensional temporally dependent advection-dispersion equation in finite homogeneous porous media

Dilip kumar Jaiswal, R.R.Yadav and Hareesh Kumar Yadav
Department of Mathematics and Astronomy,
Lucknow University, Lucknow-226007, Uttar Pradesh, India.

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ABSTRACT
Analytical solutions are obtained for advection-dispersion equation in one-dimension longitudinal finite domain. The solute dispersion parameter is assumed temporally dependent along uniform flow. The retardation coefficient and first order decay term which inversely proportional to the dispersion coefficient is also considered. Initially the space domain is solute free. Solution are obtained for two cases, first one for uniform input and second one for increasing input condition.

Introduction

A quantitative understanding of the transport of pollutants in groundwater is of great importance from the environmental perspective. Some environmental pollution scenarios involving groundwater contamination are very real. Effect of contamination depends on nature and levels of the toxicants, sometimes causing serious health hazards even at low level, therefore most of the times remediation becomes a necessity for achieving sustainability. If groundwater becomes polluted it is very difficult to rehabilitate. The slow rate of groundwater flow and low microbiological activities limit any self-purification processes which takes place in days or weeks in surface water systems can take decades in groundwater. Mathematical modeling of contaminant behavior in porous media is considered to be a powerful tool for a wide range of pollution problems related with groundwater quality rehabilitation.

Reliable analytical solutions for solute transport in saturated/unsaturated porous media are required for many application purposes such as the optimal management of agricultural practices, to preserve soils or assessing groundwater pollution risks. Numerous approaches to solving the advection-dispersion equations have been suggested in the literature. Kitagawa (1934) was perhaps the first to perform experiments on dispersion phenomena. Transverse dispersion coefficients are typically evaluated by the interpretation of steady-state transverse concentration profiles of conservative solutes in parallel flow. Ogata (1970) was the first to give a non-dimensional analytical solution to the solute transport equation in porous media. Eldor and Dagon (1972) developed an approximate solution for dispersion for dispersion in two-dimensional flow. Bruch and Street (1967) obtain a series solution for two-dimensional transport in finite system.Corapciglu and Hossain (1986) developed a two-dimensional sharp interface model by assuming the mass flow rates in horizontal and vertical directions are constant and independent. Valocchi (1989) studied transport of a kinetically sorbing solute under conditions of horizontal flow where sorbing reaction varied as an arbitrary function in vertical direction. Beltman, Boesten and Van der Zee (1995) studied pesticide transport through the unsaturated zone described by an analytical solution of the convection-dispersion equation assuming steady water flow, a linear sorption isotherm and first-order transformation kinetics. Wu et al. (1997) developed another analytical model for nonlinear adsorptive transport through layered soils ignoring the effects of the dispersion term. Analytical study of contaminant transport from a finite source in a finite–thickness aquifer is most useful in hydrological and environmental sciences and engineering but rarely investigated in previous studies. Park and Zhan (2001) provided analytical solutions of contaminant transport from one–, two–, and three–dimensional finite sources in a finite-thickness aquifer using Green's function method.

Ataie-Ashtiani et al. (2001, 2002) studied the influence of tidal fluctuation effects on groundwater dynamics and contaminant transport in unconfined coastal aquifers. Hoehn and Cirpka (2006) presented temperature time series collected at two sites in the alpine floodplain aquifers of the Brenno river in Southern Switzerland. Kumar et al. (2009) obtained analytical solutions for one-dimensional advection–diffusion equation with variable coefficients in a longitudinal finite initially solute free domain, for temporally and spatially dependent dispersion problems.

In the present study, the solute dispersion parameter is considered temporally dependent along uniform flow. The retardation coefficient and first order decay term which inversely proportional to the dispersion coefficient is also considered. Initially the space domain is solute free. The input condition is assumed at the origin of the domain. The second condition is considered at the end of the domain of flux type. Analytical solutions of advection-dispersion equation in one-dimension longitudinal finite domain are obtained for two cases, first one for uniform input and second one for increasing input condition

Tele:
E-mail addresses: yadav_rr2@yahoo.co.in,
dilip3jais@gmail.com

Laplace transformation technique is used to get both the solutions.

Mathematical Formulation and Solution

The governing parabolic partial differential equation describing the concentration distributions with first order decay in a one-dimensional porous medium is,

$$R\frac{\partial c}{\partial t}=\frac{\partial}{\partial x}\left(D(x,t)\frac{\partial c}{\partial x}-u(x,t)c\right)-\gamma(x,t)c \qquad (1)$$

where $D(x,t)$ is the longitudinal dispersion coefficient, $u(x,t)$ is the seepage velocity, c is the concentration at position x and time t. γ is the first order decay, it means degradation of contaminants is approximated as a first order function of contaminants, and geochemical factor limiting biodegradation are not explicitly considered and $1+K_d=R$, which is the retardation coefficient accounting for equilibrium linear sorption processes where K_d is distribution coefficient which is defined as ratio of the adsorbed contaminant concentration to the dissolved contaminants. In case no adsorption $K_d=0$.

Let us assume that $D(x,t)=D_0 f_1(x,t)$ and $u(x,t)=u_0 f_2(x,t)$, are the function of position or time. If both the parameters are independent to independent variables x and t, then these are called constant dispersion and uniform flow velocity respectively. The first order decay term is considered inversely proportional to the dispersion coefficient i.e.,

$$\gamma(x,t)=\frac{\gamma_0}{f_1(x,t)}$$

So under these considerations the partial differential equation (1) may be written as,

$$R\frac{\partial c}{\partial t}=\frac{\partial}{\partial x}\left(D_0 f_1(x,t)\frac{\partial c}{\partial x}-u_0 f_2(x,t)c\right)-\gamma_0 c/f_1(x,t) \qquad (2)$$

where, D_0, u_0 and γ_0 are constants and dimension of these constants are L^2T^{-1}, LT^{-1} and T^{-1} respectively.

Let us introduce a new independent variable X by a transformation (Jaiswal et al. 2009; Kumar et al. 2010)

$$\frac{\partial X}{\partial x}=\frac{1}{f_1(x,t)} \text{ or } X=\int\frac{dx}{f_1(x,t)} \qquad (3)$$

Equation (2) becomes,

$$Rf_1(x,t)\frac{\partial c}{\partial t}=\frac{\partial}{\partial X}\left(D_0\frac{\partial c}{\partial X}-u_0 f_2(x,t)c\right)-\gamma_0 c$$

Unsteady Dispersion along Uniform Flow

Let $f_1(x,t)=f(mt)$ and $f_2(x,t)=1$ where m is a flow resistance coefficient whose dimension is inverse of the time variable t. $f(mt)$ is chosen such that $f(mt)=1$ for $m=0$ or $t=0$, Thus $f(mt)$ is an expression of non-dimensional variable (mt). Then from equation (3), we have,

$$X=\int\frac{dx}{f_1(x,t)}=\int\frac{dx}{f(mt)}=\frac{x}{f(mt)} \qquad (5)$$

Equation (4) becomes,

$$Rf(mt)\frac{\partial c}{\partial t}=\frac{\partial}{\partial X}\left(D_0\frac{\partial c}{\partial X}-u_0 c\right)-\gamma_0 c \qquad (6)$$

Let us introduce a new time variable T, by the following transformation (Crank,1975),

$$T=\int_0^t\frac{dt}{Rf(mt)} \text{ or } \frac{\partial T}{\partial t}=\frac{1}{Rf(mt)} \qquad (7)$$

so

$$\frac{\partial c}{\partial t}=\frac{\partial c}{\partial T}\frac{\partial T}{\partial t}=\frac{1}{Rf(mt)}\frac{\partial c}{\partial T} \qquad (8)$$

The partial differential equation (6) reduces into constant coefficients as,

$$\frac{\partial c}{\partial T}=D_0\frac{\partial^2 c}{\partial X^2}-u_0\frac{\partial c}{\partial X}-\gamma_0 c \qquad (9)$$

Uniform Input Condition

The concentration in porous domain at time $t=0$ is solute free, it means there is no concentration at the initial stage. The concentration at $x=0$ is C_0 for time $t>0$ and concentration gradient at $x=L$ is considered zero for all time $t\geq 0$. Thus initial and boundary conditions for equation (1) in mathematical form in a finite domain may be written as,

$$c(x,t)=0,\ x\geq 0,\ t=0 \qquad (10)$$

$$c(x,t)=C_0,\ x=0,\ t>0 \qquad (11)$$

$$\frac{\partial c(x,t)}{\partial x}=0,\ x=L,\ t\geq 0 \qquad (12)$$

The practical significance of the boundary condition (11) is the constant concentration at source of the boundary i.e., $x=0$ and second boundary (12) indicate that the concentration gradient at $x=L$ is zero. In other words the condition (12) is based on the assumption that the concentration is macroscopically continuous at outlet and that no dispersion occcurs outside the aquifer/domain. This condition is reasonable if the transport conditions on the exit side of the domain are comparable to conditions within the domain.

These conditions in terms of new space and time variable may be written as,

$$c(X,T)=0,\ X\geq 0, T=0 \qquad (13)$$

$$c(X,T)=C_0,\ X=0, T>0 \qquad (14)$$

$$\frac{\partial c(X,T)}{\partial X}=0,\ X=\frac{L}{f(mt)}, T\geq 0 \qquad (15)$$

Now introducing a new dependent variable K by following transformation,

$$c(X,T)=K(X,T)\exp\left[\frac{u_0}{2D_0}X-\left\{\frac{u_0^2}{4D_0}+\gamma_0\right\}T\right] \qquad (16)$$

Then the set of equations (9), (13-15) reduced into,

$$\frac{\partial K}{\partial T}=D_0\frac{\partial^2 K}{\partial X^2} \qquad (17)$$

$$K(X,T)=0,\ X\geq 0,\ T=0 \qquad (18)$$

$$K(X,T)=C_0\exp(\alpha^2 T), X=0, T>0\ \alpha^2=\left\{\frac{u_0^2}{4D_0}+\gamma_0\right\} \qquad (19)$$

$$\frac{\partial K(X,T)}{\partial X}+\frac{u_0}{2D_0}K=0,\ X=\frac{L}{f(mt)}, T\geq 0 \qquad (20)$$

Applying Laplace transformation on equations (17) to (20), we have,

$$p\bar{K} = D_0 \frac{d^2\bar{K}}{dX^2} \qquad (21)$$

$$\bar{K}(X,p) = \frac{C_0}{(p-\alpha^2)}, \; X = 0 \qquad (22)$$

$$\frac{\partial \bar{K}(X,p)}{\partial X} + \frac{u_0}{2D_0}\bar{K} = 0, \; X = \frac{L}{f(mt)} \qquad (23)$$

Thus the general solution of equation (21) may be written as,

$$\bar{K}(X,p) = C_1 exp(-X\sqrt{p/D_0}) + C_2 exp(X\sqrt{p/D_0}) \; (24)$$

Using condition (22) and (23) in general solution (24), we may get,

$$C_1 = \frac{C_0}{(p-\alpha^2)} \frac{\left(\frac{u_0}{2D_0}+\sqrt{\frac{p}{D_0}}\right)exp\left(\frac{L}{f(mt)}\sqrt{\frac{p}{D_0}}\right)}{\left\{\left(\frac{u_0}{2D_0}+\sqrt{\frac{p}{D_0}}\right)exp\left(\frac{L}{f(mt)}\sqrt{\frac{p}{D_0}}\right)-\left(\frac{u_0}{2D_0}-\sqrt{\frac{p}{D_0}}\right)exp\left(-\frac{L}{f(mt)}\sqrt{\frac{p}{D_0}}\right)\right\}}$$

and

$$C_2 = -\frac{C_0}{(p-\alpha^2)} \frac{\left(\frac{u_0}{2D_0}-\sqrt{\frac{p}{D_0}}\right)exp\left(-\frac{L}{f(mt)}\sqrt{\frac{p}{D_0}}\right)}{\left\{\left(\frac{u_0}{2D_0}+\sqrt{\frac{p}{D_0}}\right)exp\left(\frac{L}{f(mt)}\sqrt{\frac{p}{D_0}}\right)-\left(\frac{u_0}{2D_0}-\sqrt{\frac{p}{D_0}}\right)exp\left(-\frac{L}{f(mt)}\sqrt{\frac{p}{D_0}}\right)\right\}}$$

Substituting the value of C_1 and C_2 in Eq. (24), the particular solution in the Laplacian domain may be written as,

$$\bar{K}(X,p) = \frac{C_0\left(\frac{u_0}{2D_0}+\sqrt{\frac{p}{D_0}}\right)exp\left\{-\left(X-\frac{L}{f(mt)}\right)\sqrt{\frac{p}{D_0}}\right\}}{(p-\alpha^2)\left\{\left(\frac{u_0}{2D_0}+\sqrt{\frac{p}{D_0}}\right)exp\left(\frac{L}{f(mt)}\sqrt{\frac{p}{D_0}}\right)-\left(\frac{u_0}{2D_0}-\sqrt{\frac{p}{D_0}}\right)exp\left(-\frac{L}{f(mt)}\sqrt{\frac{p}{D_0}}\right)\right\}}$$

$$-\frac{C_0\left(\frac{u_0}{2D_0}-\sqrt{\frac{p}{D_0}}\right)exp\left\{\left(X-\frac{L}{f(mt)}\right)\sqrt{\frac{p}{D_0}}\right\}}{(p-\alpha^2)\left\{\left(\frac{u_0}{2D_0}+\sqrt{\frac{p}{D_0}}\right)exp\left(\frac{L}{f(mt)}\sqrt{\frac{p}{D_0}}\right)-\left(\frac{u_0}{2D_0}-\sqrt{\frac{p}{D_0}}\right)exp\left(-\frac{L}{f(mt)}\sqrt{\frac{p}{D_0}}\right)\right\}} \qquad (25)$$

Taking inverse Laplace transform of equation (25) and using transformations (16), (7) and (5), the analytical solution of advection-dispersion equation in one-dimension longitudinal finite domain, which is reported by van Genuchten and Alves (1982), is

$$c(x,T) = C_0 F_1(x,T)/F_2(x,T) \qquad (26)$$

where
$$F_1(x,T) = \frac{1}{2} exp\left[\frac{(u_0-v_0)\frac{x}{f(mt)}}{2D_0}\right] erfc\left\{\frac{\frac{x}{f(mt)}-v_0T}{2(D_0T)^{1/2}}\right\}$$

$$+\frac{1}{2} exp\left[\frac{(u_0+v_0)\frac{x}{f(mt)}}{2D_0}\right] erfc\left\{\frac{\frac{x}{f(mt)}+v_0T}{2(D_0T)^{1/2}}\right\}$$

$$+\frac{(v_0-u_0)}{2(v_0+u_0)} exp\left[\frac{(u_0+v_0)\frac{x}{f(mt)}-2v_0L}{2D_0}\right] erfc\left\{\frac{\left(2L-\frac{x}{f(mt)}\right)-v_0T}{2(D_0T)^{1/2}}\right\}$$

$$+\frac{(v_0+u_0)}{2(v_0-u_0)} exp\left[\frac{(u_0-v_0)\frac{x}{f(mt)}+2v_0L}{2D_0}\right] erfc\left\{\frac{\left(2L-\frac{x}{f(mt)}\right)+v_0T}{2(D_0T)^{1/2}}\right\}$$

$$-\frac{u_0^2}{2\gamma_0 D_0} exp\left[\frac{u_0 L}{D_0}-\gamma_0 T\right] erfc\left\{\frac{\left(2L-\frac{x}{f(mt)}\right)+u_0T}{2(D_0T)^{1/2}}\right\}$$

$$F_2(x,T) = 1 + \frac{(v_0-u_0)}{2(v_0+u_0)} exp\left[\frac{-v_0L}{D_0}\right]$$

and $\quad v_0 = u_0\left(1+\frac{4\gamma_0 D_0}{u_0^2}\right)^{1/2}, T = \int_0^t \frac{dt}{Rf(mt)}$.

Increasing Input Condition

The pollutant concentration may not be uniform. It may be increases due to some responsible activities. This type of condition is taken to be of third type or mixed type i.e.,

$$-D(x,t)\frac{\partial c}{\partial x} + u(x,t)c(x,t) = u_0 C_0, \; x = 0, t > 0 \qquad (27)$$

It means the third-type boundary condition allows for solute concentration at the inflow boundary to be lower than C_0 initially and then to increase as more solute enters the system. Over time, the concentration gradient across the boundary $\frac{\partial c}{\partial x}$ decreases as the concentration at the inflow boundary approaches C_0.

Using the transformations (5), (7) and (16) and applying Laplace transformation on equation (27), it becomes,

$$-D_0\frac{d\bar{K}}{dX} + \frac{u_0}{2}\bar{K} = \frac{u_0 C_0}{(p-\alpha^2)}, \quad X = 0 \qquad (28)$$

Using condition (28) in place of (22) in general solution (24), we get,

$$C_1 = \frac{u_0 C_0\left(\frac{u_0}{2D_0}+\sqrt{\frac{p}{D_0}}\right)exp\left(\frac{L}{f(mt)}\sqrt{\frac{p}{D_0}}\right)}{D_0(p-\alpha^2)\left\{\left(\frac{u_0}{2D_0}+\sqrt{\frac{p}{D_0}}\right)^2 exp\left(\frac{L}{f(mt)}\sqrt{\frac{p}{D_0}}\right)-\left(\frac{u_0}{2D_0}-\sqrt{\frac{p}{D_0}}\right)^2 exp\left(-\frac{L}{f(mt)}\sqrt{\frac{p}{D_0}}\right)\right\}}$$

and

$$C_2 = -\frac{u_0 C_0\left(\frac{u_0}{2D_0}-\sqrt{\frac{p}{D_0}}\right)exp\left(-\frac{L}{f(mt)}\sqrt{\frac{p}{D_0}}\right)}{D_0(p-\alpha^2)\left\{\left(\frac{u_0}{2D_0}+\sqrt{\frac{p}{D_0}}\right)^2 exp\left(\frac{L}{f(mt)}\sqrt{\frac{p}{D_0}}\right)-\left(\frac{u_0}{2D_0}-\sqrt{\frac{p}{D_0}}\right)^2 exp\left(-\frac{L}{f(mt)}\sqrt{\frac{p}{D_0}}\right)\right\}}$$

Substituting these value of C_1 and C_2 in Eq. (24), the particular solution in the Laplacian domain may be written as,

$$\bar{K}(X,p) = \frac{u_0 C_0\left(\frac{u_0}{2D_0}+\sqrt{\frac{p}{D_0}}\right)exp\left(-X+\frac{L}{f(mt)}\sqrt{\frac{p}{D_0}}\right)}{D_0(p-\alpha^2)\left\{\left(\frac{u_0}{2D_0}+\sqrt{\frac{p}{D_0}}\right)^2 exp\left(\frac{L}{f(mt)}\sqrt{\frac{p}{D_0}}\right)-\left(\frac{u_0}{2D_0}-\sqrt{\frac{p}{D_0}}\right)^2 exp\left(-\frac{L}{f(mt)}\sqrt{\frac{p}{D_0}}\right)\right\}}$$

$$-\frac{u_0 C_0\left(\frac{u_0}{2D_0}-\sqrt{\frac{p}{D_0}}\right)exp\left(X-\frac{L}{f(mt)}\sqrt{\frac{p}{D_0}}\right)}{D_0(p-\alpha^2)\left\{\left(\frac{u_0}{2D_0}+\sqrt{\frac{p}{D_0}}\right)^2 exp\left(\frac{L}{f(mt)}\sqrt{\frac{p}{D_0}}\right)-\left(\frac{u_0}{2D_0}-\sqrt{\frac{p}{D_0}}\right)^2 exp\left(-\frac{L}{f(mt)}\sqrt{\frac{p}{D_0}}\right)\right\}} \qquad (29)$$

Taking inverse Laplace transform of equation (29) and using transformations (16), (7) and (5), the analytical solution of advection-dispersion equation in one-dimension longitudinal finite domain, which is reported by van Genuchten and Alves (1982), is

$$c(x,T) = C_0 F_3(x,T)/F_4(x,T) \qquad (30)$$

where

$$F_3(x,T) = \frac{u_0}{(u_0+v_0)} exp\left[\frac{(u_0-v_0)\frac{x}{f(mt)}}{2D_0}\right] erfc\left\{\frac{\frac{x}{f(mt)}-v_0T}{2(D_0T)^{1/2}}\right\}$$

$$+\frac{u_0}{(u_0-v_0)} exp\left[\frac{(u_0+v_0)\frac{x}{f(mt)}}{2D_0}\right] erfc\left\{\frac{\frac{x}{f(mt)}+v_0T}{2(D_0T)^{1/2}}\right\}$$

$$+\frac{u_0^2}{2\gamma_0 D_0} exp\left[\frac{u_0\frac{x}{f(mt)}}{L}-\gamma_0 T\right] erfc\left\{\frac{\frac{x}{f(mt)}-u_0T}{2(D_0T)^{1/2}}\right\}$$

$$+\frac{u_0^2}{2\gamma_0 D_0}\left[\frac{u_0\left(2L-\frac{x}{f(mt)}\right)}{D_0}+\frac{u_0^2 T}{D_0}+3+\frac{u_0^2}{\gamma_0 D_0}\right]exp\left[\frac{u_0 L}{D}-\gamma_0 T\right]erfc\left\{\frac{\left(2L-\frac{x}{f(mt)}\right)+u_0 T}{2(D_0 T)^{1/2}}\right\}$$

$$-\frac{u_0^3}{\gamma_0 D_0}\left(\frac{T}{\pi D_0}\right)^{1/2} exp\left[\frac{u_0 L}{D_0}-\gamma_0 T-\frac{1}{4 D_0 T}\left(2L-\frac{x}{f(mt)}+\gamma_0 T\right)^2\right]$$

$$+\frac{u_0(v_0-u_0)}{(v_0+u_0)^2} exp\left[\frac{(v_0+u_0)\frac{x}{f(mt)}-2v_0 L}{2D_0}\right] erfc\left\{\frac{\left(2L-\frac{x}{f(mt)}\right)-v_0 T}{2(D_0 T)^{1/2}}\right\}$$

$$-\frac{u_0(v_0+u_0)}{(u-u_0)^2} exp\left[\frac{(v_0-u_0)\frac{x}{f(mt)}-2v_0 L}{2D_0}\right] erfc\left\{\frac{\left(2L-\frac{x}{f(mt)}\right)+v_0 T}{2(D_0 T)^{1/2}}\right\}$$

$$F_4(x,T)=1-\frac{(v_0-u_0)^2}{(v_0+u_0)^2} exp\left[\frac{-v_0 L}{D_0}\right]$$ and

$$v_0=u_0\left(1+\frac{4\gamma_0 D_0}{u_0^2}\right)^{1/2},\quad T=\int_0^t \frac{dt}{Rf(mt)}.$$

Numerical Example and Discussions

The concentration distribution of the present problem in both cases is obtained for finite domain.

The concentration values are evaluated from the analytical solutions described by equation (26) for uniform input and equation (30) for increasing input in a finite domain $0 \le x(L) \le 10$ (m).

The other input parameters are considered as: L=10.0 (m), $C_0 = 1.0$, $D_0 = 1.29$(m^2/day), $u_0 = 1.05$ (m/day). In addition to these, $m = 0.1(\text{day}^{-1})$, $\gamma_0 = 0.04$ and $R = 1.37$ have been chosen. The solutions are computed for different time t (day) $= 2.1$, 2.7 and 3.4.

The Figures (1) and (3) are drawn for an increasing function $f(mt) = (1+mt)^2$. Comparison have been done between decreasing $f(mt) = exp\,(-mt)$ and increasing functions $f(mt) = (1+mt)^2$, at time $t = 2.7$ (day) for uniform and increasing input concentration are shown in Figures (2) and (4), respectively.

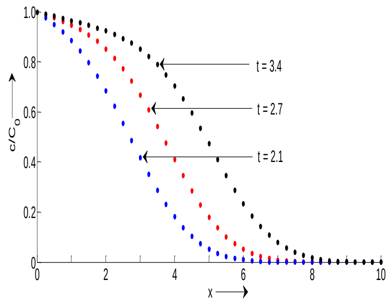

Fig. 1. Distribution of solute concentration of increasing function for uniform input.

In Fig. (1), the concentration distribution behavior for different time at particular position are different and increases with increasing time where as Fig. (3) is drawn for increasing input concentration at same time and behavior of the concentration at the particular position are increasing nature and is higher for higher time.

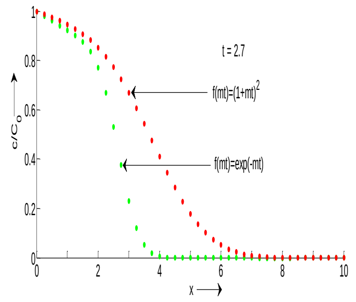

Fig. 2. Comparison between increasing and decreasing function for uniform input.

Figs. (2) and (4) shows, solute concentrations are higher for increasing function than decreasing function at that time where as decreasing function decreases rapidly in comparison to increasing function at same position. The values of concentrations are coinciding after certain position in all figures. Decay term vary inversely with dispersion coefficient, i.e., when dispersion increases then decay term decreases and vice-versa.

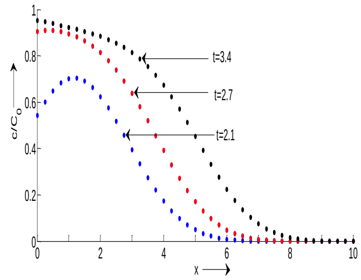

Fig. 3. Distribution of solute concentration of increasing function for increasing input.

The time dependent behavior of solutes in subsurface is of interest for many practical problems where the concentration is observed or needs to be predicted at fixed positions and solute transport involving sequential first order decay reactions frequently occurs in soil and groundwater systems. The accuracy of the numerical solutions are validated by direct comparison with the analytical solutions (26) and (30).

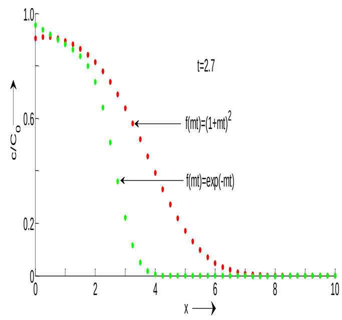

Fig. 4. Comparison between increasing and decreasing function for increasing input.

Conclusions

Analytical solutions are obtained for uniform and increasing input source. Advection-dispersion equation is considered one-dimensional and porous domain is finite. The solute dispersion parameter is considered temporally dependent. Retardation coefficient and first order decay term are also considered. Flow velocities and hydrodynamic dispersion coefficients are key parameters for description of fluid and solute transport in porous media. Analytical solutions of advection-dispersion equation are of fundamental importance in understanding and describing physical phenomena of the problems, which is usually difficult to generate through numerical calculations.

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