



## Thermal Engineering

Elixir Thermal Engg. 33 (2011) 2287-2290

Elixir  
ISSN: 2229-712X

# Thermal conductivity model for nanofluid based on static and modified Brownian motion model

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## ARTICLE INFO

### Article history:

Received: 24 February 2011;

Received in revised form:

22 March 2011;

Accepted: 30 March 2011;

### Keywords

Thermal conductivity,  
Particle volume fraction,  
Modified Brownian motion,  
Particle critical size,  
Nanofluids viscosity,  
Nanolayer thickness.

## ABSTRACT

This paper proposes a new nanofluid thermal conductivity analytical model based on the combination of static and modified Brownian motion mechanism. This is applicable for spherical nanoparticles, the particle volume fraction of 0.005, and critical size nanoparticles. This model is compared with  $Al_2O_3$ /water and CuO /water based nanofluid using existing thermal conductivity models and the experimental results in the open literature. This model deviates 2-5% with the existing Brownian motion theoretical model and experimental results. It is found that the Brownian motion contribution is significant only when the particle size is less than that of critical size and nominal particle volume fraction. It concludes that higher the particle volume fraction leads to lowering the Brownian motion velocity of particles in base fluids resulting degrading the nanoconvection.

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## Introduction

Nanofluid is a combination of suspended nanosized particles and base carrier fluid. The thermal conductivity of heat transfer fluids contributes to the development of energy-efficient heat transfer equipments. Since all other cooling options have been exhausted, nanofluid is the only option remaining with the possibility of increased heat transfer capability of the present systems. Therefore developing a novel heat transfer fluids becomes an urgent need in order to meet out the high capacity cooling requirements. Choi (1995) developed a novel fluids with suspended nanoparticles in the base fluid and named as nanofluid. Choi (1995) reported that the nanofluids have superior thermal properties to the conventional fluids. This is due to large surface area, larger number density as their size decreases, and higher surface potential to absorb and transfer heat. Nanofluids are proposed for a wide variety of industries, ranging from transportation to energy production and medical applications. The theoretical models to predict thermal conductivity of heat transfer nanofluid can be grouped into two models, such as static model and dynamic model. The static model accounts for the particle shape, nanolayer thickness and thermal conductivity of base fluids and particle volume fraction. The dynamic model accounts for the movement of particles, particle size, concentration and temperature.

### Classical models

The conventional Maxwell model was developed by using potential theory to predict the effective thermal conductivity of micro/millimeter sized particle at low volume fraction Maxwell (1873). It has the limitations of spherical shape particles and statistically homogeneous medium. Hamilton –Crosser (1962) is

the modification of Maxwell model with the inclusion of empirical shape factor  $n=3/\psi$  for spherical and cylindrical medium shapes. Where  $\psi$  is sphericity, defined as the surface area of sphere with the volume equal to that of the particles. This is valid for micrometer and millimeter sized particles. Bruggeman model (1999) is based on mean field approach to show the effect of interaction among the randomly distributed particles and binary homogeneous spherical nanoparticles. It has no limitation of particle volume fraction. This model considered the interaction among the particles. This model matches well Maxwell model at low volume fraction. Xuan *et al.*, (2000), is the Wasp model which is a macroscopic system model. This model was developed by renovating HC model (1962) with empirical shape factor is equal to one. It fits with the Maxwell model but not specified the particular shape of nanoparticles.

The classical models were found to be unable to predict the anomalously high thermal conductivity of nanofluids. This is because they do not include the effect of temperature dependant nanofluid, effects of particle size, interfacial layer of the particle fluids, nanoparticles cluster / aggregate and Brownian motion of particles. It is reported that the conduction is the mode of heat transfer for anomalous thermal conductivity of nanofluids.

### Existing conduction models

#### Static models

These models were formulated by assuming that the nanoparticles are stationary in the base fluids and thermal transport is due to conduction. These models were developed by incorporating the other influential parameters which did not included into the classical models. Maxwell- Garnett (1904) presents Maxwell – Garnett's model by considering the

nanoparticles are isolated in the base medium and there is no interaction between the nanoparticles. This model was developed based on the effective medium theory and two components systems of spherical particle and base fluids particles. Pak and Choi (1998) developed a thermal conductivity model under the assumptions that the convective heat transfer enhancement is mainly due to dispersion of suspended nanoparticles. From the literature review of static model, there are eight influencing parameters that affect the thermal conductivity of nanofluids: particle volume fraction, particle material, particle size, particle shape, base fluids, temperature, effect of interfacial layer, base fluid thermal conductivity and nanolayer. Moreover the limitation of particle size, and optimum particle volume fraction were not dealt in depth in calculating effective thermal conductivity.

#### Dynamic models

These models are based on the fact that the nanoparticles have lateral and random motion of particles in the base fluids. The movement of particles causing the collision between nanoparticles, nanoconvection which leads to the enhanced thermal conductivity Xuan and Li (2003). Das (2003) is the first to develop a dynamic model that takes into account the effects of Brownian motion. This model cannot exactly predict the strong temperature dependant nanofluids thermal conductivity data obtained by Das (2003) and Patel (2003) and Jang and Choi (2004) presented model which is based on conduction and convection caused by Brownian motion. This model takes the effect of four important modes: Collision between the base fluids molecules which represents the base fluids thermal conductivity: Thermal diffusion in the base fluids with the effect of Kapitza resistance: Collision between the nanoparticles due to Brownian motion: Thermal interactions of dynamic or dancing nanoparticles. This model is able to predict the size dependant, temperature dependant and concentration dependant. They claimed that the nanoconvection is the key role in enhancing thermal conductivity of nanofluids. Bao Yang (2008) developed a model based on the diffusive conduction and Brownian model and reported that the Brownian motion is the cause for improved thermal conductivity of nanofluids. Ravikanth S Vajjha (2009) investigated the effect of Brownian motion along with conduction model and reported the Brownian motion contributes to the enhanced thermal conductivity of nanofluids. This model failed to include the effect of viscosity of fluids and the critical radius of particles. Murshed (2009) presented a model based on the combination of static and dynamic mechanism. This model is formulated by modified Brownian motion and DLVO potential. The first term of this model includes the particle volume fraction, nanolayer together with particle size and the second term indicates interaction between the particles. the third term includes the modified Brownian motion and surface chemistry. They suggested this model is valid for the volume fraction of 0.005. They considered the interfacial layer as a separate compound.

Keblinski (2005) developed a model based on conduction model and reported the conduction due to particles interaction the model for enhanced thermal conductivity. Willam Evans (2006), Kumar (2004), Shukla, Vijay K Dhir (2005), Chu Nie, Marlow, Hassan (2008), Jurij Avsec (2007), and Yu, Choi, (2003) developed a effective thermal conductivity model based on the movement of nanoparticles. They suggested the Brownian motion is the cause for enhanced thermal conductivity. Their model did not include the effect of very smaller particle size.

Choi, (2003) suggested that the nanolayer impact is significant for small particles ( $r - h$ ). They claimed that the three to eight fold increase in the enhancement of thermal conductivity when the particle size is less than the critical size (10nm). Therefore a new optimum level is reached by replacing more volume fraction of particles with the particles of critical size. Shukla R K, Vijay K Dhir (2005) model was modified by Chandrasekar (2009) and developed the model noted in Eqn. 1. They included the cumulative effect of nanolayer, Brownian motion, and particles volume fraction. It has two terms; first term represents the contributions to the macroscopic Maxwell model: the second term represents the contribution to the Brownian motion of nanoparticles. They reported that the Brownian motion is enhanced when the particle size is decreased. Shukla Vijay K Dhir (2005) and Chandrasekar (2009) presented the nanofluid thermal conductivity model based on the Weber formula and the influence of the effect of particle shape, nanolayer thickness and Brownian motion of nanoparticles.

The second term of Eqn.1 model contains the influencing parameters on Brownian motion such as particles volume fraction, particle size and viscosity of base fluids. According to the second term of model Shukla Vijay K Dhir (2005) the possibility of Brownian movement is limited. Because the viscosity of nanofluids increases when the particle volume fraction is increased. As a result the particles hardly move in the base fluids according the linear viscosity model over the particle volume fraction. Brownian motion is suppressed when particle volume fraction is increased.

$$\frac{k_{eff}}{k_f} = \left[ \frac{k_p + (n-1)k_f + (n-1)(1+\beta)^3 \phi(k_p - k_f)}{k_p + (n-1)k_f - (1+\beta)^3 \phi(k_p - k_f)} \right] + \frac{C\phi(T - T_o)}{\mu k a^4} \quad (1)$$

Therefore the optimum particle volume fraction is essential for significant motion of particles and little agglomeration of nanoparticles. Lee (1999) reported that when particle size decreases, the surface area to volume ratio is three orders of magnitude greater than that of larger size particles. Due to that dramatic enhancement of thermal conductivity is expected because of desirable situation of higher Brownian motion velocity.

#### Proposed model

The proposed model is based on static mechanism and modified Brownian motion proposed by Murshed (2009) and Shukla, Vijay K Dhir (2005).

The inter particles separation distance  $d_s$  was not taken as the particles assumed is not complex particles as mentioned in Murshed (2009).

The mixture contains nanoparticles, interfacial layer and base fluids. The solid and liquid with nanolayer are in thermal equilibrium and Kapitza resistance was not taken into account as the particle volume fraction is low.

$$\frac{k_{eff}}{k_f} = \left[ \frac{k_p + (n-1)k_f + (n-1)(1+\beta)^3 \phi(k_p - k_f)}{k_p + (n-1)k_f - (1+\beta)^3 \phi(k_p - k_f)} \right] + \frac{1}{2} \rho_c \rho_f k_f \left( \sqrt{\frac{3k_B T (1 - 1.5\gamma^3 \phi)}{2\pi \rho_f \gamma^3 r_p^3}} \right) \quad (2)$$

The proposed model, Eqn.2, has two terms: first term represents the contribution due to the macroscopic model; the second term represents the effect of modified Brownian motion with the inclusion of nanolayer thickness  $h$  and particle radius  $r_p$ . Where  $\gamma = 1 + h / r_p$ .  $\beta$  is the ratio of nanolayer thickness to particle radius.

### Viscosity, Brownian motion velocity, and particle radius relation

Hiemenz, (1986).reported the particle sedimentation velocity and the particle size can be related by using the Stokes law.

$$V = \frac{2R^2}{9\mu}(\rho_p - \rho_l)g \quad (3)$$

Where V is the sedimentation velocity of particles: R is the radius of the spherical particles::  $(\rho_p - \rho_l)$  is the density difference between the nanoparticles and the base fluids: g is the acceleration due to gravity .According to this equation, the sedimentation velocity is minimum when a) the particle size is reduced b) viscosity of the base fluids is increased, and c)the density difference is reduced. Therefore particle size plays a major role in enhancing suspension stability. As per the colloid chemistry, when the particle size is reduced to critical size, the sedimentation velocity is stopped. This is due to the particles start moving when sedimentation and diffusion are in equilibrium. There is a negative impact when the critical sized particles dispersed as the particles are highly charged. This state of particles has the tendency to aggregate and settle out easily. Therefore the key to prepare stable nanofluids is to use smaller nanoparticles and to prevent the aggregation of the small nanoparticles simultaneously. Many nanofluids viscosity model revealed that the viscosity increases when the particle volume fraction is increased. The Eqn.4 is commonly used for calculating Brownian motion velocity.

$$u_p = \frac{2k_B T}{\pi \mu d_p^2} \quad (4)$$

This relation indicates the Brownian motion velocity increases when viscosity is decreased and the particle size is decreased. Lee (2007) discussed various parameters on nanofluids thermal conductivity and reported that Brownian motion also contributes to the enhanced thermal conductivity of nanofluids .They related the random motion velocity  $C_{BM}$  of particles with the diffusion

coefficient  $D_o$  and dynamic viscosity as  $C_{BM} = \frac{D_o}{l_{BF}}$  , By

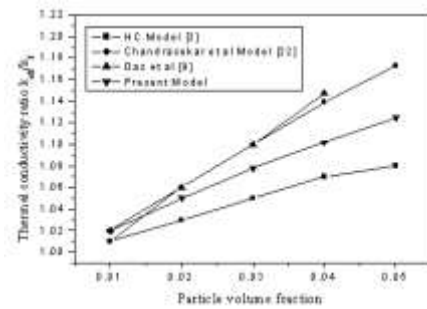
Stokes -Einstein <sup>25</sup> law,  $D_o = \frac{K_B T}{3\pi\mu d_{nano}}$  , where  $D_o$  is the

Einstein diffusion coefficient ( $m^2/s$ ), T is the temperature,  $l_{BF}$  is the base fluid mean free path and  $d_{nano}$  .  $K_B$  is the Boltzmann constant. According to this equation the Brownian motion velocity is higher when the viscosity is lower and particles diameter is smaller. Uhlenbeck (1930) presented the random velocity of nanoparticles was developed as  $V \propto T^{0.5} / d_p^{1.5}$  . As per this, the particle motion velocity V is inversely proportional to the particle radius  $d_p$  .Therefore an optimum effective particle size is needed for achieving the significant Brownian motion and kinetic stability.

### Results and Discussion

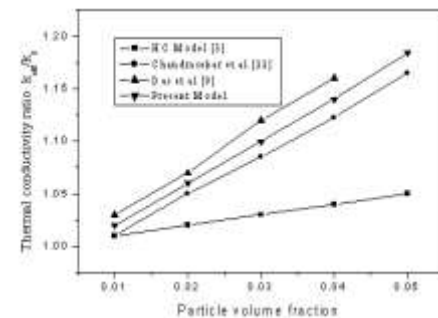
#### Effect of nanolayer thickness

The proposed model is validated by the experimental results of Das S K, (2003) and they used 38nm of particle size of  $Al_2O_3$  / water and CuO /water nanofluids. It is seen from Figs.1 and 2 indicate the theoretical model closely approach the experimental results.

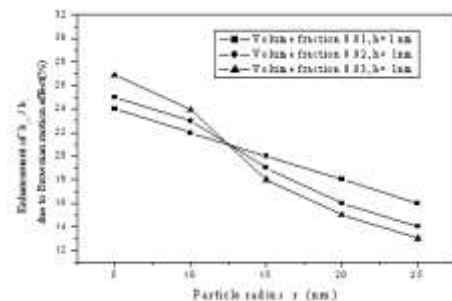


**Fig 1. Variation of thermal conductivity ratio of  $Al_2O_3$  /water nanofluids with particle volume fraction.**

The deviation between them may be due to particle size and temperature of nanofluids and the dispersion technique. It is clear that there is wide deviation between the present model and the classical HC model. This is because of the HC Model did not consider the size of the particle, nanolayer thickness and effect of nanoconvection into account for calculating thermal conductivity. Therefore it is clear that the particle volume fraction, shape and thermal conductivities are the only depending factors for enhanced thermal conductivity of nanofluids. Also the random motion of the particle at low volume fraction, temperature, viscosity and critical size of particles also influence the enhanced thermal conductivity of nanofluids. The results in Fig.2 can be interpreted as the maximum enhancement is 20% at the particle volume fraction of 0.05 is the sum of effect of particle shape, the effect of nanolayer thickness and the effect of Brownian motion of nanoparticles.



**Fig 2. Variation of Thermal conductivity ratio of CuO /water nanofluids with particle volume fraction**



**Fig 3. Variation of particles size with thermal conductivity ratio due to Brownian motion**

#### Effect of particle size on Brownian motion

Fig.3 represents the effect of particle size on contribution to the enhancement of thermal conductivity of nanofluids. It shows that the Brownian motion contribution is higher at lesser particles size when particle volume fraction is fixed. It is seen that when particle volume fraction is increased the effect of Brownian motion is significant. From fig.3 the changes in

Brownian motion is in the range of 10 nm to 15nm and this range is the critical size. The critical size of particles can be defined as the size at which the brownian motion is considerable when the particle volume fraction and temperature are fixed. The Brownian motion is suppressed when particle size and particle volume fraction are increased. This may be due to the fact that the Brownian motion velocity is inversely proportional to the particles diameter. Moreover the viscosity is increased when particles loading are increasing.

### Conclusion

This investigation presents a new thermal conductivity model for nanofluids. This model is based on the static mechanism and the effect of modified Brownian motion based on the critical radius of particles. This model is validated by the existing experimental results for  $\text{Al}_2\text{O}_3/\text{water}$  and  $\text{CuO}/\text{water}$  nanofluids and found that it deviates in a narrow range. It is found that the effect of nanoconvection is higher due to the critical sized particle. This model deviates 2-5% with the existing Brownian motion theoretical model and experimental results. This model is applicable for particles with equal or less than the critical size and valid for the optimum volume fraction. Further work is needed to quantitatively optimize the particle volume fraction and include the effect of surface charge state while particle size is decreased.

### Nomenclature

$c_p$  specific heat, J/kgK,  
 $T$  Temperature, K  
 $T_o$  Limiting temperature.K  
 $N$  Shape factor  
 $k$  Thermal conductivity, W/mK

### Subscripts

$F$  Base fluids  $eff$  Effective  
 $nf$  Nanofluids  $p$  Particle

### Greek letters

$\phi$  Volume fraction (%)  
 $\rho$  Density, kg/  $\text{m}^3$   
 $\mu$  Dynamic viscosity, kg/ $\text{m}^2\text{s}$

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