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An innovative aid for weibull analysis

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ABSTRACT

Reliability assessment popularly known as Weibull analysis is an exercise primarily concerned with reliability estimation and prediction with which failure process of a system can better be analyzed. Many situations like modeling failure processes, decision of effective maintenance policy, estimation of maintenance float factors, downtime assessment and reduction greatly rely on the parameters of the life distributions fitted to the failure data. As the Weibull distribution involves transcendental equation, determining parameters is quite difficult task. To this effect, a Nomograph is developed and presented in this paper to closely determine the Weibull parameters. The paper articulates the development of Nomograph and procedure for its implementation. Results and discussions carried out proves statistically that the results obtained using the Nomograph are better than the commonly used its counterparts.

Introduction

Maintenance management, a combination of various activities to minimize the deleterious effects of failure, aims to retain equipment in operable condition or restore to operable condition. In order to achieve the two primary goals of down time control and productivity control, issues related to maintenance systems caught the attention of researchers and practitioners. The systems such as Breakdown Maintenance (BDM), Preventive Maintenance (PM), Condition-based Maintenance (CBM), Reliability Centered Maintenance (RCM) and Total Productive Maintenance (TPM) are identified with considerable attention in literatures. The starting point for all these techniques is failure data analysis which involves identifying the trends in product or system failure and using which one can attempt to correct them or compensate for them, thereby improving the reliability. In Reliability engineering, determination of burn-in plays a key role in provisions of warranty. As pointed out by Jenab *et al.* (2010), fitting probability distributions, like Weibull distribution to data related to electronic components, is an essential activity in warranty forecasting model and lifetime analysis.

Notations

ANN : Artificial Neural Network

BDM : Breakdown Maintenance

β : Shape parameter of Two- parameter Weibull Distribution

PM : Preventive Maintenance

C.V : Coefficient of Variation

E(T) : Expected value of T

f(t) : pdf of random variable T

F(t) : Distribution function of t

h(t) : hazard rate function

ln : Natural logarithm

MLE : Maximum Likelihood Estimator

μ : mean of a random variable

pdf : Probability density function

RCM : Reliability Centered Maintenance

R^2 : coefficient of determination

TPM : Total Productive Maintenance

TTF : Time to failure

R.V : Random Variable

σ : Standard deviation of a random variable

t : specific value of R.V, T

T : R.V TTF

θ : Scale parameter of Two- parameter Weibull Distribution

V(T) : Variance of T

WPP : Weibull Probability Plot

X : ideal value of the parameter

Y : predicted value of the parameter

Literature Review

The lifetime distribution is named “Weibull distribution”, after a Swedesh mechanical engineer Walodie Weibull who published perhaps the first paper on it in 1939. The probability density for Weibull family is given in the Equation (1)

$$f(t) = \beta \frac{t^{\beta-1}}{\theta^{\beta}} e^{-(t/\theta)^{\beta}}, \quad t \geq 0, \quad \beta > 0, \quad \theta > 0 \tag{1}$$

where, β and θ are shape and scale parameters respectively.

Weibull analysis is primarily concerned with reliability estimation and prediction of the system under investigation. Fitting distribution to the failure data with appropriate parameters enables to decide right maintenance strategy. Pandit (1978) and Ramalingam *et al.* (1978) carried out investigations pertaining to the statistical analysis of reliability using data available in manufacturing processes. The authors found that

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normal, Weibull and exponential distributions can be fitted to tool life data for turning, milling, drilling and cutting operations. Lowe and Lewis (1983) estimated maintenance float factors fitting Weibull distribution, using Weibull probability plot (WPP). Wayne (1985) observed while carrying out reliability assessment on mechanical components that most of the bearing life data from the field is adequately represented by Weibull distribution with shape parameter 1.5. Pan *et al.* (1986) proposed a mathematical model for predicting the system reliability of cutting tools for an automated machining system with various carbide parts using Weibull distribution. Abdul-Nour *et al.* (1998) while implementing RCM in an aluminium plant found that the amount and type of maintenance applied depend strongly on the life model fitted of the equipments. Beichelt (2001) analyzed a policy for optimal scheduling replacement intervals of technical systems on the basis of maintenance cost parameter. The author validated the policies proposed using Raileigh (a specific case of Weibull distribution) and Maxwell distributions. Lai *et al.* (2003) developed a general model to describe bathtub-shaped hazard-rate function. An advantage of the model is that the model parameters can be estimated easily based on a Weibull probability plot (WPP) that serves as a tool for model identification.

Abbasi *et al*. (2008) remarked that Weibull distribution plays important role in reliability studies and they have many applications in engineering. According to the authors, estimating parameters of three-parameter Weibull distribution is quite difficult, and to that effect the authors developed an approach that takes the advantage of artificial neural networks (ANN) exploiting the concept of the moment method to estimate Weibull parameters. Further, it was demonstrated the power of the proposed ANN-based method by conducting extensive simulation study and compared the results with maximum likelihood estimator (MLE) and two moment-based methods. Yeh *et al.* (2009) proposed a maintenance scheme for leased equipment, using hazard-rate reduction method and derived an optimal PM policy that minimizes expected total cost. The authors developed an efficient algorithm to derive the optimal PM policy, and a closed-form solution was obtained for the case where the lifetime distribution of the equipment is Weibull. This algorithm was applied to various numerical examples.

Weibull Distribution

The mean, variance, distribution function and hazard rate for the Weibull distribution are given in Equations (2), (3), (4) and (5) respectively.

$$E(T) = \theta\Gamma(1+(1/\beta)) \qquad (2)$$

$$V(T) = \theta^2\left[\Gamma(1+(2/\beta)) - \Gamma(1+(1/\beta))^2\right] \qquad (3)$$

$$F(t) = 1 - e^{-(t/\theta)^{\beta}} \qquad (4)$$

$$h(t) = \beta\,\frac{t^{\beta-1}}{\theta^{\beta}} \qquad (5)$$

The uniqueness of Weibull model is that a family of curves can be generated for different values of shape and scale parameters, which can be used to cover the entire bathtub profile piecewise and therefore, they can be used to describe any failure process. The major bottleneck in the reliability assessment program is determining these parameters. The following sections briefly discuss four methods commonly used for estimating these parameters.

Weibull Probability Plot (WPP)

This technique involves plotting time to failure, T against distribution function F(t) using appropriate scale. Equation (6) is used for plotting and this can be obtained by taking natural logarithm on Equation (4) two times and rearranging.

$$\ln\ln\{1/[1-F(t)]\} = \beta\ln(t) - \beta\ln(\theta) \qquad (6)$$

The graphical representation of Equation (6) is known as Weibull Probability Plot (WPP) i.e., ln.ln(.) on ordinate and lnt on Abcissa. If the collected failure data fits to a straight line after the required transformation as suggested by the Equation (6), it is inferred that the life model follows Weibull distribution. Then, correspondingly the parameters can be evaluated from the plot.

Maximum Likelihood Estimator (MLE)

This method involves construction of likelihood function for the given sample observation using probability density function (pdf) possessing the parameters to be estimated. The values of the parameters are calculated using the property that the likelihood function must be the maximum.

Method of Moments

This method uses the property that theoretical moments are equal to the observed moments. Equating as much number of moments as the number of parameters to be estimated and thereupon solving we get the parameters.

Chi-square Test

This method for a set of proposed parameters computes expected frequency which in turn is compared with the observed frequency. The deviations are summed up as a single numeric value referred as observed total deviation. If this observed deviation is lesser than the standard chi-square value for a desired significance level, the set of parameters proposed are accepted, else the parameters are adjusted and the procedure of testing is repeated. Statgraphics plus Version 6.0 incorporates this facility is used in this study for the comparative analysis.

The Problem on Hand

The two parameters viz. shape parameter β and scale parameter θ can be related through mean (μ) and variance (σ^2) but the expressions are unfortunately transcendental, which introduces difficulty in establishing a closed form solution. Xie (2002) mentioned that among different methods available to estimate the parameters of Weibull distribution, WPP and MLE are used commonly. Methods like MLE and method of moments will not yield closed form solutions. As reported by William and Douglas (1980), the method of moments does not always generate an estimate that is compatible with the knowledge of the situation. Therefore, graphical approaches perhaps gained their importance. Jiang and Murthy (1995) observed that often when the failure data are plotted on WPP yields a curve rather a straight-line whereby the parameter estimation looses its accuracy.

The estimation of Weibull parameters not only models the failure process but also helps in deciding effective maintenance policy, estimation of maintenance float factors, management of downtime etc. Therefore, there was considerable motivation to develop a suitable decision aid to determine Weibull parameters β and θ more closely and also easily. This motivation gave an impetus to develop a Nomograph, the details of which are presented in the subsequent sections of the paper.

Development of Nomograph

Using Equations (2) and (3) as it is and solving them for β and θ is difficult. It was found logical to use the ratio of standard deviation to mean which is the Coefficient of Variation (CV). By doing so, we gain an advantage of eliminating the scale parameter θ and the resulting relation is given by Equation (7).

$$\frac{\sigma}{\mu} = \frac{\sqrt{\Gamma(1+(2/\beta)) - \Gamma(1+(1/\beta))^2}}{\Gamma(1+(1/\beta))} \tag{7}$$

Now the problem reduces to uni-variate, i.e., CV over β. But, still, the equation remains transcendental. Therefore, an attempt was made to overcome this difficulty by means of a graphical approach. This resulted in the development of a very useful Nomograph as an aid, the details of which are as follows.

The Nomograph as a Decision Aid

The Equation (7) represents a functional relationship between CV and β. The graphical representation of this relationship results in a Nomograph. For a given β obtained from the Nomograph, scale parameter, θ can be obtained using the Equation (8) which is the rearrangement of Equation (2).

$$\theta = \frac{\mu}{\Gamma(1+(1/\beta))} \tag{8}$$

In order to develop the Nomograph, it was necessary to have data ideally following Weibull distribution. For this purpose, twenty data sets, each consisting of 5000 data points were simulated using Statgraphics Plus Version 6.0. Care has been taken to nullify the effect of seed. (Bloch and Geitner, 1993) found that most of the engineering problems involve shape parameter greater than 0.5 while, most of the wear processes assume β = 4.0 which closely represents normal distribution. Hence, the twenty sets were generated having β in the range of 0.5 to 6.0. The salient measures of these twenty sets generated are shown in Table 1.

Using these twenty sets of data, the graph obtained with CV along the Ordinate and β along the Abscissa is plotted and the Nomograph obtained is as shown in Figure 1. The Nomograph can be now used to estimate β and θ for the given set of data values. For the calculated C.V for the given data β can be obtained from the Nomograph. Then using Equation (8), the parameter θ can be obtained.

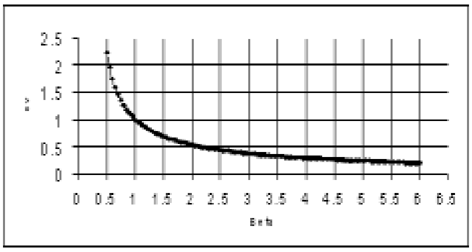


Figure 1. Nomograph

Results and Discussion

For the twenty sets of data generated β and θ were calculated using the three different methods viz. Weibull probability plot, Chi-square test with 5% confidence using Statgraphics Plus Version 6.0 and the Nomograph developed. The values obtained for β and θ are referred as β_1, β_2 and β_3, and θ_1, θ_2 and θ_3 respectively for these methods which are presented in Table 2. To ascertain the merit of a method, a comparative analysis has been carried out as detailed in the following sections. The mean squared error of each of the methods in the estimation of β and θ is shown in the last row of the Table 2.

The smaller the mean squared error value clearly reveals the better performance of the Nomograph. The superior performance is more significant in the estimation of β. The results obtained by three methods were further subjected to scatter plots to strengthen the investigation.

Scatter Plots

In the scatter plot, the ideal value of the parameter used in simulation, X is plotted against the value obtained using various methods, Y. The points lying on this line reveals that the values obtained are accurate and the departure will indicate the degree of closeness of values obtained. Therefore, the straight line Y = X is known as 'accuracy line'. The coefficient of determination (R^2) conforms to the adequacy of the fit. The results of the scatter plots on β obtained using all the three methods are shown in Figures 2 through 4 with the respective regression equations. Similar analysis was carried out for θ as well, which are shown in Figures 5 through 7.

It is quite evident from the Figures 2 through 4 that the regression model particularly in the case of the Nomograph is fitting well, as the coefficient of determination is unity. The Y intercepts for all the cases are low but the Nomograph in particular, possesses the least value. The slope of the regression line approaches unity indicating that the parameters, especially β, obtained from Nomograph is expected to be more accurate. The results and discussion presented in the preceding sections provided sufficient encouragement to use Nomograph to evaluate β and θ for Weibull distribution.

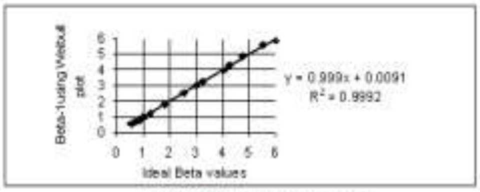


Figure 2. Scatter plot analysis of β_1

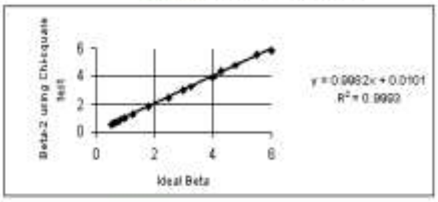


Figure 3. Scatter plot analysis of β_2

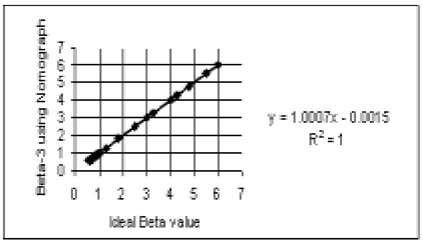


Figure 4. Scatter plot analysis of β_3

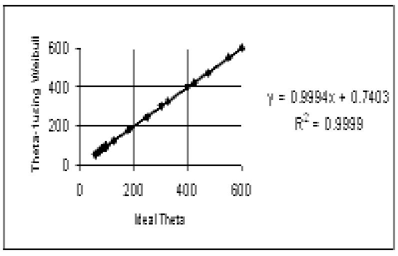


Figure 5. Scatter plot analysis of θ_1

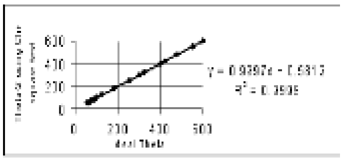


Figure 6. Scatter plot analysis of θ2

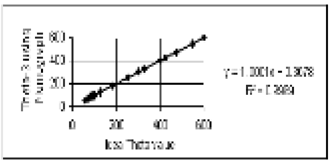
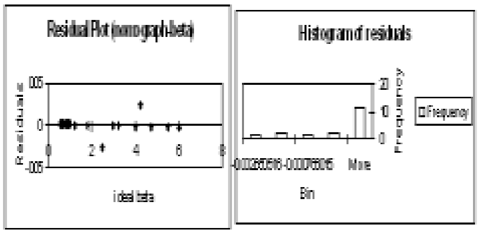


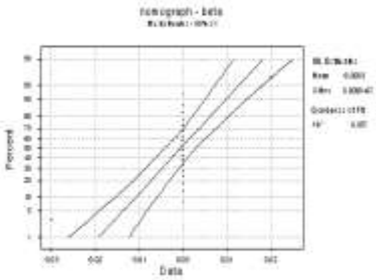
Figure 7. Scatter plot analysis of θ1

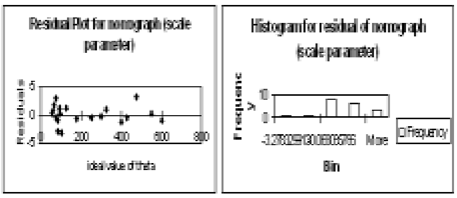
Model Adequacy

To ascertain the statistical adequacy of the model, F and t-tests for significance of regression and residual analysis were carried on the parameters evaluated using Nomograph. Minitab software was used for this purpose. The details of F and t-tests along with residual analysis on shape and scale parameters obtained using the Nomograph, are presented in Table 3 and Table 4 respectively. The results of the tests conform to the adequacy of the model.

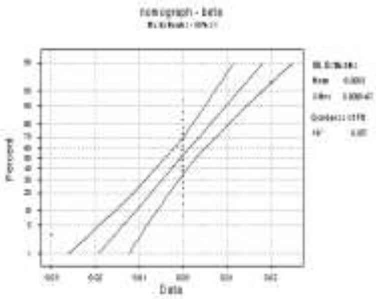


Normal plot for residuals of shape parameter using Nomograph





Normal plot for residuals of shape parameter using Nomograph



Conclusions

Weibull distribution plays key roles in maintenance management, especially in the area of Reliability Centered Maintenance (RCM). Moreover basic failure processes can be represented successfully by family of Weibull distribution and hence Weibull distribution is effective in life modeling with its two parameters viz. shape parameter β and scale parameter θ. The commonly used methods like WPP, chi-square tests etc. do not provide closer solution. The Nomograph developed in this study provides closer solution to the parameter estimation and it is easy to implement due its simplicity. The values obtained using the Nomograph were found to be closer as compared to its counterparts and the consistency in accuracy was statistically investigated and found satisfactory.

References

1. Abdul-Nour, G., Beaudoin, H., Ouellet, P., Rochette, R. Lambert, S.: A reliability based maintenance policy: a case study. Computers in Industrial Engg. 35, 591--594 (1998).
2. Abbasi, B., Rabelo, L., Hosseinkouchack, M.: Estimating parameters of the 3 parameter Weibull distribution using ANN. European Journal of Industrial Engg. 2, 428--445 (2008).
3. Beichelt, F.A.: Replacement policy based on limiting the cumulative maintenance cost. International Journal of Quality & Reliability Management. 18, 76-83 (2001).
4. Bloch, H.P., Geitner, F.K..: An Introduction to Machinery Reliability Assessment. Van Nostrand Reinhold, New York (1993).
5. Jenab, K., Kazeminia, A., Liu, D.S.: A computational intelligent approach to estimate the Weibull parameters. Int. J. Industrial and Systems Engineering. 6, 62--78 (2010).
6. Jiang, R., Murthy, D.N.P.: Reliability modeling involving two-Weibull distributions. Reliability Engineering & System Safety. 47, 187--98 (1995).
7. Lai, C.D., Xie, M., Murthy, D.N.P.: A modified Weibull distribution. IEEE Transactions on Reliability. 52, 33--37 (2003).
8. Lowe, P.H, Lewis, W.: Reliability analysis based on the Weibull distribution: an application to maintenance float factors. Int. Journal on Production Research. 21, 461--470 (1983).
9. Pan, J.N, Kolarik, W.J, Lambert, B.K.: Mathematical model to predict the system reliability of tooling for automated machining systems. International Journal on Production Research 24, 493--501 (1986).
10. Pandit, S.M.: Data dependent system approach to stochastic tool life and reliability. Transaction of the ASME. 100, 318--322 (1978).
11. Ramalingam, S., Peng, Y.I. Watson, J.D.: Tool life distributions. Part3. Mechanism of single injury tool failure and tool life distribution in interrupted cutting. Journal of Engineering for Industry. 100, 193-200 (1978).
12. Wayne, N.: Weibull analysis of reliability data with few or no failures. Journal of Quality Technology. 17, 141--146 (1985).
13. William, W.H., Douglas, C.M.: Probability and statistics in Engineering and Management Science, John Wieley & Sons, USA (1980).
14. Xie, M., Tang, Y, Goh, T.N.: A modified Weibull extension with bathtub-shaped failure rate function. Reliability Engineering & System Safety. 76, 279--285 (2002).
15. Yeh, R.H., Kao, K., Chang, W.L.: Optimal preventive maintenance policy for leased equipment using hazard rate reduction. Comp. and Industrial Engg. 57, 304--309 (2009).

Table 1. The salient measures of the simulated data

Set	Parameters used for data generation		Measures of life data generated		
	β	θ	σ	μ	CV
1	0.55	55	182.72	95.04	1.92
2	0.60	60	157.43	91.18	1.730
3	0.65	65	146.99	91.63	1.600
4	0.70	70	132.83	88.43	1.500
5	0.75	75	128.63	93.22	1.380
6	0.80	80	112.80	90.12	1.250
7	0.85	85	107.55	89.89	1.200
8	0.90	90	105.52	96.24	1.100
9	0.95	95	101.62	97.81	1.040
10	1.00	100	96.32	97.10	0.990
11	1.25	125	93.72	117.85	0.795
12	1.80	180	91.77	159.82	0.574
13	2.50	250	94.31	221.72	0.425
14	3.00	300	97.89	268.91	0.364
15	3.25	325	98.44	292.40	0.337
16	4.00	400	102.22	361.81	0.283
17	4.25	425	100.62	386.10	0.260
18	4.75	475	103.54	438.17	0.236
19	5.50	550	106.18	508.43	0.209
20	6.00	600	110.56	555.30	0.199

Table 2. Comparison of the values of βs and θs obtained by the three methods

set	Parameters estimated using						Squared error of estimates obtained by					
	WPP		Chi-square test		Nomograph		WPP		Chi-square test		Nomograph	
	β_1	θ_1	β_2	θ_2	β_3	θ_3	β_1	θ_1	β_2	θ_2	β_3	θ_3
1	0.557	56.78	0.556	56.85	0.55	55.74	4.9E-05	3.1684	3.6E-5	3.4225	0	0.5476
2	0.596	60.09	0.596	60.18	0.6	61.03	1.6E-05	0.0081	1.6E-5	0.0324	0	1.0609
3	0.641	67.14	0.647	66.59	0.65	67.06	8.1E-05	4.5796	9.0E-6	2.5281	0	4.2436
4	0.678	69.3	0.687	68.62	0.7	69.87	0.00048	0.49	0.00017	1.9044	0	0.0169
5	0.757	78.29	0.753	78.35	0.75	78.26	4.9E-05	10.8241	9.0E-6	11.2225	0	10.6276
6	0.815	80.24	0.812	80.34	0.8	79.54	0.00023	0.0576	0. 14E-3	0.1156	0	0.2116
7	0.869	83.37	0.863	83.32	0.85	82.47	0.00036	2.6569	0.00017	2.8224	0	6.4009
8	0.888	92.23	0.904	91.72	0.9	91.43	0.00014	4.9729	1.6E-05	2.9584	0	2.0449
9	0.979	96.62	0.976	96.64	0.95	95.45	0.00084	2.6244	0.00068	2.6896	0	0.2025
10	0.994	97.68	1	97.25	1	97.1	3.6E-05	5.3824	0	7.5625	0	8.41
11	1.268	126.89	1.264	126.86	1.25	126.53	0.00032	3.5721	0.0002	3.4596	0	2.3409
12	1.784	180.14	1.8	179.68	1.8	179.65	0.00026	0.0196	0	0.1024	0	0.1225
13	2.546	249.49	2.52	249.94	2.475	249.88	0.00212	0.2601	0.0004	0.0036	63E-5	0.0144
14	3.02	300.95	2.98	301.18	3	300.1	0.0004	0.9025	0.0004	1.3924	0	0.01
15	3.26	326.18	3.28	326.12	3.25	326.34	1.0E-04	1.3924	0.0009	1.2544	0	1.7956
16	3.96	398.99	3.96	399.42	4	399.17	0.0016	1.0201	0.0016	0.3364	0	0.6889
17	4.34	424.00	4.34	423.99	4.275	424.99	0.0081	1	0.0081	1.0201	63E-5	1.0E-04
18	4.88	477.83	4.82	478.28	4.75	478.56	0.0049	8.0089	0.0049	10.7584	0	12.6736
19	5.569	551.25	5.569	550.34	5.5	550.43	0.00476	1.5625	0.00476	0.1156	0	0.1849
20	5.847	599.84	5.847	599.84	6	599.22	0.02341	0.0256	0.02341	0.0256	0	0.6084
Sum							0.04825	52.5282	0.04591	53.7269	125E-5	52.2058
Mean Squared Error (MSE)							0.00241	2.62641	0.0023	2.68635	6.25E-5	2.61029

Table 3. Summary output of shape parameter from the Nomograph

Regression Statistics	
Multiple R	0.9999903
R Square	0.9999806
Adjusted R Square	0.9999795
Standard Error	0.0082309
Observations	20

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	62.77	62.77	926520.3	7.31E-44
Residual	18	0.0012	6.77E-05		
Total	19	62.7712			

t-test

	Coefficients	Std Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	-0.0015	0.0029	-0.523	0.607	-0.0077	0.0046
X Variable 1	1.0007	0.001	962.559	7.31E-44	0.9985	1.0029

Table 4. Summary output of scale parameter from the Nomograph

Regression	*Statistics*
Multiple R	0.99996
R Square	0.99992
Adjusted R Square	0.99992
Standard Error	1.65229
Observations	20

ANOVA

	df	SS	MS	F	Significance F
Regression	1	626956	626956	229648	2.0695E-38
Residual	18	49.1412	2.73007		
Total	19	627005			

t-test

	Coefficients	Std Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	0.36779	0.58973	0.62366	0.5407	-0.8712	1.6068
slope	1.00011	0.00209	479.216	2.1E-38	0.9957	1.0045