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Planar geometry ion acoustic shock waves in electron-positron-ion quantum plasmas

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ABSTRACT

We have studied the formation of quantum ion acoustic shock waves (QIASWs) in a three component unmagnetized plasma, whose constituents are electrons, ions and immobile positrons. The effects of both the dissipation due to the plasma kinematic viscosities and the dispersion caused by the Bohm potential are taken into account. Employing reductive perturbation method, we have obtained the deformed Korteweg-deVries Burger (dKVB) equation for quantum ion acoustic shock wave in planar geometry. From our numerical analysis, we have studied the effect of the quantum parameter H and ion kinematic viscosities (η_{i0}) of the planar dKdVB on different values of the positron concentration P.

Introduction

The study of quantum plasmas has grown rapidly in recent years due to the fact that, both plasma and quantum effects can coexist in nature. Where that happens, quantum effects are excepted to play a significant role on the dynamics of plasmas particles. The much attention on quantum plasma is because of the interesting and important applications in micro and nanoscale systems [1-3], quantum dots and quantum wires [4], dense astrophysical environments [5-6], as well as in laser produced plasmas [7]. Making use of fluid description, some interesting properties of nonspin and spin quantum plasmas were uncovered [8,9].

The Schrödinger-Poisson model, the Wigner-Poisson model and the quantum hydrodynamic (QHD) model, are the three well known mathematical formulations to describe the dynamics of quantum plasmas. The fluid model for plasmas is generalized by QHD model while taking into account the macroscopic variables only (i.e. the density and fluid velocity, the stress tensor and the electrostatic potential). These models have been discussed in detail in Refs. [10,11]. Quantum plasmas obey the Fermi-Dirac distribution leading to the Fermi pressure and new forces arising due to Bohm potential play a vital role [12].

The study of linear and nonlinear ion-acoustic waves has received great deal of attention in recent years, due to the impressive developments in quantum plasmas [13-15]. Many authors have studied the effects of quantum diffraction and Fermi pressure on linear and nonlinear electrostatic waves in dense electron-positron-ion plasmas [16-17]. It is found that Bohm potential leads to the wave dispersion due to quantum correlation of density fluctuations associated with wave-like nature of the charge carries. However, both dispersion and dissipation may play necessary roles in some quantum plasma system. Such vital roles have been studied in different quantum plasma systems for the formation of ion-acoustic shocks (where the dissipation is due to kinematic viscosity) [18-19].

Recently, Sahu and Roychoudhury [18] studied the properties of quantum ion acoustic shock waves taking into account the quantum-mechanical effects for two species plasma (electrons and ions) in both planar and nonplanar geometry. Since in many astrophysical environments there exists a small number of ion along with the electrons and positrons, therefore, it is important to study linear and non-linear behaviour of plasma waves in electron-positron-ion (e-p-i) plasmas. It is therefore of interest to examine the effects of kinematic viscosity as well as the quantum mechanical effects on the formation of shock waves instead of solitary wave solution on e-p-i plasmas.

In this paper, we have considered a three component plasma comprising electrons-positrons and ions in planar geometry. By using standard reductive perturbation method, the quantum ion acoustic shock waves is described by deformed Korteweg-deVries Burger's (dKdVB) equation, where the Burger term appears due to the kinematic viscosities of the plasma constituents.

Basic Equations and Quantum Ion Acoustic Shock Solutions

We have followed exactly the procedure of Sahu and Roychoudhury [18] in investigating nonlinear propagation of ion acoustic shock waves for a three-species quantum plasma system which is made up of electrons, positrons and ions in a planar geometry. We consider the propagation of quantum ion acoustic shock waves (QIASWS) in an unmagnetized collisonless quantum plasma with electrons, ions and Immobile positrons. The dynamics of QIASWs in our quantum plasma is governed by the following set of hydrodynamic equations:

$$\frac{\partial n_e}{\partial t}+\frac{\partial}{\partial x}(n_e u_e)=0, \qquad (1)$$

$$\frac{\partial n_i}{\partial t}+\frac{\partial}{\partial x}(n_i u_i)=0, \qquad (2)$$

$$\frac{\partial u_e}{\partial t}+u_e\frac{\partial u_e}{\partial x}=\frac{e}{m_e}\frac{\partial \phi}{\partial x}-\frac{1}{m_e n_e}\frac{\partial p_e}{\partial x}+\frac{\eta^2}{2m_e^2}\frac{\partial}{\partial x}\left(\frac{\partial^2\sqrt{n_e}/\partial x^2}{\sqrt{n_e}}\right)+\mu_e\frac{\partial^2 u_e}{\partial x^2}, \qquad (3)$$

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$$\frac{\partial u_i}{\partial t}+u_i\frac{\partial u_i}{\partial x}=\frac{-e}{m_i}\frac{\partial \phi}{\partial x}+\frac{\eta^2}{2m_i^2}\frac{\partial}{\partial x}\left(\frac{\partial^2\sqrt{n_i}/\partial x^2}{\sqrt{n_i}}\right)+\mu_i\frac{\partial^2 u_i}{\partial x^2} \quad (4)$$

$$\frac{\partial^2\phi}{\partial x^2}=\frac{e}{\epsilon_0}\left(n_e-n_p-n_i\right). \quad (5)$$

Where $n_\alpha, u_\alpha, m_\alpha$ are respectively the density (with equilibrium value $n_{\alpha 0}$), velocity and mass for electrons $(\alpha=e)$, positrons $(\alpha=p)$ and ions $(\alpha=i)$; η is the Planck's constant divided by 2π; ϕ is the electrostatic wave potential; p_e is the electron pressure; x and t are respectively the space and time variables, and $\mu_e(\mu_i)$ are the coefficient of electron (ion) kinematic viscosity. For simplicity, the pressure effects for ions are neglected. At equilibrium the overall charge neutrality condition reads

$$n_{e0}=n_{p0}+n_{io}. \quad (6)$$

We assume that the ions are cold, and electrons obey the following pressure law [10, 20].

$$p_e=\frac{m_e V_{F_e}^2}{3n_{e0}^2}n_e^3 \quad (7)$$

where $V_{F_e}=\sqrt{2K_B T_{F_e}/m_e}$ is the electron Fermi thermal speed, T_{F_e} is the particle Fermi termperature given by $K_B T_{F_e}=\eta^2\left(3\pi^2\right)^{2/3} n_{e0}^{2/3}/2m_e, K_B$ is the Boltzmann's constant. Now introducing the following normalizations:

$$\bar{x}=w_{pi}\,x/t,\bar{t}=w_{pi}t,\bar{n}_\alpha=n_\alpha/n_{\alpha 0},\bar{u}_\alpha=u_\alpha/c_s$$

$$\bar{\phi}=e\phi/\left(2K_B T_{F_e}\right), \quad (8)$$

Where $\alpha=e,i$ and $w_{p\alpha}=\sqrt{n_{\alpha 0}\,e^2/\epsilon_0\, m_\alpha}$ is the α - particle plasma frequency, $c_s=\sqrt{2K_B T_{F_e}/m_i}$ is the quantum ion acoustic velocity; we obtain the following normalized set of basic equations as;

$$\frac{\partial n_e}{\partial t}+\frac{\partial}{\partial x}\left(n_e u_e\right)=0 \quad (9)$$

$$\frac{\partial n_e}{\partial t}+\frac{\partial}{\partial x}\left(n_e u_e\right)=0 \quad (10)$$

$$\frac{m_e}{m_i}\left(\frac{\partial u_e}{\partial t}+u_e\frac{\partial u_e}{\partial x}\right)=\frac{\partial \phi}{\partial x}-n_e\frac{\partial n_e}{\partial x}+\frac{H^2}{2}\frac{\partial}{\partial x}\left(\frac{\partial^2\sqrt{n_e}/\partial x^2}{\sqrt{n_e}}\right)$$

$$+\eta_e\frac{m_e}{m_i}\frac{\partial^2 u_e}{\partial x^2} \quad (11)$$

$$\frac{\partial u_i}{\partial t}+u_i\frac{\partial u_i}{\partial x}=-\frac{\partial \phi}{\partial x}+\frac{m_e}{m_i}\frac{H^2}{2}\frac{\partial}{\partial x}\left(\frac{\partial^2\sqrt{n_i}/\partial x^2}{\sqrt{n_i}}\right)$$

$$+\eta_i\frac{\partial^2 u_i}{\partial x^2} \quad (12)$$

$$\frac{\partial^2\phi}{\partial x^2}=\mu n_e+\left(1-\mu\right)n_p-n_i \quad (13)$$

Where $p=n_{p0}/n_{e0}$ connected through the charge neutrality condition [Eqn. (6)] $\mu=1/(1-p)$ and the nondimensional quantum parameter $H=\eta w_{pe}/\left(2K_B T_{F_e}\right)$ (the ratio between the electron Plasmon energy and the electron Fermi energy) proportional to quantum diffraction, and $\eta_{e,i}=\mu_{e,i}\,w_{pi}/c_s^2$. Now, integrating once Eqn. (11) with the boundary conditions viz. $n_e\to 1,\partial n_e/\partial x\to 0$ and $\phi\to 0$ at $\pm\infty$ will considering the fact that $m_e/m_i<<1$ we have

$$\phi=-\frac{1}{2}+\frac{n_e^2}{2}-\frac{H^2}{2\sqrt{n_e}}\frac{\partial^2\sqrt{n_e}}{\partial x^2}. \quad (14)$$

In order to investigate the propagation of QIASWs and to derive the required governing equation in our electron-positron-ion quantum plasma, we stretch the independent variables as $T=\epsilon^{\frac{1}{2}}\left(x-t\right),\tau=\epsilon^{\frac{3}{2}}t$ with $\eta_i=\epsilon^{\frac{1}{2}}\eta_{i0}$, while η_{i0} is a finite quantity of the order of unity and the dependent variables are expanded as

$$n_\alpha=1+\epsilon n_{\alpha 1}+\epsilon^2 n_{\alpha 2}+... \quad (15)$$

$$u_i=\epsilon u_{i1}+\epsilon^2 u_{i2}+... \quad (16)$$

Where α=e,i and ϵ is a small nonzero parameter proportional to the amplitude of perturbation. With respect to the above expansion of n_e [Eqn. (15)], ϕ from equation (14) can be expanded as

$$\phi=\epsilon\left(n_{e1}-\frac{H^2}{4}\frac{\partial^2 n_{e1}}{\partial x^2}\right)+\frac{\epsilon^2}{2}\left[n_{e1}^2+2n_{e2}+\frac{H^2}{4}\left\{n_{e1}\frac{\partial^2 n_{e1}}{\partial x^2}-\frac{\partial^2 n_{e2}}{\partial x^2}+\left(\frac{1}{2}\frac{\partial n_{e1}}{\partial x}\right)^2\right\}\right] \quad (17)$$

Making use of Eqn (17), we can develop a power series of ϵ for the continuity equation (10), momentum equation (12) and Poisson's equation (13) in order to obtain a system of equations written as:

$$\frac{\partial}{\partial \xi}\left(u_{i1}-n_{i1}\right)+\epsilon\left\{\frac{\partial n_{i1}}{\partial \tau}+\frac{\partial}{\partial \xi}\left(u_{i2}-n_{i2}+n_{i1}u_{i1}\right)\right\}=0\left(\epsilon^2\right) \quad (18)$$

$$\frac{\partial}{\partial \xi}\left(n_{e1}-u_{i1}\right)+\epsilon\left\{\frac{\partial u_{i1}}{\partial \tau}+\frac{\partial u_{i2}}{\partial \xi}+u_{i1}\frac{\partial u_{i1}}{\partial \xi}-\frac{H^2}{4}\frac{\partial^3 n_{e1}}{\partial \xi^3}\right\}$$

$$+\epsilon\left\{\frac{1}{2}\frac{\partial}{\partial \xi}\left(n_{e1}^2+2n_{e2}-\eta_{io}\right)\frac{\partial^2 n_{i1}}{\partial \xi^2}\right\}=0\left(\epsilon^2\right) \quad (19)$$

$$\mu\left(n_{p1}-n_{e1}\right)+n_{i1}-n_{pi}+\left[\mu\left(n_{p2}-n_{e2}\right)+n_{i2}-n_{p2}+\frac{\partial^2 n_{ei}}{\partial\xi^2}\right]\in=0\left(\in^2\right) \qquad (20)$$

The zeroth-order terms of the above equations together with the assumption that u_{i1} and n_{i1} vanishes as $\xi \to 0$ yields;

$$n_{e1}=n_{i1}=u_{i1}=u(\xi,\tau). \qquad (21)$$

Considering the 1st – order terms in equation (18 – 20) and making use of equation (21), we have

$$\frac{\partial u}{\partial\tau}+\frac{\partial}{\partial\xi}\left(u_{i2}-n_{i2}+u^2\right)=0 \qquad (22)$$

$$\frac{\partial u}{\partial\tau}+\frac{\partial}{\partial\xi}\left(n_{e2}-u_{i2}+u^2-\frac{H^2}{4}\frac{\partial^2 u}{\partial\xi^2}\right)-\eta_{io}\frac{\partial^2 u}{\partial\xi^2}=0 \qquad (23)$$

and

$$\frac{\partial^2 u}{\partial\xi^2}=n_{e2}\mu+(1-\mu)n_{p2}-n_{i2} \qquad (24)$$

Finally, eliminating all the second order quantities from equations (22 – 24), we obtain the modified deformed KdVB equation for quantum ion acoustic waves as

$$\frac{\partial u}{\partial\tau}+2u\frac{\partial u}{\partial\xi}+\frac{\mu}{1+\mu}\left(\frac{1}{\mu}-\frac{H^2}{4}\right)\frac{\partial^3 u}{\partial\xi^3}-\frac{\mu\eta_{io}}{1+\mu}\frac{\partial^2 u}{\partial\xi^2}=0. \qquad (25)$$

In the observe of positrons that is setting p=0, the earlier result of planar geometry for quantum ion acoustic shock waves in electron-ion plasma are completely recovered [18] as

$$\frac{\partial u}{\partial\tau}+2u\frac{\partial u}{\partial\xi}+\frac{1}{2}\left(1-\frac{H^2}{4}\right)\frac{\partial^3 u}{\partial\xi^3}-\frac{\eta_{io}}{2}\frac{\partial^2 u}{\partial\xi^2}=0. \qquad (26)$$

Numerical Analysis and Discussions

Making use of tanh method [21, 22], we obtain the traveling solution for the modified deformed KdVB equation (25) as

$$\phi(\xi,\tau)=a_0+a_1\tanh\{\alpha(\xi-V\tau)\}+a_2\tanh^2\{\alpha(\xi-V\tau)\} \qquad (27)$$

where V is the shock wave velocity and the coefficients a_0, a_1 and a_2 given as below:

$$a_0=\frac{1}{2}\left[V+12A\alpha^2\right]$$

$$a_1=-6B\alpha/5$$

$$a_2=-6A\alpha^2 \qquad (28)$$

and

$$A=\frac{1}{2\left(1-\frac{P}{2}\right)}\left[1-P-\frac{H^2}{4}\right]$$

$$B=\frac{\eta_{i0}}{2\left(1-\frac{P}{2}\right)}$$

$$\alpha=B/10A \qquad (29)$$

We find that all the coefficients A, and B are modified by the inclusion of positron when compared with reference [18]. In the absence of the viscosity term equation (25) reduces to the usual KdV equation for the propagation of QIASWs, while for H=2 (for which p=0) it reduces to a purely Burger's equation.

The numerical results of the stationary solution equation (25) are presented in the following profiles.

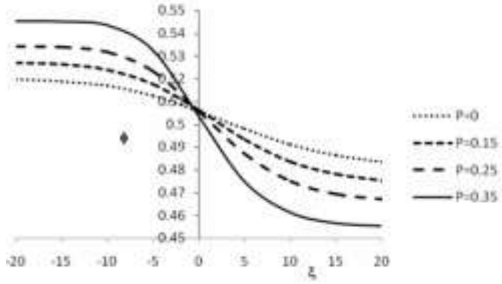

Figure 1: Numerical solution of equation 25 for different values of p where η_{io} =0.5, V=1, H=1, τ=-3

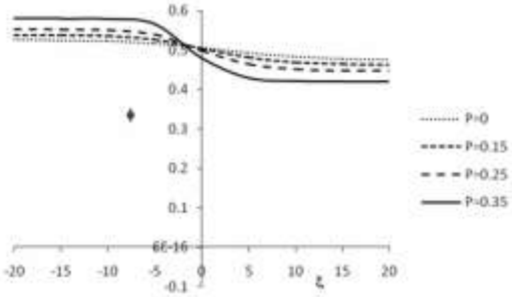

Figure 2: Numerical solution of equation 25 for different values of p where η_{io} =0.5, V=1, H=1.3, τ=-3

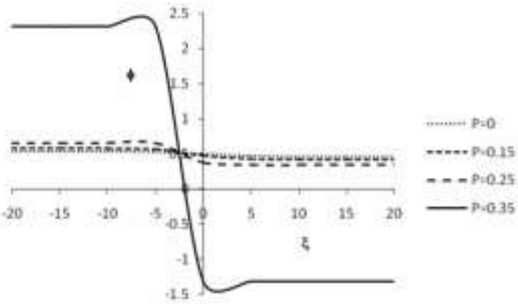

Figure 3: Numerical solution of equation 25 for different values of p where η_{io} =0.5, V=1, H=1.6, τ=-3

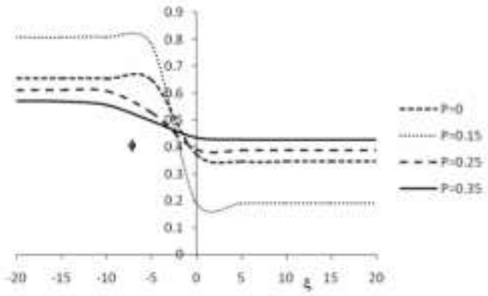

Figure 4: Numerical solution of equation 25 for different values of p where η_{io} =0.5, V=1, H=1.9, τ=-3

For figures (5-7), we present plots of the numerical solution of equation (25) for different values of η_{i0} and fixed values of V=1, H=1.5 and τ=-3. From figure 5(for $\eta_{i0}=0.2$), we can see that the shock height increases appreciably as the positron concentration increases. While for figure (6) (for η_{i0} =0.4), we have observed that for smaller positron concentration (P=0, 0.15, 0.25) the shape of the developed shock structures are closer to each other. But shock shape for P=0.35 stands out from the rest of the plot. Finally, when η_{i0} is set equal to 0.6 as in figure (7), the shock wave height increases with increase positron concentration.

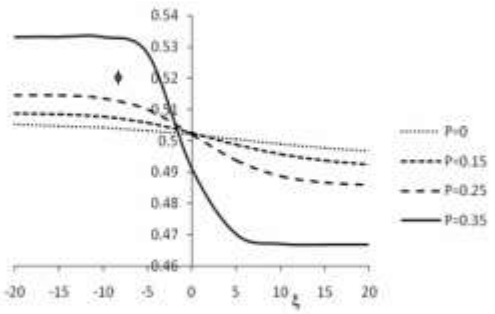

Figure 5: Numerical solution of equation 25 for different values of p where η_{io} =0.2, V=1, H=1.5, τ=-3

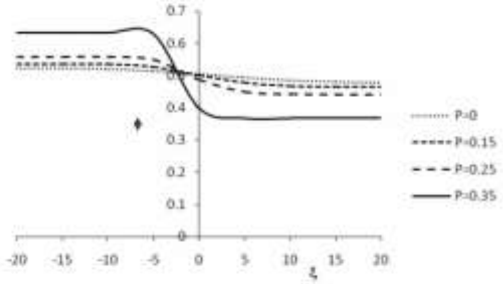

Figure 6: Numerical solution of equation 25 for different values of p where η_{io} =0.4, V=1, H=1.5, τ=-3

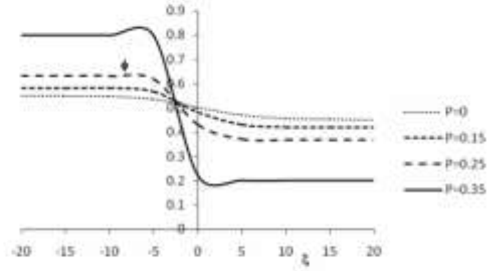

Figure 7: Numerical solution of equation 25 for different values of p where η_{io} =0.6, V=1, H=1.5, τ=-3

Conclusion

We have investigated the quantum ion acoustic waves in an umagnatized three component plasma consisting of electrons, positrons and ions. We have made use of the standard reductive perturbation method in deriving the planar geometry dKdVB equation. Both the dissipative (due to kinematic viscosity) and dispersive (due to Bohm potential) effects are taken into consideration for the formation of QIA shock structures. It is also observed that; the shock structures are also modified by the effect of H, η_{i0} and P. Our numerical, analysis reveals that large amplitude shock structure occurs comparatively, at larger values of H($\geq$ 1.3), η_{i0} *($\geq$ 0.5), P($\geq$ 0.35).* where as for small amplitude, shock structures occur at lower values of H(~1), η_{i0}(~0.2) and P=(~0.15). Our quantum plasma model could be of interest in astrophysical and laser produced plasmas due to the significant modifications of the shock structures.

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References

[1] Manfredi, G.: Fields Inst. Commun. 46, 263 (2005).
[2] Bonitz M; J. Phys. A. Math. Gen. 36, 5921 (2003).
[3] Manfredi, G. and Hervieux, P. A.: Appl. Phys. Lett. 91, 061108 (2007).
[4] Jung, Y. D.: Phys. Plasmas, 8, 3842 (2001)
[5] Khan, S. A. and Haque, Q.: Chin. Phys. Lett. 25,4329 (2008).
[6] Kremp, D., Bornath, Th., Bonitz, M. and Schlanges, M.: Phys. Rev. E, 60,4725 (1999)
[7] Manfredi, G and Feix, M. R.: J. Plasma Phys. 53, 6466 (1996).
[8] Manfredi, G. and Haas, F.: Phys. Rev. B, 64, 075316 (2001).
[9] Marklund, M. and Brodin, G.: Phys. Rev. Lett, 98,025001 (2007).
[10] Haas, F., Manfredi, G and Goedert, J.: Brazilian J. Phys. 33, 128(2003).
[11] Haas, F., Garcia, L. G., Goedert, J. and Manfredi, G.: Phys. Plasmas 10, 3858 (2003).
[12] Garcia, L. G., Haas, F., de Oliveira, L. P. L. and Goedert, J.: Phys, Plasmas 12, 012302(2005).
[13] Khan, S. A. and Mushtaq, A.: Phys. Plasmas 14, 083703 (2007).
[14] Khan, S. A., Mahmood, S. and Mirza Arshad M.: Phys. Lett. A 372, 14836 (2008).
[15] Ali, S., Moslem, W. M., Shukla, P. K. and Schlickeiser, R.: Phys. Plasmas 14, 082307 (2007).
[16] Mushtaq A. and Khan, S. A.: Phys. Plasmas 14, 052307 (2007).
[17] Khan, S. A. and Masood, W.: Phys. Plasmas 15, 062301 (2008).
[18] Sahu, B. and Roychondhury, R.: Phys. Plasmas 14, 072310 (2007).
[19] Roy, K., Misra, A. P. and Chatterjee, P.: Phys. Plasmas 15, 032310(2008).
[20] L. D. Landau, E. M. Lifshitz, Statistical Physics, Oxford University Press, Oxford, 1998.
[21] R. Roychoudhury, Das, G. C. and Sarma J.: Phys. Plasmas 6,2721(1999).
[22] Wazzan, L.: Proc. Pakistan Acad. Sci. 44,117 (2007).