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Discrete Mathematics

Elixir Dis. Math. 35 (2011) 2921-2924

ISSN: 2229-712X

Certain new summation formulae of hypergeometric function

Salahuddin

Department of Mathematics, P.D.M College of Engineering, Bahadurgarh , Haryana , India.

ARTICLE INFO

Article history:

Received: 11 April 2011;

Received in revised form: 20 May 2011;

Accepted: 27 May 2011;

Keywords

Gaussian Hypergeometric function,

Recurrence relation,

Bailey summation theorem.

ABSTRACT

The main objective of the present paper is to establish certain new formulae in the aspects of Hypergeometric function.

2000 MSC NO: 33C05, 33C20 , 33C60, 33C70.

Introduction

Generalized Gaussian Hypergeometric function of one variable is defined by

$$ {}_AF_B(a_1,a_2,\ldots,a_A;b_1,b_2,\ldots b_B;z) = \sum_{k=0}^{\infty} \frac{(a_1)_k(a_2)_k\cdots(a_A)_k\, z^k}{(b_1)_k(b_2)_k\cdots(b_B)_k\, k!} \quad (1) $$

or

$$ {}_AF_B((a_A);(b_B);z) \equiv {}_AF_B((a_j)^A_{j=1};(bj)^B_{j=1};z) = \sum_{k=0}^{\infty} \frac{((a_A))_k\, z^k}{((b_B))_k\, k!} \quad (2) $$

where the parameters b_1 , b_2 , ….,b_B are neither zero nor negative integers and A , B are non negative integers.

Contiguous Relation is defined by

[Andrews p.363(9.16) , E.D. p.51(10), H.T.F.I. p.103(32)]

$$ (a-b)\,{}_2F_1(a, b ; c; z) = a\,{}_2F_1(a+1,b; c; z) - b\,{}_2F_1(a,b+1; c; z) \quad (3) $$

Recurrence relation is defined by

$$ \Gamma(z+1) = z\,\Gamma(z) \quad (4) $$

Bailey summation theorem[Prud, p.491(7.3,7.8)]

$$ {}_2F_1\left(a, 1-a ; c; \tfrac{1}{2}\right) = \frac{\Gamma\left(\frac{c}{2}\right)\Gamma\left(\frac{c+1}{2}\right)}{\Gamma\left(\frac{c+a}{2}\right)\Gamma\left(\frac{c+1-a}{2}\right)} = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-1}\Gamma\left(\frac{c+a}{2}\right)\Gamma\left(\frac{c+1-a}{2}\right)} \quad (5) $$

Main Summation Formulae

$$ {}_2F_1\left(a,-a-23; c; \tfrac{1}{2}\right) = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c+23}} \Bigg[\frac{-167423153990400a+125014938806880a^2-18823735705512a^3}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} + $$

$$ \frac{-134443747888 4a^4+186077235570a^5+13005587615a^6-179145666a^7-30583857a^8-696210a^9}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} $$

$$ + \frac{-235a^{10}+138a^{11}+a^{12}+167423193907200c-377813573406720ac+151164048930048a^2c-9013101392064a^3c}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} $$

$$ + \frac{-1710587307264a^4c+38492217456a^5c+7743811152a^6c+152164320a^7c-4630560a^8c-182160a^9c-1584a^{10}c}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} $$

$$ + \frac{252798755143680c^2-290589129908736ac^2+67736468852992a^2c^2+8229489571 2a^3c^2-58745701308 8a^4c^2}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} $$

$$ + \frac{-11551045464a^5c^2+1162888888a^6c^2+44320080a^7c^2+196080a^8c^2-8280a^9c^2-72a^{10}c^2+158248967602176c^3}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} $$

$$ + \frac{-112564370873856ac^3+14911175689344a^2c^3+727478742144a^3c^3-81963996576a^4c^3-3842361600a^5c^3}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} $$

$$ + \frac{35555520a^6c^3+3400320a^7c^3+36960a^8c^3+55185546149888c^4-25526419162752ac^4+1695839633248a^2c^4}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} $$

$$ + \frac{168601358688a^3c^4-4292203832a^4c^4-386554560a^5c^4-3528560a^6c^4+77280a^7c^4+840a^8c^4}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} $$

$$ + \frac{12057504645120c^5-3635785359360ac^5+79422486528a^2c^5+17774152704a^3c^5+73565184a^4c^5-16321536a^5c^5}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} $$

$$ + \frac{-236544a^6c^5+1748439531520c^6-334925832192ac^6-2704808960a^2c^6+987452928a^3c^6+16726528a^4c^6}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} $$

$$ + \frac{-247296a^5c^6-3584a^6c^6+172855001088c^7-19930927104ac^7-544794624a^2c^7+27979776a^3c^7+608256a^4c^7}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} $$

$$ + \frac{11712036864c^8-739450368ac^8-28493568a^2c^8+317952a^3c^8+6912a^4c^8+535265280c^9-15544320ac^9}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} $$

$$ + \frac{-675840a^2c^9+15769600c^{10}-141312ac^{10}-6144a^2c^{10}+270336c^{11}+2048c^{12}}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} $$

$$ + \frac{-8(-7039647014400+10292368252320a-3461543784072a^2+172246260780a^3+37343273550a^4)}{\Gamma\left(\frac{c+a+23}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} $$

$$ + \frac{-8(-721531965a^5-162065211a^6-3240930a^7+95700a^8+3795a^9+33a^{10}-15352587210240c)}{\Gamma\left(\frac{c+a+23}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} $$

$$ + \frac{-8(14277071009952ac-2963329986984a^2c-17602551820a^3c+24832967230a^4c+513537353a^5c)}{\Gamma\left(\frac{c+a+23}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} $$

$$ + \frac{-8(-48159209a^6c-1853110a^7c-8240a^8c+345a^9c+3a^{10}c-12478566431232c^2+7801459901664ac^2)}{\Gamma\left(\frac{c+a+23}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} $$

$$ + \frac{-8(-948255977416a^2c^2-49182611016a^3c^2+5107157594a^4c^2+243547920a^5c^2-2172940a^6c^2-212520a^7c^2)}{\Gamma\left(\frac{c+a+23}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} $$

$$ + $$

Tele:

E-mail addresses: sludn@yahoo.com,vsludn@gmail.com

$$\frac{-8(-2310a^{8}c^{2}-534101546035 2c^{3}+2267617072352ac^{3}-140159707848a^{2}c^{3}-14461922888a^{3}c^{3})}{\Gamma\left(\frac{c+a+23}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

+

$$\frac{-8(350706202a^{4}c^{3}+32315920a^{5}c^{3}+295540a^{6}c^{3}-6440a^{7}c^{3}-70a^{8}c^{3}-1383437102080c^{4}+393301413120ac^{4})}{\Gamma\left(\frac{c+a+23}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

+

$$\frac{-8(-7874155520a^{2}c^{4}-1871876160a^{3}c^{4}-8106560a^{4}c^{4}+1700160a^{5}c^{4}+24640a^{6}c^{4}-232301808640c^{5})}{\Gamma\left(\frac{c+a+23}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

+

$$\frac{-8(42729163008ac^{5}+371375872a^{2}c^{5}-123802560a^{3}c^{5}-2098880a^{4}c^{5}+30912a^{5}c^{5}+448a^{6}c^{5}-26147337216c^{6})}{\Gamma\left(\frac{c+a+23}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

+

$$\frac{-8(2933796096ac^{6}+80631936a^{2}c^{6}-4080384a^{3}c^{6}-88704a^{4}c^{6}-1991463936c^{7}+123595008ac^{7}+4764288a^{2}c^{7})}{\Gamma\left(\frac{c+a+23}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

+

$$\frac{-8(-52992a^{3}c^{7}-1152a^{4}c^{7}-101291520c^{8}+2914560ac^{8}+126720a^{2}c^{8}-3294720c^{9}+29440ac^{9}+1280a^{2}c^{9})}{\Gamma\left(\frac{c+a+23}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

+

$$\frac{-8(-61952c^{10}-512c^{11})}{\Gamma\left(\frac{c+a+23}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}\Bigg] \qquad (5)$$

$${}_2F_1\left(a,-a-24;\ c;\ \frac{1}{2}\right)=\frac{\sqrt{\pi}\,\Gamma(c)}{2^{c+24}}\Bigg[\frac{-33484634789760 0a+265325367396960a^{2}-48906562552488a^{3}}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}$$

$$\frac{-9884400270 44a^{4}+456572472690a^{5}+13283988815a^{6}-1027452594a^{7}-51835857a^{8}-355890a^{9}+16325a^{10}}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}$$

+

$$\frac{282a^{11}+a^{12}+334846387814400c-770922757140480ac+335199615860736a^{2}c-30606869408576a^{3}c}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}$$

+

$$\frac{-2651022197200a^{4}c+182437576640a^{5}c+13010262692a^{6}c-32654356a^{7}c-14160760a^{8}c-251000a^{9}c}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}$$

+

$$\frac{-348a^{10}c+12a^{11}c+505597510287360c^{2}-602790837594624ac^{2}+158726624744256a^{2}c^{2}}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}$$

+

$$\frac{-4891578059600a^{3}c^{2}-1152080696800a^{4}c^{2}+8383983076a^{5}c^{2}+2772982604a^{6}c^{2}+48973960a^{7}c^{2}}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}$$

+

$$\frac{-679000a^{8}c^{2}-19740a^{9}c^{2}-84a^{10}c^{2}+316497935204352c^{3}-237502302350336ac^{3}+37880115459200a^{2}c^{3}}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}$$

+

+

$$\frac{483254751200a^{3}c^{3}-199400831840a^{4}c^{3}-4064477592a^{5}c^{3}+197794240a^{6}c^{3}+5922000a^{7}c^{3}+21280a^{8}c^{3}}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}$$

$$\frac{-280a^{9}c^{3}+110371092299776c^{4}-54909050969600ac^{4}+4982414441600a^{2}c^{4}+243156959360a^{3}c^{4}}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}$$

+

$$\frac{-15689908640a^{4}c^{4}-624144640a^{5}c^{4}+2296000a^{6}c^{4}+210560a^{7}c^{4}+1120a^{8}c^{4}+24115009290240c^{5}}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}$$

+

$$\frac{-8006175784960ac^{5}+350327986176a^{2}c^{5}+31839851008a^{3}c^{5}-444138240a^{4}c^{5}-34361600a^{5}c^{5}-220416a^{6}c^{5}}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}$$

+

$$\frac{1792a^{7}c^{5}+3496879063040c^{6}-760158271488ac^{6}+8977457664a^{2}c^{6}+2055594240a^{3}c^{6}+9542400a^{4}c^{6}}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}$$

+

$$\frac{-758016a^{5}c^{6}-5376a^{6}c^{6}+345710002176c^{7}-47175131136ac^{7}-423459840a^{2}c^{7}+69081600a^{3}c^{7}+783360a^{4}c^{7}}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}$$

+

$$\frac{-4608a^{5}c^{7}+23424073728c^{8}-1866600960ac^{8}-39072000a^{2}c^{8}+1082880a^{3}c^{8}+11520a^{4}c^{8}+1070530560c^{9}}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}$$

+

$$\frac{-44032000ac^{9}-1111040a^{2}c^{9}+5120a^{3}c^{9}+31539200c^{10}-529408ac^{10}-11264a^{2}c^{10}+540672c^{11}-2048ac^{11}}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}$$

+

$$\frac{4096c^{12}}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}$$

+

$$\frac{1295295050649600-1896647335752960a+637891170745248a^{2}-28840153053144a^{3}}{\Gamma\left(\frac{c+a+25}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

+

$$\frac{-7853213302484a^{4}+125780769150a^{5}+39740201759a^{6}+967242738a^{7}-30821217a^{8}-1618590a^{9}-20107a^{10}}{\Gamma\left(\frac{c+a+25}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

+

$$\frac{6a^{11}+a^{12}+2881193222799360c-2712374334775296ac+567894958060032a^{2}c+6887093615936a^{3}c}{\Gamma\left(\frac{c+a+25}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

+

$$\frac{-5511685512400a^{4}c-141457445504a^{5}c+13185695684a^{6}c+627401236a^{7}c+3663560a^{8}c-212680a^{9}c}{\Gamma\left(\frac{c+a+25}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

+

$$\frac{-3516a^{10}c-12a^{11}c+2418876921028608c^{2}-1548190647885312ac^{2}+191273453601216a^{2}c^{2}}{\Gamma\left(\frac{c+a+25}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

+

$$\frac{11857723403600a^{3}c^{2}-1230562632640a^{4}c^{2}-70453570852a^{5}c^{2}+664320524a^{6}c^{2}+90640760a^{7}c^{2}+1407560a^{8}c^{2}}{\Gamma\left(\frac{c+a+25}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

+

$$\frac{-420a^{9}c^{2}-84a^{10}c^{2}+1082575376154624c^{3}-476929280533504ac^{3}+3003119069 1200a^{2}c^{3}+3688238041120a^{3}c^{3}}{\Gamma\left(\frac{c+a+25}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

+

$$\frac{-92886862880a^{4}c^{3}-10906822248a^{5}c^{3}-128757440a^{6}c^{3}+3969840a^{7}c^{3}+81760a^{8}c^{3}+280a^{9}c^{3}}{\Gamma\left(\frac{c+a+25}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

+

$$\frac{297280550465536c^{4}-89201015564800ac^{4}+1743086610560a^{2}c^{4}+534842385280a^{3}c^{4}+3178776160a^{4}c^{4}}{\Gamma\left(\frac{c+a+25}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

+

$$\frac{-725123840a^{5}c^{4}-15014720a^{6}c^{4}+4480a^{7}c^{4}+1120a^{8}c^{4}+53811029606400c^{5}-10682374651904ac^{5}}{\Gamma\left(\frac{c+a+25}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

+

$$\frac{-123516008448a^{2}c^{5}+41696087552a^{3}c^{5}+907818240a^{4}c^{5}-19054336a^{5}c^{5}-521472a^{6}c^{5}-1792a^{7}c^{5}}{\Gamma\left(\frac{c+a+25}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

+

$$\frac{6669524516864c^{6}-832718874624ac^{6}-28013078016a^{2}c^{6}+1740291840a\%3\,c^{6}+54055680a^{4}c^{6}-16128a^{5}c^{6}}{\Gamma\left(\frac{c+a+25}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

+

$$\frac{-5376a^{6}c^{6}+575608061952c^{7}-41563152384ac^{7}-2053032960a^{2}c^{7}+32663040a^{3}c^{7}+1336320a^{4}c^{7}}{\Gamma\left(\frac{c+a+25}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

+

$$\frac{4608a^{5}c^{7}+34569351168c^{8}-1243061760ac^{8}-77226240a^{2}c^{8}+23040a^{3}c^{8}+11520a^{4}c^{8}+1416560640c^{9}}{\Gamma\left(\frac{c+a+25}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

+

$$\frac{-18145280ac^{9}-1479680a^{2}c^{9}-5120a^{3}c^{9}+37756928c^{10}}{\Gamma\left(\frac{c+a+25}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

+ $\Bigg]$ (6)

Derivations of formula (5) :

Putting b= -a-23 , z= $\frac{1}{2}$ in known formula (2) , we get

$(2a+23)\ {}_2F_1(a,\ -a-23\ ;\ c;\ \frac{1}{2}) = a\ {}_2F_1(a+1,-a-23;\ c;\ \frac{1}{2}) + (a+23)\ {}_2F_1(a,-a-22;\ c;\ \frac{1}{2})$

Now using the derived result from Bailey theorem, we get

$$\text{L.H.S}=a\,\frac{\sqrt{\pi}\,\Gamma(c)}{2^{c+22}}\Bigg[\frac{918086400-16950835202400a+10584564059112a^{2}-788004913276a^{3}}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a-1}{2}\right)}$$

$$\frac{-165169095330a^{4}+1321910185a^{5}+784810026a^{6}+28009857a^{7}-14190a^{8}-16205a^{9}-258a^{10}-a^{11}}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a-1}{2}\right)}$$

+

$$\frac{16953607710720c-34322089570560ac+10321461272704a^{2}c+83462693328a^{3}c-107449852752a^{4}c}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a-1}{2}\right)}$$

+

$$\frac{-3959356212a^{5}c+153673716a^{6}c+10350360a^{7}c+153320a^{8}c-180a^{9}c-12a^{10}c+23740996624896c^{2}}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a-1}{2}\right)}$$

+

$$\frac{-2297856848172 8ac^2+3445932208176a^2c^2+233789551776a^3c^2-18791551548a^4c^2-1360139172a^5c^2}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a-1}{2}\right)}$$

$$+\frac{-9864120a^6c^2+776840a^7c^2+15300a^8c^2+60a^9c^2+13447533058048c^3-7506060079488ac^3}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a-1}{2}\right)}$$

$$+\frac{477207540512a^2c^3+66369220512a^3c^3-511663208a^4c^3-144916800a^5c^3-2866640a^6c^3+3360a^7c^3}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a-1}{2}\right)}$$

$$+\frac{280a^8c^3+4142892175104c^4-1384574392896ac^4+14122659264a^2c^4+7988963024a^3c^4+142853760a^4c^4}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a-1}{2}\right)}$$

$$+\frac{-5430880a^5c^4-141120a^6c^4-560a^7c^4+780012052480c^5-153255137280ac^5-3609980416a^2c^5}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a-1}{2}\right)}$$

$$+\frac{463771392a^3c^5+13782272a^4c^5-16128a^5c^5-1792a^6c^5+94790019072c^6-10272636416ac^6-462180096a^2c^6}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a-1}{2}\right)}$$

$$+\frac{11570944a^3c^6+446208a^4c^6+1792a^5c^6+7591993344c^7-397550592ac^7-23665152a^2c^7+27648a^3c^7}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a-1}{2}\right)}$$

$$++\frac{4608a^4c^7+398301696c^8-7428864ac^8-566784a^2c^8-2304a^3c^8+13168640c^9-15360ac^9-5120a^2c^9}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a-1}{2}\right)}$$

$$\frac{248832c^{10}+1024ac^{10}+2048c^{11}}{\Gamma\left(\frac{c+a+24}{2}\right)\Gamma\left(\frac{c-a-1}{2}\right)}$$

$$+\frac{-2851577326080+967114476000a+2853158930376c^2-982050969284a^3}{\Gamma\left(\frac{c+a+23}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

$$+\frac{-6981112410a^4+9920202935a^5+370911618a^6-13212417a^7-920310a^8-14035a^9+6a^{10}+a^{11}}{\Gamma\left(\frac{c+a+23}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

$$+\frac{-3054322925568c-5050109817600ac+5026194167296a^2c-643673353680a^3c-52738828272a^4c}{\Gamma\left(\frac{c+a+23}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

$$+\frac{3027879540a^5c+260501556a^6c+2913000a^7c-118840a^8c-2700a^9c-12a^{10}c+1494542100480c^2}{\Gamma\left(\frac{c+a+23}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

$$+\frac{-6912286139200ac^2+2667302562096a^2c^2-92181897504a^3c^2-21532454748a^4c^2-120774108a^5c^2}{\Gamma\left(\frac{c+a+23}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

$$+\frac{36712200a^6c^2+916600a^7c^2+2340a^8c^2-60a^9c^2+2723939992576c^3-3344243372160ac^3+633783341600a^2c^3}{\Gamma\left(\frac{c+a+23}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

$$+\frac{12500339040a^3c^3-3012309608a^4c^3-91761600a^5c^3+1084720a^6c^3+50400a^7c^3+280a^8c^3+1307381028096c^4}{\Gamma\left(\frac{c+a+23}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

$$+\frac{-827035667904ac^4+74721788736a^2c^4+4492818736a^3c^4-153767040a^4c^4-8116640a^5c^4-47040a^6c^4}{\Gamma\left(\frac{c+a+23}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

$$+\frac{560a^7c^4+326202646528c^5-118176092160ac^5+3941407232a^2c^5+456771840a^3c^5+234752a^4c^5-241920a^5c^5}{\Gamma\left(\frac{c+a+23}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

$$+\frac{-1792a^6c^5+48932911104c^6-10205794816ac^6-919296a^2c^6+20943104a^3c^6+231168a^4c^6-1792a^5c^6}{\Gamma\left(\frac{c+a+23}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

$$+\frac{4648697856c^7-531348480ac^7-9730560a^2c^7+414720a^3c^7+4608a^4c^7+281977344c^8-15795456ac^8}{\Gamma\left(\frac{c+a+23}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

$$+\frac{-400896a^2c^8+2304a^3c^8+10588160c^9-230400ac^9-5120a^2c^9+224256c^{10}-1024ac^{10}+2048c^{11}}{\Gamma\left(\frac{c+a+23}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

$$+\quad]$$

$$+(a+23)\frac{\sqrt{\pi}\,\Gamma(c)}{2^{c+22}}\Bigg[\frac{28158588057600-39821204949120a+11955529243008a^2-128825821416a^3}{\Gamma\left(\frac{c+a+23}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

$$\frac{-160985846180a^4-2799925610a^5+589651209a^6+27570627a^7+120210a^8-13680a^9-247a^{10}-a^{11}}{\Gamma\left(\frac{c+a+23}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

$$+\frac{61410348840960c-54305472223872ac+9468059685408a^2c+470948730896a^3c-85699475992a^4c}{\Gamma\left(\frac{c+a+23}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

$$+\frac{-4672646524a^5c+85526756a^6c+9118760a^7c+154400a^8c-60a^9c-12a^{10}c+49914265724928c^2}{\Gamma\left(\frac{c+a+23}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

$$+\frac{-29100634231680ac^2+2645251786080a^2c^2+295577986288a^3c^2-12164943448a^4c^2-1285490052a^5c^2}{\Gamma\left(\frac{c+a+23}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

$$+\frac{-14878640a^6c^2+656600a^7c^2+14760a^8c^2+60a^9c^2+21364061841408c^3-8260028209024ac^3}{\Gamma\left(\frac{c+a+23}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

$$+\frac{276436005408a^2c^3+67024140064a^3c^3+169823192a^4c^3-127662080a^5c^3-2882320a^6c^3+1120a^7c^3}{\Gamma\left(\frac{c+a+23}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

$$+\frac{280a^8c^3+5533748408320c^4-1389450548992ac^4-8934903488a^2c^4+7366041984a^3c^4+167910960a^4c^4}{\Gamma\left(\frac{c+a+23}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

$$+\frac{-4595920a^5c^4-137200a^6c^4-560a^7c^4+929207234560c^5-144699061248ac^5-4918466560a^2c^5}{\Gamma\left(\frac{c+a+23}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

$$+\frac{408516864a^3c^5+13836032a^4c^5-5376a^5c^5-1792a^6c^5+104589348864c^6-9315339264ac^6-494233600a^2c^6}{\Gamma\left(\frac{c+a+23}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

$$+\frac{9804032a^3c^6+437248a^4c^6+1792a^5c^6+7965855744c^7-350155776ac^7-23720448a^2c^7+9216a^3c^7+4608a^4c^7}{\Gamma\left(\frac{c+a+23}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

$$+\frac{405166080c^8-6302208ac^8-559872a^2c^8-2304a^3c^8+13178880c^9-5120ac^9-5120a^2c^9+247808c^{10}+1024ac^{10}}{\Gamma\left(\frac{c+a+23}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

$$+\frac{2048c^{11}}{\Gamma\left(\frac{c+a+23}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}$$

$$+\frac{-7610141548800a+5664039006240a^2-862754799384a^3-50618670580a^4+7467040370a^5}{\Gamma\left(\frac{c+a+22}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}$$

$$+\frac{438809595a^6-6355587a^7-793890a^8-14040a^9-5a^{10}+a^{11}+7610145177600c-16808965194240ac}{\Gamma\left(\frac{c+a+22}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}$$

$$+\frac{6614345582784a^2c-407540891632a^3c-64080638752a^4c+1532361068a^5c+237007316a^6c+3767960a^7c}{\Gamma\left(\frac{c+a+22}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}$$

$$+\frac{-95080a^8c-2580a^9c-12a^{10}c+11144936816640c^2-12438124883712ac^2+2816392769280a^2c^2}{\Gamma\left(\frac{c+a+22}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}$$

$$+\frac{-7962118672a^3c^2-20409810848a^4c^2-321937308a^5c^2+30366560a^6c^2+895720a^7c^2+2880a^8c^2-60a^9c^2}{\Gamma\left(\frac{c+a+22}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}$$

$$+\frac{6686546853888c^3-4562724765568ac^3+579141303072a^2c^3+23612015392a^3c^3-2538975208a^4c^3}{\Gamma\left(\frac{c+a+22}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}$$

$$+\frac{-97227200a^5c^3+739760a^6c^3+48160a^7c^3+280a^8c^3+2204499968000c^4-9624226018048ac^4}{\Gamma\left(\frac{c+a+22}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}$$

$$+\frac{60401179328a^2c^4+5027680896a^3c^4-113909040a^4c^4-7822640a^5c^4-50960a^6c^4+560a^7c^4+447863848960c^5}{\Gamma\left(\frac{c+a+22}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}$$

$$+\frac{-124690728960ac^5+2574892544a^2c^5+453449472a^3c^5+1417472a^4c^5-231168a^5c^5-1792a^6c^5}{\Gamma\left(\frac{c+a+22}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}$$

$$+\frac{59117076480c^6-10142060544ac^6-62343680a^2c^6+20000512a^3c^6+240128a^4c^6-1792a^5c^6+5169905664c^7}{\Gamma\left(\frac{c+a+22}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}$$

$$+\frac{-510661632ac^7-10947072a^2c^7+396288a^3c^7+4608a^4c^7+297369600c^8-14986752ac^8-407808a^2c^8+2304a^3c^8}{\Gamma\left(\frac{c+a+22}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}$$

$$+\frac{10813440c^9-220160ac^9-5120a^2c^9+225280c^{10}-1024ac^{10}+2048c^{11}}{\Gamma\left(\frac{c+a+22}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}$$

$$+\quad]$$

On simplification, the formula (9) is derived.

On the same way we can prove the formula (10).

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