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A new crossover genetic algorithm based on trust region method

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ABSTRACT
This paper proposes a new crossover for the shortage of genetic algorithm, which is called Same-position reverse Crossover, replaces the mutation operator in genetic algorithm for trust region method. Finally, proves convergence of algorithm. Numerical experiment shows that the new method is effective.

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Introduction

Genetic algorithm is proposed by Professor John Holland, it is summarized up a simulation evolutionary algorithm. now genetic algorithm has a great advantage in practical science[1].But its local optimization ability is inferior to others [2]. It shows two parts in follows: (1) the efficiency of GA is low in search process. (2) The local search of GA is worst.

But trust region method has a better search ability on local and rapid convergence. So this paper proposes a genetic algorithm based on modified trust region and numerical experiment shows that the new method is effective and attractive.

Trust region method

Consider unrestraint optimizing problem

$$\min f(x), x \in R^n \tag{1}$$

Its quadratic model is

$$q_k(d) = f(x_k) + g_k^T d + \frac{1}{2} d^T G_k d, \quad \|d\|_2 \le \delta_k \tag{2}$$

Where $g_k = \nabla f(x_k)$, $G_k = \Delta^2 f(x_k)$, $\|\bullet\|$ refers to the Euclidean norm, δ_k is called the trust region radius.

The trust region method is usually executed by solving quadratic model, called the trust region subproblem

$$\min q_k(d), \ s.t. \|d\|_2 \le \delta_k \tag{3}$$

Let d_k be the solution of (3). The actual reduction of the objection function is defined by $Ared_k = f(x_k) - f(x_k + d_k)$ and the predictive reduction is defined by $pred_k = q_k(0) - q_k(d_k)$.The ratio between these two reductions is defined by $r_k = \dfrac{Ared_k}{pred_k}$.

As is known, the value of r_k plays a key role in the trust region method to select x_{k+1} and δ_k.The more r_k is close to 1, the more the objective function declines, we take $x_{k+1} = x_k + d_k$ and enlarge the trust region radius δ_{k+1}. Otherwise, one rejects the trial step, reduces the trust region radius and resolves the subproblem.

Trust region method are described as follows: (algorithm 1)

1) Choose x_1, d_1, ε, $k := 1$.

2) Calculate g_k, G_k, if $\|g_k\| < \varepsilon$, stop; else, go to3).

3) Solve problem (3) optimizing point d_k.

4) Compute $f(x_k + d_k)$ and r_k.

5) If $r_k < \frac{1}{4}$, $\delta_{k+1} = \frac{\|d\|_2}{4}$. If $r_k < \frac{3}{4}$ and $\delta_k = \|d_k\|_2$, order $\delta_{k+1} = 2\delta_k$; else, $\delta_{k+1} = \delta_k$.

6) If $r_k \le 0$, $x_{k+1} = x_k$, else, $x_{k+1} = x_k + d_k$.

7) $k := k + 1$, turn to 2).

Same-position reverses Crossover

The paper raises Same-position reverse crossover based on the method in reference [5], Same-position reverse Crossover which means that it takes the same position gene to change each other and reverses both of them when they are cross. Sum up new individuals to keep it diversity. Operation figure 1 and 2 are given as follows: two sirgeneration chromosome, which participate in cross operation are $p1$ 、 $p2$,the subgenerations after cross are $c1$ $c2$

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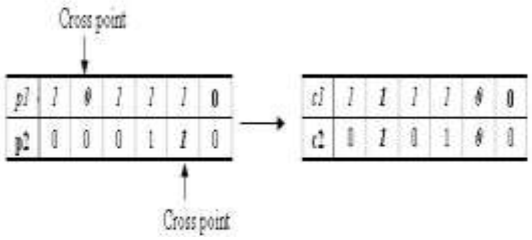

Fig.1 one-point cross

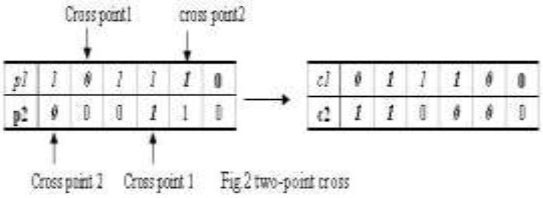

Fig 2 two-point cross

New crossover genetic algorithms

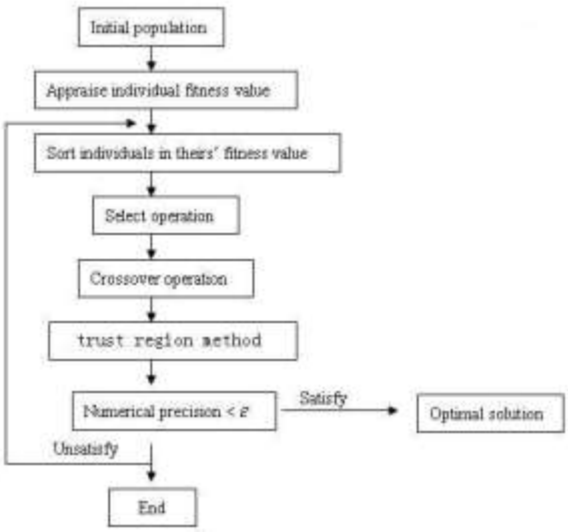

Fig.3 algorithm process

3.1 Use real coding, that is to say the chromosome coding of $X = \{x_1, x_2, ..., x_n\}^T$ is $x_1 x_2 ... x_n$.

3.2 use algorithm 1 to search and choose initial solutions by taking x_1 as initial point.

3.3 Fitness function: set $F(X) = -f(X)$.

3.4 Order: According to the fitness value to order from high to low sorting.

3.5 Selection Operator: keep excellent individual of parent, select fitness relative low individuals for filial generation individual X_i.

$$P_s^i = \frac{F(X_i)+1}{\sum_{j=1}^{n}\left[F(X_j)\right]+1}$$

3.6. Crossover: select two point same-position reserve crossovers.

3.7 According to optimal solution x_k for the initial points to start Algorithm 1.

3.8 Terminal condition: turn to 2) to when optimal solution reaches certain numerical precision; or output it.

Numerical experiments

We use the following function to test the algorithm.

$$\min f(x) = \frac{-20}{0.09+(x_1-6)^2+(x_2-6)^2} + x_1^{\ 2} + x_2^{\ 2}$$

$$s.t. x_1^2 + x_2 > 0 . \qquad (4)$$

$$\frac{x_1}{2} + 2x_2 < 0$$

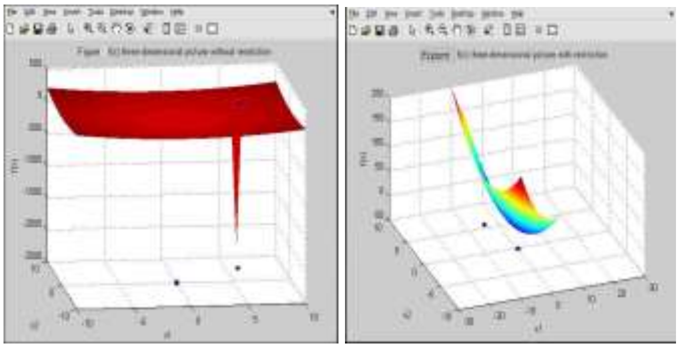

Fig 4 picture without restriction
Fig 5 picture with restriction

Using correction trust region solve the problem. Set $\alpha_1 = 0.5$, $\alpha_2 = 2$, $\beta_1 = 0.25$, $\beta_2 = 0.75$, $\Delta_0 = 0.1$, $\Delta_{\min} = 10^{-4}$, $\varepsilon = 10^{-6}$. The Maximum generation is 40. The probability is 1.0. The comparison results are shown in TABLE 1.

Convergences

The Convergence of genetic algorithm usually depends on iteration population created by algorithm converges to a steady state.

Definition 1 Suppose $\{f_k\}$ is real value sequence of random variables, it stands for the best fitness value in a certain state. Genetic algorithm converge to global optimization value f^*, if and only if, $\lim_{k\to\infty} P\{\left|f_k - f^*\right| \le \phi\} = 1$ for any $\forall \phi > 0$, where $f^* = \max\{f(X_k) | X_k \in W \subset R\}$.

Definition 2 ψ_ϕ is neighborhood of optimization X^*, set $\psi_\phi = \{X_k | X_k \in W \subset R, f(X_k) - f(X^*) < \phi\}$.

Definition 3 $A_k = \{e | X_k \in \psi_\phi\}$ is satisfied incident, it means X_k enter into incident ψ_ϕ.

Lemma 1 if arithmetic satisfies two conditions as follows, it constricts to global optimization by probability.

x' is achievement By cross operation and trust region method through x been changed. Both of them are in feasible region.

$\{X_k\}$ is monotonous.

Theorem 1 Suppose $\{X_k\}$ is the point sequence computed by arithmetic 1, we have that it is convergent to global optimization by probability.

Prove From the algorithm, we have $f(X_k) - f(X^*) \ge f(X_{k+1}) - f(X^*)$, $k = 0,1,2,...$

Namely $A_k \subset A_{k+1}$, $k = 0,1,2,...$ The event can not be repeated.

Thus $P(A_k) = P(\cap_{k=1}^{\infty} A_k)$. For given $\xi \in (0,1)$, there exists M making $P(A_k) \ge 1 - \xi$ when $k > M$.

So we have $P\{e \mid e \notin A_k, k > M\} = 1 - P\{e \mid e \in A_k, k > M\} < \xi$.

For any $\forall \phi > 0$, we have $P\{e \mid f(X_k) > f(X^*) + \phi\} \le P\{e \mid X_k \notin \psi_\phi, k > M\} \le \xi^{k-M}$.

Thus $\lim P\{e \mid f(X_k) > f(X^*) + \phi\} = 0$. This algorithm is convergence according to probability.

Conclusions

Because the basic genetic algorithm cannot converge to the global optimal solution, trust region method is effective for solving optimization problem and has good convergence property; many scholars have combined with them and study it. The paper uses real coding and combines trust region with genetic algorithm, finely, obtains very good experimental results. But there also exists some problems in two method combined above should be worth further studying.

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Table 1 part results of genetic algorithm with correction trust region

generation	x_1	x_2	f(x)	generation	x_1	x_2	f(x)
1	0.01869	-0.01857	-0.27683	8	5.99679	-5.99837	71.80309
2	2.59829	-2.69870	13.82507	9	5.99675	-5.99839	71.80285
3	2.99967	-3.99467	24.77175	10	5.99570	-5.99979	71.80709
4	4.99658	-5.99798	60.80368	11	5.99775	-5.99859	71.81726
5	5.99600	-2.99057	44.64837	12	5.99874	-5.99794	71.85071
6	5.99673	-3.89865	50.95631	13	5.99874	-5.99794	71.85071
7	5.99650	-5.99865	71.80298	14	5.99874	-5.99794	71.85071