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Estimating the size of a crack in a rotating beam using embedded modeling

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ABSTRACT

This paper presents an embedded modeling approach for estimating the crack size in a rotating beam by predicting the vibrations of the cracked beam. The model embeds a non-linear switching function into a finite element model of the beam to characterize the effect of crack breathing on the local stiffness of the beam. Solving the model enables the prediction of the vibrations of the cracked beam and the evaluation of the modal frequencies of the vibrating signal using Chirp-z transform. Inputs to the model include the vibrations of the un-cracked beam to calibrate the model at the beginning, i.e., no need for run-to-fail tests. The model is validated and refined utilizing experimental data.

Introduction

The article presents a model for investigating the diagnosis of a rotating beam (blade). Numerous cases of mechanical failures caused by fatigue crack are presented in the literature. In 1970, metallurgical examination revealed that a fatigue crack found in a rotating turbine blade in an electric power generator was the major cause of failures [1].

In the literature, little existing work about crack assessment of rotating beams. In [2], the author developed a theory on vibration of cracked shafts and included an extension to cracked beams and turbine blades. Lee [3] used a microphone as a non-contact sensor and established the utility of chirp-z transform as a signal processing tool to estimate the vibration modal frequencies. Moreover, the author built a neural network empirical model relating the estimated vibration mode frequencies and the crack size. A significant limitation of this model is that it does not have much generality due to its dependence on run-to-fail data of a beam with specific material properties and geometry that adds to the experimental cost. Lee's experimental data is used in this paper to verify the results of the developed model. Batayneh [4] presents an embedded modeling for diagnosis and prognosis of a rotating beam.

In [5], [6], and [7], the authors presented empirical vibration models for man-made notch beam. Unlike a real crack, man-made notch does not "breath". Bachschmid et. al, [8] described the breathing of a crack in a rotating horizontal shaft. Doyle [9] and Rizos et. al. [10] investigated analytically and experimentally the effect of man-made notches on the beam's natural frequencies. However, generalization of such models to an actual crack is not straightforward because of crack breathing. In this study, the breathing effect is taken into consideration when developing the finite element model for the rotating beam.

Researchers proposed various techniques to monitor a defect in a structure by watching changes in some secondary phenomenon [11]. Such phenomena include the characteristic vibration signature or the global/local properties of natural frequencies and mode shapes of the structure. Most analytical, numerical, and experimental papers related to vibration analysis of a cracked structure are well described in the two survey papers [12] and [2]. The presence of a crack in a structure weakens it and increases its local flexibility, which is a function of the crack geometry and the stress intensity factor [13] and [14]. Due to the increase in the local flexibility, the local stiffness decreases. Since natural frequencies of the structure are proportional to the local stiffness, the natural frequencies decrease by crack growth. Rizos et al. [10] and Doyle [9] showed that the change amplitude in natural frequency depends on the location and geometry of the crack.

Cawley and Adams [15] and [16] used the change in the natural frequencies to estimate the location and depth of a crack. Chondros and Dimarogonas [13] used the first three vibration mode frequencies to calculate the crack location. The authors developed monographs for calculating the depth of the crack at different conditions of the beam. Rizos et al. [10] used changes of the natural frequencies of a cantilever beam to determine the crack location and size. Narkis [5] experimentally verified the results of Rizos et al.'s. Moreover, the author developed finite element calculations for the same case. Kim and Stubbs [17] presented a practical method to non-destructively locating and estimating the size of a crack using changes in natural frequencies of a structure. Lin [18] used an analytical transfer matrix method to solve the direct and inverse problems of simply supported beams with an open crack modeled as a rotational spring with sectional flexibility connecting two beams.

In the literature, not enough experimental data on the frequency shift of a structure due to growth of a fatigue crack are available. For instance, most of the simulation results of models developed for cantilever beams have been compared with the results from Wendtland's experiment [19]. In Wendtland's experiments, different shapes of geometry and

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different types of boundary conditions were used to investigate changes in the natural frequency of a beam. However, Wendtland used a machined slot instead of a real crack. On the other hand, Gomez and Silva [20] and [21] and Silva and Gomez [22] used real cracks to study the frequency changes at various crack sizes and crack locations. In their experiments, the authors used the free-free beam to eliminate the possible damping effects caused by the boundary conditions.

In this article, we employ embedded modeling to predict the cracked beam vibration at different crack sizes. Utilizing predicted vibrations, we estimate modal frequencies using chirp-z Transform. Subsequently, a curve fitting is developed to establish relationships between the vibration mode frequencies and the crack size. The theoretical relationships are then used to estimate the crack size from empirical modal frequencies observed from the real beam. The proposed model uses the modal frequencies as opposed to directly adjusting crack size in the beam model to match its responses to that of the real beam.

Proposed Method

The article builds on the embedded modeling method presented in Fan [23] to predict vibrations in a cracked beam. The proposed model accounts for the crack breathing phenomena in diagnosing the size of the crack in the rotating beam. The model assumes symmetric crack propagation for low speed applications. Hence, centrifugal forces are minimal and are neglected in the study. A summary of the embedded modeling method presented in [23] follows.

Embedded Modeling Method

Assume that the dynamics of a real system are described by

$$\dot{x} = \mathrm{F}(x, f), \qquad \mathrm{s} = \mathrm{h}(x) \tag{1}$$

Where x is the n-dimensional system state vector, f is the force function; s is the m-dimensional vector of the system's physical variables measured by transducers; and h is the function relating the state vector x and the measurable vector s. The model of the real system takes the following form

$$\dot{\hat{x}} = \hat{\mathrm{F}}(\hat{x}, f; \theta), \qquad \hat{\mathrm{s}} = \mathrm{h}(\hat{x}) \tag{2}$$

Where, the circumflex stands for approximation, and θ is the p-dimensional vector of partially unknown model parameters.

One can show that the model output $\hat{\mathrm{s}}$ will approach the real system's output s if $\hat{\mathrm{F}}$ approaches F provided that the initial error is small. Therefore, the objective is to find $\hat{\mathrm{F}}$ that minimizes the difference between the system and the model outputs. Where, the total error E is given by

$$E = \frac{1}{2}\int_{t_0}^{t_f} e^T e\, dt \quad (continuous) \quad \text{or} \quad E = \frac{1}{2}\sum_{i=1}^{M} e_i^T e_i \quad (discrete) \tag{3}$$

Where, $e = s - \hat{s}$, $e_i = s(i) - \hat{s}(i)$, M is the number of samples, and t_o and t_f define the interval. Equation (3) represents the objective function that can be minimized using non-linear programming or by an iterative procedure starting from an initial guess

$$\theta^{k+1} = \theta^{k} - \alpha\, \mathrm{R}^{k}\, \mathrm{G}^{k} \tag{4}$$

Where, G^K is the gradient of E at (θ^k, x_0^0); R^K is a positive-definite square matrix and α defines the step size. The product $R^K G^k$ determines the search direction. Different R^K matrices yield different gradient based updating schemes, such as steepest descent and Newton's method, each with a different set of efficiencies, robustness characteristics, and computational cost. This study used the Levenberg-Marquardt method for its robustness. To obtain G^K, one takes the derivative of E with respect to θ, which yields

$$G_\theta = \sum_{i=1}^{M} e_i^T \frac{\partial e_i}{\partial \theta} = -\sum_{i=1}^{M}\left(\frac{\partial \hat{s}_i}{\partial \theta}\right)(s_i - \hat{s}_i) \tag{5}$$

Note from Equation 5 that the gradient G is calculated from the error e and the $m \times p$ Jacobian matrix that contains the partial derivative of the approximated output ŝ with respect to parameters θ. The Jacobian is denoted as J hereinafter. Taking the derivative of ŝ with respect to θ yields J as

$$J = \frac{\partial \hat{s}}{\partial \theta} = \left(\frac{d\mathrm{h}}{d\hat{x}}\right)^T \frac{\partial \hat{x}}{\partial \theta} \tag{6}$$

However, the partial derivative of $\hat{x}$ with respect to θ is not readily available. In order to obtain it, one must solve the sensitivity equation, which in this case, is the derivative of the model (Equation 2), with respect to θ.

$$\frac{d\left(\frac{\partial \hat{x}}{\partial \theta}\right)}{dt} = \frac{\partial \hat{F}}{\partial \hat{x}}\frac{\partial \hat{x}}{\partial \theta} + \frac{\partial \hat{F}}{\partial \theta} \quad \text{or} \quad \dot{\xi} = \hat{F}'_x \xi + \hat{F}'_\theta \tag{7}$$

Where $\xi = \partial\hat{x}/\partial\theta$, $\hat{F}'_x$ and $\hat{F}'_\theta$ are the partial derivative of $\hat{F}$ with respect to x and θ, respectively. The solution to Equation 7 is the needed $\partial\hat{x}/\partial\theta$ which has an initial value as a zero matrix since the initial condition, $\hat{x}(t_0) = \hat{x}_0$, does not depend on the parameters of the model.

Formulating an Embedded Model for a Beam

In engineering dynamics, a beam with one end fixed is approximated using a lumped-mass system as:

$$\mathrm{M}\ddot{\mathrm{z}} + \mathrm{K}(\mathrm{z} - \mathrm{z}_0) = \mathrm{f(t)} \tag{8}$$

where M and K represent the mass and stiffness matrices, respectively; $\mathrm{f(t)}$ represents the forcing function; z represents the generalized deflection vector and z_0 represents the deflection vector due to an initial bow in the beam. Since the stiffness matrix; K is the only term in Equation 8 that is a function of the area moment of inertia, it will be the only term affected by the presence of a crack in the beam. To reflect the dependency of stiffness matrix on various factors, it will be designated with a prime. Consequently, Equation 8 becomes:

$$\mathrm{M}\ddot{\mathrm{z}} + \mathrm{K}'(\mathrm{z} - \mathrm{z}_0) = \mathrm{f(t)} \tag{9}$$

Fitting Equation 9 to the state space form of Equation 2 yields the following set of equations

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -M^{-1}K' & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix} f(t)$$
$$s = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad (10)$$

Where, $x_1 = z - z_0$; $x_2 = \dot{x}_1 = \dot{z}, \dot{x}_2 = \ddot{x}_1 = \ddot{z}$. The reduction in the stiffness can be expressed as:

$$K' = K - \Delta K \qquad (11)$$

Where ΔK is the stiffness reduction matrix, which is found to be a complicated matrix function of the change in the moment of Inertia (I) due to the presence of the crack. If the change in the moment of inertia occurs, then the vibration of the beam will be modified, which would allow one to observe such change via the vibrations of the beam. Consequently, the purpose of the crack detection diagnostics would be to determine if there is an abnormality in the vibration of the signal obtained from the beam, and consequently the modal frequencies of the vibration signal and if so, determine the size of the crack based on the change in the modal frequencies of the vibrational signal.

Simulation Study

This study embeds a stiffness reduction function into a regular Finite Element (FE) beam model so that it can be used to describe the behavior of a cracked beam. The presence of a crack in the beam causes a reduction in its local stiffness, which can be denoted as ΔK .

However, ΔK is not a constant value, it depends on the configuration of the beam. As the beam vibrates, the crack experiences the so called “breathing effect,” which is the cyclic switching between open (under tensile stress) and closed (under compressive stress) and the transition between the two. When the crack is closed, there is no stiffness reduction, even with the presence of the crack i.e. $\Delta K = 0$. When the crack is fully open, the stiffness reduction matrix (ΔK) approaches its maximum value corresponding to the crack size.

In the transitional stage, the stiffness reduction matrix will assume a value between zero and the maximum value that depends on the vibration of the crack at that moment and the slope of the beam’s vibration at the crack point. From the slop of the beam’s vibration one can tell whether the crack is closed or open and from the vibration of the beam one can tell the amount of opening.

For the beam used in this study, the crack is most likely to occur in the root of the beam, because of the stress concentration in that region.

To this end, a Finite Element Model (FEM) of the beam is developed where the beam is divided into several segments, or elements.

The root element has a smaller length than the other elements because of the presence of the crack. Furthermore, the stiffness reduction matrix is embedded in the FEM of the beam to take the crack breathing effect into consideration. Each element’s mode shape is approximated using a cubic equation to ensure the continuity of the deflection and the slope at the nodes of the model.

The stiffness reduction matrix is the difference between the stiffness matrix of the crack-free beam and the stiffness matrix of the cracked beam. The stiffness matrix for an element on the beam can be expressed as:

$$\mathbf{k}^{(e)} = E^{(e)} I^{(e)} \mathbf{A}$$
$$A = \frac{1}{(L^e)^3} \begin{bmatrix} 12 & 6L^e & -12 & 6L^e \\ 6L^e & 4(L^e)^2 & -6L^e & 2(L^e)^2 \\ -12 & -6L^e & 12 & -6L^e \\ 6L^e & 2(L^e)^2 & -6L^e & 4(L^e)^2 \end{bmatrix} \qquad (12)$$

Where E represents the modulus of elasticity of the beam, which is constant regardless of the presence of the crack; L^e represents the length of the element, which is constant when chosen for each element of the beam; matrix A is constant for any element regardless of the presence of the crack; and $I^{(e)}$ represents the area moment of inertia of an element and it is the only term in the stiffness matrix equation that is affected by the size of the crack. Therefore, the stiffness reduction matrices can be written as:

$$\Delta k^{(e)} = E \Delta I^{(e)} A \qquad (13)$$

Where $\Delta I^{(e)}$ is the reduction of area moment of inertia. The beam considered in this study is a simple beam with a rectangular cross section. Consequently, its area moment of inertia can be calculated as:

$$I = \frac{1}{12} \frac{m}{\rho L} t^2 \qquad (14)$$

Where m represent the mass density per length of the beam; ρ represent the density of the beam kg/m^3; L represent the length of the beam element and t represents the thickness of the beam. Equation 14 can be rewritten with normalized variables. Let’s define $\Delta \tilde{I}^{(e)}$ as:

$$\Delta \tilde{I}^{(e)} = \frac{\Delta I^{(e)}}{I^{(e)}} \times 100 \quad (\%) \quad or$$
$$\Delta I^{(e)} = \frac{\Delta \tilde{I}^{(e)} I^{(e)}}{100} \qquad (15)$$

Substitute back into equation 13 yields:

$$\Delta k^{(e)} = \frac{E I^{(e)}}{100} \Delta \tilde{I}^{(e)} A \qquad (16)$$

The crack size in this study is defined as the percentage of the remaining thickness of the beam in the presence of a crack to the total thickness of the beam. For a rectangular cross sectional beam (Figure1) the crack size is calculated as:

$$a = \frac{t'}{t} \times 100 \qquad (17)$$

Where a is the size of the crack, t' is the remaining thickness of the beam due to the presence of the crack, and t is the thickness of the uncracked beam.

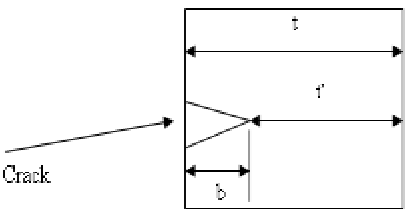

Figure 1: Cross-Sectional Area of a Rectangular Beam

Now let I_{old}, I_{new} be the old and new area moments of inertia respectively. The old area moment of inertia is the area moment of inertia of the beam when there is no crack on it. The new area moment of inertia is the area moment of inertia of the beam when there is a crack. These two terms can be represented as:

$$I_{old} = \frac{1}{12}\frac{m}{\rho \mathrm{L}}t^2, \;\; or \;\; t = \sqrt{12 I_{old}\frac{\rho L}{m}}$$
$$I_{new} = \frac{1}{12}\frac{m}{\rho \mathrm{L}}t'^2, \;\; or \;\; t' = \sqrt{12 I_{new}\frac{\rho L}{m}} \qquad (18)$$

By substituting t and t' back into Equation 17, the crack size of the beam with rectangular cross section becomes:

$$\frac{a}{100} = \sqrt{\frac{I_{new}}{I_{old}}} \qquad (19)$$

As shown in Equation 19, the crack size equal to the square root of the new area moment of inertia divided by the old area moment of inertia. A similar procedure can be followed to derive the relationship between the crack size and the area moment of inertia for beams of different cross sectional shapes. Equation 19 illustrates the crack model used in this article. Now a relation between the crack size, *a*, and the stiffness reduction matrix when the crack is fully open can be derived using Equations 19, 11 and 12. The relation is as follows:

$$\Delta k = E^{(e)} I_{old} A\left(1 - \left(\frac{a}{100}\right)^2\right) \qquad (20)$$

Equation 20 represents a nonlinear stiffness reduction matrix that is embedded into the finite element beam model when the crack is fully open. One should take into consideration the state of the crack when evaluating the stiffness reduction matrix (whether it is fully closed, fully open or partially open). The value of the stiffness reduction matrix when the crack is partially open depends on the vibration of the crack point and the slope of the vibration signal of the beam at the crack point at that moment. The stiffness reduction matrix is nonlinearly related to the amount of opening of the crack due to breathing. From the literature, previous studies ignore the breathing effect. This unknown nonlinear function depends on many factors including eccentricity, gravity, and the varying flow forces acting on the blade. In this study, we assume that the crack opens/closes gradually in a linear manner until it reaches its maximum/minimum value when it is fully open/close half way through the cycle. Next section presents the results of applying this model into a rotating beam.

Crack Size Estimation and Experimental Results

First an FEM for the beam is developed. The model embeds a non-linear switching function into a regular FEM of the beam to characterize the effect on local stiffness due to crack breathing. In this model, the crack is assumed to be in the root element of the beam. Solving such a model enables the prediction of vibration of the cracked beam at different crack sizes, and consequently enables the evaluation of the modal frequencies using Chirp z transform. Once the model is validated and refined as appropriate, it will be used to estimate the crack size. Input to the model is limited to the vibration of the uncracked beam to calibrate the model at the beginning, i.e., no need for run-to-fail tests. This approach is considered indirect because it uses modal frequencies and does not try to find the crack size that minimizes the discrepancies between beam vibrations predicted by the model and measured by the transducer.

The model is validated using the experimental results done by Lee [3]. Lee used a microphone as a non-contact sensor to pick up the acoustic signal excited by a rectangular cross sectional beam with the dimensions shown in figure 2. is made of 6061-T6 aluminum and its dimension is 2mm thick, 2.3mm wide and 109mm long from the center of the mounting hole. The beam is mounted to the shaft, its free-vibrating length is 102mm. Its density is 2700kg/m^3 and Young's modulus is 68.95×10^9 *Pa*. He then established the utility of chirp-z transform as a signal processing tool to estimate the vibration mode frequencies, and built a neural network empirical model relating the estimated vibration mode frequencies and the crack size.

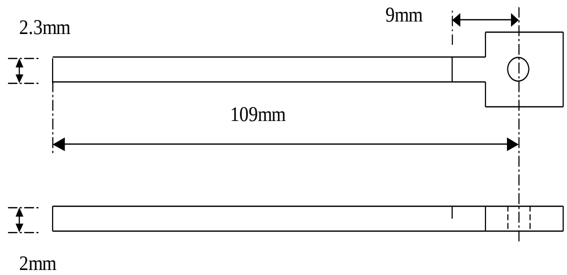

Figure 2: Dimension of the Beam Used in Fatigue Test (Lee, 2003)

Figure 3 illustrates a schematic view of the rotating beam fatigue experimental setup designed by Lee [3]. The experimental setup consists of a shaft driven by a DC motor (No. 3) through a belt. A rotating beam (No. 1) is clamped to the flat area near the middle of the shaft. Once every rotation, the beam hits the Teflon tip of an impulse hammer (No. 2) that measures the impact force. The impact excites beam vibration that results in crack initiation and propagation in the beam. The location of the crack is controled by the V-notch used to initiate the crack. A condenser microphone (No. 4) is placed nearby to pick up the pressure wave excited by the vibration of the beam. The output voltage of the condenser microphone is proportional to the sound wave pressure intensity. The output is amplified and then digitized by a PC-based data acquisition system (No. 5). A counter (No. 9) keeps an accurate count of the number of impacts by using a reflective optic sensor (No. 7) that produces one pulse per revolution.

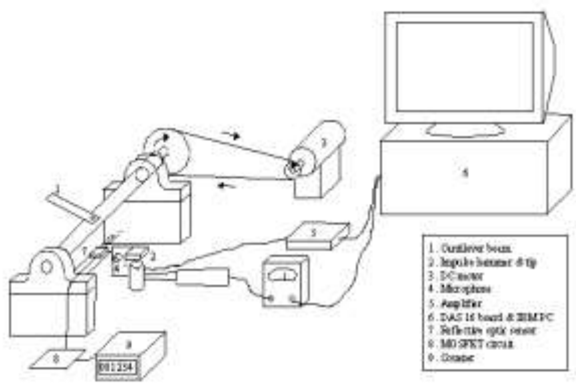

Figure 3: The experimental setup presented in [3]

The nominal beam's length is assumed to be the length of the beam from the center of the mounting hole to the tip of the beam. However, a calibration scheme was developed to minimize the discrepancy between the vibration measured from the real beam and predicted by the model for the crack-free case by adjusting the effective length of the model beam. The output of the finite element model represents the model beam's response, which was then processed by the signal processing technique to evaluate the mode frequencies of the beam. This process is repeated for different crack sizes in order to collect information about the mode frequencies for a wide range of crack sizes. Subsequently, the relation between both the first and second mode frequencies and the crack sizes are developed using a curve fitting technique. Chirp-z transform is used in this study to extract the beam's modal frequencies from its vibrating signals. The chirp-z transform algorithm is a means by which one can determine samples of the z-transform equally spaced along a contour in the z-plane. Strictly speaking, the Discrete Fourier-transform (DFT) finds these samples along the unit circle, while the chirp-z transform algorithm finds samples along a general contour, which could be part of the unit circle [25]. One of the advantages of the chirp-z transform over DFT is that a user can specify the range of frequency he/she is interested in.

The beam modeling yields two curves of modal frequency versus crack size, one for each mode. Having two mode frequencies estimated from the signal of the actual beam, one has to determine the crack size using the two theoretical curves of mode frequencies vs. crack size predicted by the FEM embedded model. The general idea is to find the crack size that minimizes the discrepancies between the theoretical mode frequencies and those of the actual beam. It can be posted as a simple optimization problem having an objective function of weighted sum of the two discrepancies. Equal weights are used in this study.

To validate the proposed rotating beam crack diagnostic algorithm, a cracked rotating beam was simulated and theoretical mode frequencies were predicted over a range of crack sizes. The results are illustrated in Figure 4 along with empirical mode frequencies identified from vibro-acoustic signals of four real cracked beams collected by Lee [3] in four run-to-fail tests. The root mean square (RMS) for Test #1 was found to be 0.029mm, with an average crack size error of 4.89%. The root mean square for Test #2 was found to be 0.03mm, with an average crack size error of 3.7%. The root mean square for Test #3 was found to be 0.018mm, with an average error in crack size of 2.51%. Finally, the root mean square for Test #4 was found to be 0.041mm, with an average crack size error of 5.88%.

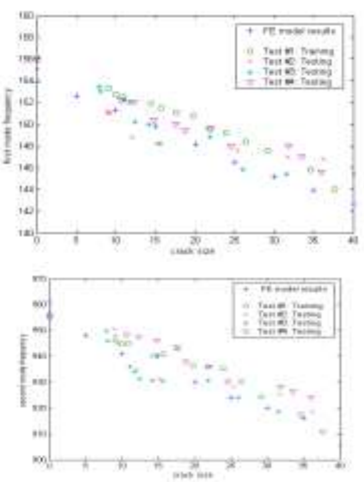

Figure 4: Crack size results from the developed model compared to the four test results

Summary and Conclusions

An embedded model that predicts the vibration of a cracked beam by considering the crack-breathing phenomenon was developed.

The crack-breathing phenomenon was included by embedding a non-linear switching stiffness function in an FEM beam model to characterize the local stiffness variation associated with crack breathing.

A calibration scheme was developed to minimize the discrepancy between the vibration measured from the real beam and predicted by the model for the crack-free case by adjusting the effective length of the model beam. The model was then used to simulate vibration under various crack sizes.

Natural frequencies of both the first and second modes were then estimated from the vibrations using chirp-z transform. Two curves of natural frequency versus crack size were then fitted from the natural frequencies of the 1st and 2nd modes respectively.

A diagnostic scheme was then established using the two curves and natural frequencies estimated from microphone measurements. The diagnosis model was evaluated using experimental data from four run-to-fail tests performed in the literature. Generally speaking, the crack sizes were estimated with good accuracy, i.e., a maximal error below 6%.

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