



Stabilization and synchronization for lu system

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ABSTRACT

In this paper, we study Lu's system, and we study the stability of equilibrium point of Lu's system. Then, we study chaos synchronization of Lu's system by using adaptive control methods.

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Keywords

Lu's system,
 Synchronization,
 Adaptive control.

Introduction

Chaos control is a new field in explorations of chaotic motions and it is crucial in applications of chaos. Until now, many different techniques and methods have been proposed to achieve chaos control, such as OGY method, impulsive control method, differential geometric method and linear state space feedback etc. In practice, there exist many examples of impulsive control systems (see [1–3]). Recently, impulsive control has been widely used to stabilize and synchronize chaotic systems (see [4–14]). In 1963, Lorenz found the first chaotic attractor in a simple three-dimensional autonomous system. So far there are many researchers who studied the chaos theory. During the last decades dynamic chaos theory has been deeply studied and applied to many fields extensively, such as secure communications, optical system, biology and so forth. The stabilization and synchronization of a class chaotic systems called Lorenz systems, Xie et al. in [10] and Sun et al. in [11] derived some sufficient conditions for the stabilization and synchronization of Lorenz systems via impulsive control with varying impulsive intervals. In 1999, Chen and Ueta found a similar but nonequivalent chaotic attractor [15], which is known to be the dual of the Lorenz system, over the last three years, there are some detailed investigations and studies of the Chen system [16–19]. Recently, Lu et al. [20] reported a new chaotic attractor, which bridged the gap between the Lorenz system and Chen system [21]. In the following we will call it Lu system. It is a typical transition system; Lu system has been analyzed in papers [22–24]. But few result of stabilization and synchronization for Lu system by using impulsive control.

Lu's system is described by

$$\begin{aligned} \dot{x} &= a(y-x) \\ \dot{y} &= -xz + cy \\ \dot{z} &= xy - bz \end{aligned} \quad (1.1)$$

where x, y, z are state variables, a, b, c are positive constants.

The objective of this paper is as follow. We investigate adaptive synchronization for Lu's system when the parameters

of the drive system are fully unknown and different with those of the response system.

Synchronization of the Lu's system

Consider two nonlinear systems:

$$\dot{x} = f(t, x) \quad (*)$$

$$\dot{y} = g(t, y) + u(t, x, y) \quad (**)$$

Where $x, y \in \mathfrak{R}^n, f, g \in C^r[\mathfrak{R}^+ \times \mathfrak{R}^n, \mathfrak{R}^n]$,

$u \in C^r[\mathfrak{R}^+ \times \mathfrak{R}^n \times \mathfrak{R}^n, \mathfrak{R}^n], r \geq 1, \mathfrak{R}^+$ is the set of non-negative real numbers. Assume that (*) is the drive system, (**) is the response system, and $u(t, x, y)$ is the control vector.

Definition 2.1. Response system and drive system are said to be synchronic if for any initial conditions $x(t_0), y(t_0) \in \mathfrak{R}^n$,

$$\lim_{t \rightarrow +\infty} \|x(t) - y(t)\| = 0.$$

In this section, we consider adaptive synchronization Lu's systems. This approach can synchronize the chaotic systems when the parameters of the drive system are fully unknown and different with those of the response system. Assume that there are two Lu's systems such that the drive system (with the subscript 1) is to control the response system (with the subscript 2). The drive and response system are given, respectively, by

$$\begin{aligned} \dot{x}_1 &= a(y_1 - x_1) \\ \dot{y}_1 &= -x_1 z_1 + c y_1 \\ \dot{z}_1 &= x_1 y_1 - b z_1 \end{aligned} \quad (2.1)$$

where the parameters a, b, c are unknown or uncertain, and

$$\begin{aligned} \dot{x}_2 &= a_1(y_2 - x_2) - u_1 \\ \dot{y}_2 &= -x_2 z_2 + c_1 y_2 - u_2 \\ \dot{z}_2 &= x_2 y_2 - b_1 z_2 - u_3 \end{aligned} \quad (2.2)$$

where a_1, b_1, c_1 are parameters of the response system which need to be estimated, and $u = [u_1, u_2, u_3]^T$ is the controller we introduced in (2.2). We choose

$$\begin{aligned} u_1 &= k_1 e_x - a(x_2 - x_1) \\ u_2 &= k_2 e_y + c(y_2 - y_1) \\ u_3 &= k_3 e_z - b(z_2 - z_1) \end{aligned} \tag{2.3}$$

where e_x, e_y, e_z are the error states which are defined as follows:

$$\begin{aligned} e_x &= x_2 - x_1 \\ e_y &= y_2 - y_1 \\ e_z &= z_2 - z_1 \end{aligned}$$

and

$$\begin{aligned} \mathcal{K} &= f_a = -\gamma x_2 e_x \\ \mathcal{K}_1 &= f_b = -\theta y_2 e_y \\ \mathcal{K}_2 &= f_c = -\delta x_2 e_z \end{aligned} \tag{2.4}$$

where $k_1, k_2, k_3 \geq 0$ and γ, θ, δ are positive real constants.

Theorem 2.1. Let $k_1, k_2, k_3 \geq 0$ be properly chosen so that the following matrix inequality holds,

$$P = \begin{pmatrix} 2k_1 & -3a & 0 \\ -3a & 2k_2 & -5(3-2b) \\ 0 & -5(3-2b) & 2k_3 \end{pmatrix} > 0 \tag{2.5}$$

or k_1, k_2, k_3 can be chosen so that the following inequalities holds,

- (i) $A = 2k_1 > 0$
- (ii) $B = 2Ak_2 - 9a^2 > 0$
- (iii) $C = 4Ak_2k_3 - 25A(3-2b)^2 - 2k_3a > 0$.

Then the two Lu's system (2.1) and (2.2) can be synchronized under the adaptive controls (2.3) and (2.4).

Proof. It is easy to see from (2.1) and (2.2) that the error system is

$$\begin{aligned} \dot{\mathcal{E}}_x &= a_1 y_2 - a y_1 - u_1 \\ \dot{\mathcal{E}}_y &= -x_2 z_2 + c_1 y_2 - u_2 \\ \dot{\mathcal{E}}_z &= x_2 y_2 - b_1 z_2 - u_3 \end{aligned} \tag{2.6}$$

Let $e_a = a_1 - a, e_b = b_1 - b, e_c = c_1 - c$. Choose the Lyapunov function as follows:

$$V(t) = \frac{1}{2} \left(\frac{1}{\gamma} e_a^2 + \frac{1}{\theta} e_b^2 + \frac{1}{\delta} e_c^2 \right).$$

Then the differentiation of V along trajectories of (2.6) is

$$\begin{aligned} \dot{V} &= \frac{1}{\gamma} e_a \dot{\mathcal{E}}_x + \frac{1}{\theta} e_b \dot{\mathcal{E}}_y + \frac{1}{\delta} e_c \dot{\mathcal{E}}_z \\ &= e_x [a_1 y_2 - a y_1 - u_1] + e_y [-x_2 z_2 + c_1 y_2 - u_2] + e_z [x_2 y_2 - b_1 z_2 - u_3] + \frac{1}{\gamma} e_a f_a + \frac{1}{\theta} e_b f_b + \frac{1}{\delta} e_c f_c \\ &= e_x [a_1 y_2 - a y_1] - e_x [a_1 x_2 - a x_1] + e_y [x_1 z_1 - x_2 z_2] + e_y [c_1 y_2 - c y_1] + e_z [x_2 y_2 - x_1 y_1] + e_z [b z_1 - b_1 z_2] \\ &\quad + \frac{1}{\gamma} e_a f_a + \frac{1}{\theta} e_b f_b + \frac{1}{\delta} e_c f_c \\ &= y_2 e_a e_x + a e_x e_y - e_a e_x + e_x^2 e_x e_y - e_y^2 + e_y e_z + x_2 e_c e_z + c(x_2 - x_1) e_z - k_2 e_y^2 - k_3 e_z^2 + \frac{1}{\gamma} e_a f_a + \frac{1}{\theta} e_b f_b + \frac{1}{\delta} e_c f_c \\ &= -2(k_1 - a) e_x^2 - 2(k_2 + 1) e_y^2 + 5(3-2b) e_y e_z + e_a \left[\frac{1}{\gamma} f_a + y_2 e_x \right] + e_b \left[\frac{1}{\theta} f_b - y_2 e_z \right] + e_c \left[\frac{1}{\delta} f_c + x_2 e_z \right] \\ &\leq -2(k_1 - 3a) e_x^2 - 3(a+2) |e_x e_y| + 5(3-2b) |e_y e_z| \\ &= -e^T P e \end{aligned}$$

where $e = \begin{bmatrix} |e_x| & |e_y| & |e_z| \end{bmatrix}^T$ and P is as in (2.5). Since $V(t)$ is positive definite and $\dot{V}(t)$ is negative semidefinite, it follows

that $e_x, e_y, e_z, a_1, b_1, c_1 \in L_\infty$. From $\dot{V}(t) \leq -e^T P e$, we can easily show that the square of e_x, e_y, e_z are integrable with respect to t , namely, $e_x, e_y, e_z \in L_2$. From (2.6), for any initial conditions, we have $\mathcal{E}_x(t), \mathcal{E}_y(t), \mathcal{E}_z(t) \in L_\infty$. By the well-known Barbalat's Lemma, we conclude that $(e_x, e_y, e_z) \rightarrow (0, 0, 0)$ as $t \rightarrow +\infty$. Therefore, in the closed-loop system, $x_2(t) \rightarrow x_1(t), y_2(t) \rightarrow y_1(t), z_2(t) \rightarrow z_1(t)$ as $t \rightarrow +\infty$. This implies that the two Lu's systems have synchronized under the adaptive controls (2.3) and (2.4).

Numerical Simulations

The numerical simulations are carried out using the Fourth-order Runge-Kutta method. The initial states are $x_1(0) = 0.5, y_1(0) = 0.5, z_1(0) = 0.5$ for the drive system and $x_2(0) = 0.2, y_2(0) = 0.2, z_2(0) = 0.2$ for the response system. The parameters of the drive system are $a = 5, b = 10, c = 5$. The control parameters are chosen as follows $k_1 = 5, k_2 = 10, k_3 = 20$ which satisfy (2.5). Choose $\gamma = \theta = \delta = 0.5$. The initial values of the parameters a_1, b_1, c_1 are all chosen to be 0. As shown in Fig. 1, the response system synchronizes with the drive system. The changing parameters of a_1, b_1, c_1 are shown in Fig. 2-4.

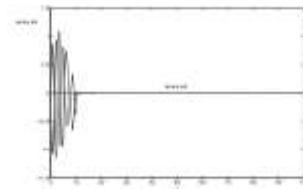


Figure 1: Synchronization error for system (2.1) and (2.2) with time t

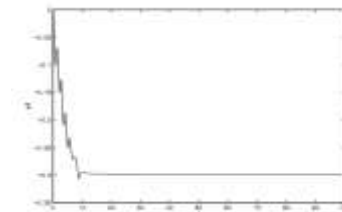


Figure 2: Changing parameter a_1 of the system (2.2) with time t.

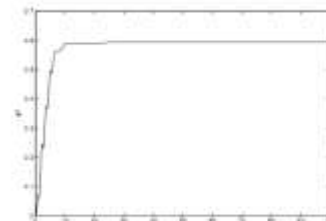


Figure 3: Changing parameter b_1 of the system (2.2) with time t

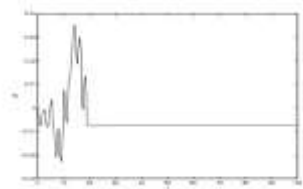


Figure 4: Changing parameter c_1 of the system (2.2) with time t.

Conclusions

In this paper, we give sufficient conditions for stability of equilibrium points of synchronization of two Lu's systems using adaptive control which control the chaotic behavior of Lu's system to its equilibrium points. Numerical Simulations are also given to verify results we obtained.

Acknowledgements

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