Available online at www.elixirpublishers.com (Elixir International Journal)

Applied Mathematics

ISSN: 2229-712X

Elixir Appl. Math. 36 (2011) 3429-3433

Wave propagation in a homogeneous isotropic generalized thermoelastic cylindrical panel

P. Ponnusamy[1] and R .Selvamani[2]

[1]Department of Mathematics Govt Arts College (Autonomous), Coimbatore, TamilNadu, India.

[2]Department of Mathematics Karunya University, Coimbatore, TamilNadu, India.

ARTICLE INFO

Article history:

Received: 13 May 2011;

Received in revised form:

6 July 2011;

Accepted: 16 July 2011;

Keywords

Cylindrical panel,

Complex eigenvalues,

Zinc material.

ABSTRACT

In this paper the three dimensional wave propagation in a homogenous Isotropic generalized thermo elastic cylindrical panel is investigated in the context of the linear theory of thermo elasticity. The analysis is carried out by introducing three displacement potentials so that the equations of motion are uncoupled and simplified. A modified Bessel function solution with complex arguments is then directly used for the case of complex eigenvalues , To clarify the correctness and effectiveness of the developed method the dispersion curves of different panel parameters are computed and presented for Zinc material with the support of MATLAB.

Introduction

The analysis of thermally induced vibration of cylindrical panel is common place in the design of structures, atomic reactors, steam turbines , supersonic aircraft, and other devices operating at elevated temperature. In the field of nondestructive evaluation, laser-generated waves have attracted great attention owing to their potential application to noncontact and nondestructive evaluation of sheet materials. The high velocities of modern aircraft give rise to aerodynamic heating, which produces intense thermal stresses, reducing the strength of the aircraft structure. In the nuclear field, the extremely high temperatures and temperature gradients originating inside nuclear reactors influence their design and operations. Moreover, it is well recognized that the investigation of the thermal effects on elastic wave propagation has bearing on many seismological application. This study may be used in applications involving nondestructive testing (NDT), qualitative nondestructive evaluation (QNDE) of large diameter pipes and health monitoring of other ailing infrastructures in addition to check and verify the validity of FEM and BEM for such problems.

The theory of thermo elasticity is well established by Nowacki [1]. Lord and Shulman [2] and Green and Lindsay [3] modified the Fourier law and constitutive relations, so as to get hyperbolic equation for heat conduction by taking into account the time needed for acceleration of heat flow and relaxation of stresses. A special feature of the Green–Lindsay model is that it does not violate the classical Fourier's heat conduction law. Vibration of functionally graded multilayered orthotropic cylindrical panel under thermo mechanical load was analyzed by X.Wang et.al [4]. Hallam and Ollerton [5] investigated the thermal stresses and deflections that occurred in a composite cylinder due to a uniform rise in temperature, experimentally and theoretically and compared the obtained results by a special application of the frozen stress technique of photoelasticity. Noda [6] have studied the thermal-induced interfacial cracking of magneto electro elastic materials under uniform heat flow. Chen et al [7] analyzed the point temperature solution for a pennay-shapped crack in an infinite transversely isotropic thermo-piezo-elastic medium subjected to a concentrated thermal load applied arbitrarily at the crack surface using the generalized potential theory. Banerjee and Pao [8] investigated the propagation of plane harmonic waves in infinitely extended anisotropic solids by taking into account the thermal relaxation time. Dhaliwal and Sherief [9] extended the generalized thermo elasticity to anisotropic elastic bodies. Chadwick [10] studied the propagation of plane harmonic waves in homogenous anisotropic heat conducting solids. Sharma and Sidhu[11] studied the propagation of plane harmonic thermo elastic wave in homogenous transversely isotropic, cubic crystals and anisotropic materials in the context of generalized thermo elasticity. Sharma[12] investigated the three dimensional vibration analysis of a transversely isotropic thermo elastic cylindrical panel. The application of powerful numerical tools like finite element or boundary element methods to these problems is also becoming important. Prevost and Tao [13] carried out an authentic finite element analysis of problems including relaxation effects.

Eslami and Vahedi [14] applied the Galerkin finite element to the coupled thermoelasticity problem in beams. Huang and Tauchert [15]derived the analytical solution for cross-ply laminated cylindrical panels with finite length subjected to mechanical and thermal loads using the extended power series method.

In this paper, the three dimensional wave propagation in a homogeneous isotropic generalized thermo elastic cylindrical panel is discussed using the linear three-dimensional theory of elasticity. The frequency equations are obtained using the boundary conditions. A modified Bessel functions with complex argument is directly used to analyze the frequency equations by fixing the length to mean radius ratio and are studied numerically for the material Zinc. The computed non-dimensional frequencies are plotted in the form of dispersion curves.

Tele:

E-mail addresses: Selvam1729@gmail.com

The Governing equations

Consider a cylindrical panel as shown in Fig.1 of length L having inner and outer radius a and b with thickness h. The angle subtended by the cylindrical panel, which is known as center angle, is denoted by α. The deformation of the cylindrical panel in the direction r, θ, z are defined by u, v and w. The cylindrical panel is assumed to be homogenous, isotropic and linearly elastic with Young's modulus E, poisson ratio ν and density ρ in an undisturbed state.

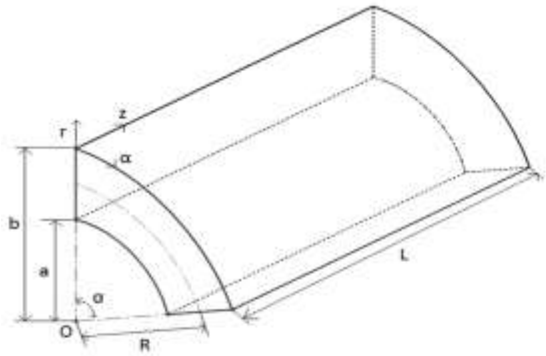

Fig.1 Geometry of the problem

In cylindrical coordinate the three dimensional stress equation of motion, strain displacement relation and heat conduction in the absence of body force for a linearly elastic medium are

$$\sigma_{rr,r} + r^{-1}\sigma_{r\theta,\theta} + \sigma_{rz,z} + r^{-1}\left(\sigma_{rr} - \sigma_{\theta\theta}\right) = \rho u_{,tt}$$

$$\sigma_{r\theta,r} + r^{-1}\sigma_{\theta\theta,\theta} + \sigma_{,rzz} + \sigma_{\theta z,z} + 2r^{-1}\sigma_{r\theta} = \rho v_{,tt} \qquad (1)$$

$$\sigma_{rz,r} + r^{-1}\sigma_{\theta z,\theta} + \sigma_{zz,z} + r^{-1}\sigma_{r\theta} = \rho w_{,tt}$$

$$K(T_{,rr} + \frac{1}{r}T_{,r} + \frac{1}{r^2}T_{,\theta\theta} + T_{,zz}) = \rho C_V T_{,t} + \rho T_{,tt} + \beta T_0(u_{,rt} + \frac{1}{r}(u_{,t} + v_{\theta t}) + w_{,tz})$$

The stress-strain relation of isotropic generalized thermo elastic material is given By Duhamel Neumann

$$\sigma_{rr} = \lambda(e_{rr} + e_{\theta\theta} + e_{zz}) + 2\mu e_{rr} - \beta(T + \eta T_{,t})$$

$$\sigma_{\theta\theta} = \lambda(e_{rr} + e_{\theta\theta} + e_{zz}) + 2\mu e_{\theta\theta} - \beta(T + \eta T_{,t}) \qquad (2)$$

$$\sigma_{zz} = \lambda(e_{rr} + e_{\theta\theta} + e_{zz}) + 2\mu e_{zz} - \beta(T + \eta T_{,t})$$

$$\sigma_{r\theta} = \mu\gamma_{r\theta} \quad \sigma_{rz} = \mu\gamma_{rz} \quad \sigma_{\theta z} = \mu\gamma_{\theta z} \quad e_{rr} = \frac{\partial u}{\partial r}$$

$$e_{\theta\theta} = \frac{u}{r} + \frac{1}{r}\frac{\partial v}{\partial \theta} \qquad (3)$$

$$e_{zz} = \frac{\partial w}{\partial z} \qquad \gamma_{r\theta} = \frac{\partial v}{\partial r} - \frac{v}{r} + \frac{1}{r}\frac{\partial u}{\partial \theta} \qquad \gamma_{rz} = \frac{\partial w}{\partial r} + \frac{\partial u}{\partial z}$$

$$\gamma_{z\theta} = \frac{\partial v}{\partial z} + \frac{1}{r}\frac{\partial w}{\partial \theta}$$

Where u, v, w are displacements along radial, circumferential and axial directions respectively. $\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}, \sigma_{r\theta}, \sigma_{\theta z}, \sigma_{rz}$ are the stress components, $e_{rr}, e_{\theta\theta}, e_{zz}, e_{r\theta}, e_{\theta z}, e_{rz}$ are strain components, T is the temperature change about the equilibrium temperature T_0, ρ is the mass density ,β_1 is the thermal stress coefficient , t is the relaxation time ,C_V is specific heat capacity , η is the generalized thermo elastic constant , K is the thermal conductivity, λ and μ are the Lame's constants.

Substituting the Eqns (3) and Eqns (2) in Eqn(1),gives the following three dimensional equation of motion and heat conduction are obtained as follows:

$$(\lambda + 2\mu)(u_{,rr} + r^{-1}u_{,r} - r^{-2}u) + \mu r^{-2}u_{,\theta\theta} + \mu u_{,zz} + r^{-1}(\lambda + \mu)v_{,r\theta}$$
$$-r^{-2}(\lambda + \mu)v_{,\theta} + (\lambda + \mu)w_{,rz} - \beta(T_{,r} + \eta T_{,rt}) = \rho u_{,tt} \qquad (4a)$$

$$\mu(v_{,rr} + r^{-1}v_{,r} - r^{-2}v) + r^{-2}(\lambda + 2\mu)v_{,\theta\theta} + \mu v_{,zz} + r^{-2}(\lambda + 3\mu)u_{,\theta}$$
$$+r^{-1}(\lambda + \mu)u_{,r\theta} + r^{-1}(\lambda + \mu)w_{,\theta z} - \beta(T_{,\theta} + \eta T_{,\theta t}) = \rho v_{,tt} \qquad (4b)$$

$$(\lambda + 2\mu)w_{,zz} + \mu(w_{,rr} + r^{-1}w_{,r} + r^{-2}w_{,\theta\theta}) + (\lambda + 2\mu)u_{,rz}$$
$$+r^{-1}(\lambda + \mu)v_{,\theta z} + r^{-1}(\lambda + \mu)u_{,z} - \beta(T_{,z} + \eta T_{,zt}) = \rho w_{,tt} \qquad (4c)$$

$$\rho c_v k(T_{,rr} + r^{-1}T_{,r} + r^{-2}T_{,\theta\theta} + T_{,zz}) = \rho\tau T_{,tt} + \rho c_v T_{,t}$$
$$+\beta T_0(u_{,rt} + r^{-1}(u_{,t} + v_{,\theta t}) + w_{,zt}) \qquad (4d)$$

Where $k = K/\rho C_v$, τ is the generalized thermo elasticity , the comma in the subscripts denotes the partial differential equation with respect to the variables.

To uncouple the Eqns (4), the displacement potential u,v,w along the radial,circumferencial and axial directions are assumed following [12]as follows

$$u = \frac{1}{r}\psi,_{\theta} - \phi,_{r} \quad v = -\frac{1}{r}\phi,_{\theta} - \psi,_{r} \quad w = -\chi,_{z} \qquad (5)$$

Using Eqns (5) in Eqns (1), we find that ϕ, χ, T satisfies the equations.

$$((\lambda + 2\mu)\nabla_1^2 + \mu\frac{\partial^2}{\partial z^2} - \rho\frac{\partial^2}{\partial t^2})\phi - (\lambda + \mu)\frac{\partial^2 \chi}{\partial z^2} = \beta\,(T + \eta T,_t) \quad (6a)$$

$$(\mu\nabla_1^2 + (\lambda + 2\mu)\frac{\partial^2}{\partial z^2} - \rho\frac{\partial^2}{\partial t^2})\chi - (\lambda + \mu)\nabla_1^{\,2}\phi = \beta(T + \eta T,_t) \quad (6b)$$

$$\nabla_1^2 T + \frac{\partial^2 T}{\partial z^2} - \frac{\tau}{C_v K}\frac{\partial^2 T}{\partial t^2} - \frac{1}{k}\frac{\partial T}{\partial t} + \frac{\beta T_0(i\omega)}{\rho C_V K}(\nabla_1^2\phi + \frac{\partial^2 \chi}{\partial z^2}) = 0 \quad (6c)$$

$$(\nabla_1^2 + \frac{\partial^2}{\partial z^2} - \frac{\rho}{\mu}\frac{\partial^2}{\partial t^2})\psi = 0 \qquad (6d)$$

Where $\nabla_1^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$

Eqn (6d) in ψ gives a purely transverse wave , which is not affected by temperature. This wave is polarized in planes perpendicular to the z-axis. We assume that the disturbance is time harmonic through the factor $e^{i\omega t}$.

Solution to the problem

The Eqns (6) is coupled partial differential equations of the three displacement components. To uncouple Eqns(6),we can write three displacement functions which satisfies the simply supported boundary conditions followed by Sharma [12]

$$\psi(r,\theta,z,t) = \overline{\psi}(r)\sin(m\pi z)\cos(n\pi\theta/\alpha)e^{i\omega t}$$

$$\phi(r,\theta,z,t) = \overline{\phi}(r)\sin(m\pi z)\sin(n\pi\theta/\alpha)e^{i\omega t} \qquad (7)$$

$$\chi(r,\theta,z,t) = \overline{\chi}(r)\sin(m\pi z)\sin(n\pi\theta/\alpha)e^{i\omega t}$$

$$T(r,\theta,z,t) = \overline{T}(r,\theta,z,t)\sin(m\pi z)\sin(n\pi\theta/\alpha)e^{i\omega t}$$

By introducing the dimensionless quantities:

$$r' = \frac{r}{R} \quad z' = \frac{z}{L} \quad \bar{T} = \frac{T}{T_0} \quad \delta = \frac{n\pi}{\alpha} \quad t_L = \frac{m\pi R}{L} \quad \bar{\lambda} = \frac{\lambda}{\mu}$$

$$\in_4 = \frac{1}{2+\bar{\lambda}}$$

$$C_1^2 = \frac{\lambda+2\mu}{\rho} \quad \Omega^2 = \frac{\omega^2 R^2}{C_1^2} \quad \tau_1 = (1+i\omega\eta) \qquad (8)$$

We obtain the system of Eqns (6) as

$$(\nabla_2^2 + k_1^2)\bar{\psi} = 0 \qquad (9a)$$

$$(\nabla_2^2 + g_3)\bar{\phi} - g_2\bar{\chi} + g_4\bar{T} = 0 \qquad (9b)$$

$$(\nabla_2^2 - g_1)\bar{\chi} + (1+\bar{\lambda})\nabla_2^2\bar{\phi} + (2+\bar{\lambda})g_4\bar{T} = 0 \qquad (9c)$$

$$\left(\nabla_2^2 - t_L^2 + \in_2 \Omega^2 - i\in_3\right)\bar{T} + i\in_1 \Omega\nabla_2^2\bar{\phi} - i\in_1 \Omega t_L^2\bar{\chi} = 0 \qquad (9d)$$

Where

$\nabla_2^2 = \frac{\partial^2}{\partial r^2}\frac{1}{r}\frac{\partial}{\partial r} - \frac{\delta^2}{r^2}$, C_1 wave velocity of the cylindrical panel.

Re writing Eq.(9), results in the following equation.

$$\begin{vmatrix} (\nabla_2^2+g_3) & -g_2 & g_4 \\ (1+\bar{\lambda})\nabla_2^2 & (\nabla_2^2-g_1) & (2+\bar{\lambda})g_4 \\ ig_5\nabla_2^2 & -ig_5t_L^2 & (\nabla_2^2-t_L^2+\in_2\Omega^2-i\Omega\in_3) \end{vmatrix} (\bar{\phi},\bar{\chi},\bar{T}) = 0 \qquad (10)$$

Where $\in_1 = \frac{T_0 R\beta^2}{\rho^2 C_V C_1 K}$; $\in_2 = \frac{\tau C_1^2}{C_V K}$; $\in_3 = \frac{C_1 R}{K}$;

$$g_1 = (2+\bar{\lambda})(t_L^2 - \Omega^2)$$

$$g_2 = \in_4 (1+\bar{\lambda})t_L^2 \quad g_3 = (\Omega^2 - \in_4 t_L^2) \quad g_4 = \frac{\beta T_0 R^2 \tau}{\lambda+2\mu}$$

$$g_5 = \in_1 \Omega$$

Eqn (10), on simplification reduces to the following differential equation:

$$\left(\nabla_2^6 + A\nabla_2^4 + B\nabla_2^2 + C\right)\bar{\phi} = 0 \qquad (11)$$

Where,

$$A = -g_1 + g_2(1+\bar{\lambda}) + g_3 - g_4 g_5 i t_L^2 + \in_2 \Omega^2 - i\in_3 \Omega$$

$$B = -g_1g_3 - g_1g_4g_5 i - g_2g_4g_5 i(2+\bar{\lambda}) + t_l^2(g_1 - g_2 - g_3) + g_4g_5 i t_L^2 + g_2\Omega^2 \in_2 (1+\bar{\lambda}) - g_2 i\in_3 \Omega(1+\bar{\lambda})$$
$$-g_2 t_L^2\bar{\lambda} + g_3\Omega(\Omega\in_2 - i\in_3) + g_1\Omega(i\in_3 - \Omega\in_2)$$

$$C = g_1g_3(t_L^2 + i\in_3\Omega - \in_2\Omega^2) + ig_3g_4g_5t_L^2(2+\bar{\lambda}) \qquad (12)$$

The solution of Eqn (11) are

$$\bar{\phi}(r) = \sum_{i=1}^{3}(A_i J_\delta(\alpha_i r) + B_i Y_\delta(\alpha_i r))$$

$$\bar{\chi}(r) = \sum_{i=1}^{3} d_i(A_i J_\delta(\alpha_i r) + B_i Y_\delta(\alpha_i r))$$

$$\bar{T}(r) = \sum_{i=1}^{3} e_i(A_i J_\delta(\alpha_i r) + B_i Y_\delta(\alpha_i r))$$

$$\bar{\psi}(r) = A_4 J_\delta(k_1 r) + B_4 Y_\delta(k_1 r) \qquad (13)$$

Where $k_1^2 = (2+\bar{\lambda})\Omega^2 - t_L^2$

Where d_i *and* e_i are computed from

$$d_i = \left[\frac{(1+\bar{\lambda})\delta_i^2 - (2+\bar{\lambda})\delta_i^2 - g_1}{g_2(2+\bar{\lambda}) - \delta_i^2 - g_3}\right] \qquad (14)$$

$$e_i = \left(\frac{\lambda+2\mu}{\beta T_0 R^2}\right)\left[\frac{\varepsilon_4\delta_i^2 + \left(\varepsilon_4(g_1+g_3) + \varepsilon_4(1+\bar{\lambda})g_2\right)\delta_i^2 + \varepsilon_4 g_1 g_3 + \delta_i^2) - g_1 g_3}{\varepsilon_4 g_3 + \varepsilon_4\delta_i^2 - g_2}\right] \qquad (15)$$

Eqn (9a) is a Bessel equation with its possible solutions are

$$\bar{\psi} = \begin{cases} A_3 J_\delta(k_1 r) + B_3 Y_\delta(k_1 r), k_1^2 > 0 \\ A_3 r^\delta + B_3 r^{-\delta}, k_1^2 = 0 \\ A_3 I_\delta(k_1' r) + B_3 K_\delta(k_1' r), k_1^2 < 0 \end{cases} \qquad (16)$$

Where $k_1'^2 = -k_1^2$,and , J_δ and Y_δ are Bessel functions of the first and second kinds respectively while, I_δ and k_δ are modified Bessel functions of first and second kinds respectively. A_i and B_i $(i = 1,2,3,4)$ are two arbitrary constants. Generally $k_1^2 \neq 0$,so that the situation $k_1^2 \neq 0$ is will not be discussed in the following . For convenience ,we consider the case of $k_1^2 > 0$,and the derivation for the case of $k_1^2 < 0$ is similar.

Boundary conditions and Frequency equation

In this section we shall derive the secular equation for the three dimensional vibrations cylindrical panel subjected to traction free boundary conditions at the upper and lower surfaces at

$r = a, b$

$$u_r = \left(-\bar{\phi}' - \frac{\delta\bar{\psi}'}{r}\right)\sin(m\pi z)\sin(\delta\theta)e^{i\omega t}$$

$$u_\theta = \left(-\bar{\psi}' - \frac{\delta\bar{\phi}'}{r}\right)\sin(m\pi z)\cos(\delta\theta)e^{i\omega t}$$

$$u_z = \bar{\chi}t_L\cos(m\pi z)\sin(\delta\theta)e^{i\omega t}$$

$$T(r,\theta,z,t) = \bar{T}(r,\theta,z,t)\sin(m\pi z)\sin(n\pi\theta/\alpha)e^{i\omega t}$$

$$\bar{\sigma}_{rr} = \left[(2+\bar{\lambda})\delta\left(\frac{\bar{\psi}'}{r} - \frac{\bar{\psi}}{r^2}\right) + (2+\bar{\lambda})\left(\frac{1}{r}\bar{\varphi}' + (\alpha_i^2 - \frac{\delta^2}{r^2}\bar{\varphi})\right) + \bar{\lambda}\left(\frac{\delta}{r^2}\bar{\psi} - \frac{1}{r}\bar{\phi}' - \frac{\delta^2}{r^2}\bar{\varphi} - \frac{\delta}{r}\bar{\psi}' - t_L^2\bar{\chi}\right)\right]$$
$$\sin(m\pi)z\cos(\delta\theta)e^{i\omega t}$$

$$\bar{\sigma}_{r\theta} = 2\left(\frac{1}{r}\bar{\psi} + (\alpha_i^2 - \frac{\delta^2}{r^2})\bar{\psi} - \frac{2\delta}{r}\bar{\varphi}' + \frac{2\delta}{r^2}\bar{\varphi} + \frac{\bar{\psi}'}{r} - \frac{\delta^2}{r^2}\bar{\psi}\right)\sin(m\pi)z\cos(\delta\theta)e^{i\omega t} \qquad (17)$$

$$\bar{\sigma}_{rz} = 2t_L\left(-\bar{\varphi}' - \frac{\delta}{r}\bar{\psi} + \bar{\chi}'\right)\cos(m\pi)z\sin(\delta\theta)e^{i\omega t}$$

$$\sigma_{rr} = \sigma_{r\theta} = \sigma_{rz} = 0, T_{,r} = 0 \qquad at \quad r = a, b \qquad (18)$$

The frequency equation is obtained as

$$\left|E_{ij}\right| = 0 \quad i,j = 1.2....8 \qquad (19)$$

$$E_{11} = (2+\bar{\lambda})\left((\delta J_\delta(\alpha_1 t_1)/t_1^2 - \frac{\alpha_1}{t_1}J_{\delta+1}(\alpha_1 t_1)) - ((\alpha_1 t_1)^2 R^2 - \delta^2)J_\delta(\alpha_1 t_1)/t_1^2\right)$$
$$+\bar{\lambda}\left(\delta(\delta-1)J_\delta(\alpha_1 t_1)/t_1^2 - \frac{\alpha_1}{t_1}J_{\delta+1}(\alpha_1 t_1)\right) + \bar{\lambda}d_1 t_L^2 J_\delta(\alpha_1 t_1) - \beta T_0 R^2 e_1\bar{\lambda}$$

$$E_{13} = (2+\bar{\lambda})\left((\delta J_\delta(\alpha_2 t_1)/t_1^2 - \frac{\alpha_2}{t_2}J_{\delta+1}(\alpha_2 t_1)) - ((\alpha_2 t_1)^2 R^2 - \delta^2)J_\delta(\alpha_2 t_1)/t_1^2\right) \qquad (20)$$
$$+\bar{\lambda}\left(\delta(\delta-1)J_\delta(\alpha_2 t_1)/t_1^2 - \frac{\alpha_2}{t_1}J_{\delta+1}(\alpha_2 t_1)\right) + \bar{\lambda}d_2 t_L^2 J_\delta(\alpha_2 t_1) - \beta T_0 R^2 e_2\bar{\lambda}$$

$$E_{15}=(2+\bar{\lambda})\left((\delta J_{\delta}(\alpha_3 t_1)/t_1^{\,2}-\frac{\alpha_2}{t_2}J_{\delta+1}(\alpha_3 t_1))-((\alpha_3 t_1)^2R^2-\delta^2)J_{\delta}(\alpha_3 t_1)/t_1^{\,2}\right)$$
$$+\bar{\lambda}\left(\delta(\delta-1)J_{\delta}(\alpha_3 t_1)/t_1^{\,2}-\frac{\alpha_2}{t_1}J_{\delta+1}(\alpha_3 t_1)\right)+\bar{\lambda}d_3 t_L^{\,2}J_{\delta}(\alpha_3 t_1)-\beta T_0R^2e_3\bar{\lambda}$$
$$E_{17}=(2+\bar{\lambda})\left((\frac{k_1\delta}{t_1}J_{\delta+1}(k_1t_1)-\delta(\delta-1)J_{\delta}(k_1t_1)/t_1^{\,2}\right)+\bar{\lambda}\left(\delta(\delta-1)J_{\delta}(k_1t_1)/t_1^{\,2}-\frac{k_1\delta}{t_1}J_{\delta+1}(k_1t_1)\right)$$
$$E_{21}=2\delta\left((\alpha_1/t_1)J_{\delta+1}(\alpha_1t_1)-\delta(\delta-1)J_{\delta}(\alpha_1t_1)\right)$$
$$E_{23}=2\delta\left((\alpha_2/t_1)J_{\delta+1}(\alpha_2t_1)-\delta(\delta-1)J_{\delta}(\alpha_2t_1)\right)$$
$$E_{25}=2\delta\left((\alpha_3/t_1)J_{\delta+1}(\alpha_3t_1)-\delta(\delta-1)J_{\delta}(\alpha_3t_1)\right)$$
$$E_{27}=(k_1t_1)^2R^2J_{\delta}(k_1t_1)-2\delta(\delta-1)J_{\delta}(k_1t_1)/t_1^2+k_1/t_1J_{\delta+1}(k_1t_1)$$
$$E_{31}=-t_L(1+d_1)\left(\delta/t_1J_{\delta}(\alpha_1t_1)-\alpha_1J_{\delta+1}(\alpha_1t_1)\right)$$
$$E_{33}=-t_L(1+d_2)\left(\delta/t_1J_{\delta}(\alpha_2t_1)-\alpha_2J_{\delta+1}(\alpha_2t_1)\right)$$
$$E_{35}=-t_L(1+d_3)\left(\delta/t_1J_{\delta}(\alpha_3t_1)-\alpha_2J_{\delta+1}(\alpha_3t_1)\right)$$
$$E_{37}=-t_L(\delta/t_1)J_{\delta}(k_1t_1)$$
$$E_{41}=e_1[(\delta/t_1)J_{\delta}(\alpha_1t_1)-(\alpha_1)J_{\delta+1}(\alpha_1t_1)]$$
$$E_{43}=e_2[(\delta/t_1)J_{\delta}(\alpha_2t_1)-(\alpha_2)J_{\delta+1}(\alpha_2t_1)]$$
$$E_{45}=e_3[(\delta/t_1)J_{\delta}(\alpha_3t_1)-(\alpha_3)J_{\delta+1}(\alpha_3t_1)]$$
$$E_{47}=0$$

In which $t_1=a/R=1-t^*/2$, $t_2=b/R=1+t^*/2$ and $t^*=b-a/R$ is the thickness -to-mean radius ratio of the panel. Obviously $E_{ij}\left(j=2,4,6,8\right)$ can obtained by just replacing modified Bessel function of the first kind in $E_{ij}\left(i=1,3,5,7\right)$ with the ones of the second kind, respectively, while $E_{ij}\left(i=5,6,7,8\right)$ can be obtained by just replacing t_1 in $E_{ij}\left(i=1,2,3,4\right)$ with t_2 .

Numerical results and discussion

The frequency Eqn (19) is numerically solved for Zinc material. For the purpose of numerical computation we consider the closed circular cylindrical shell with the center angle $\alpha=2\pi$ and the integer n must be even since the shell vibrates in circumferential full wave.

The frequency equation for a closed cylindrical shell can be obtained by setting $\delta=l\left(l=1,2,3.....\right)$ where l is the circumferential wave number in Eqns(20). The material properties of a Zinc material is taken from[12]

$\rho=7.14\times10^3 kgm^{-3}$,

$T_0=296^0K$, $\beta=1$, $K=1$, $\varepsilon_1=0.0221$ $\omega_1^*=5.01\times10^{11}\sec^{-1}$

$\mu=0.508\times10^{11}Nm^{-2}$, $\lambda=0.385\times10^{11}Nm^{-2}$,

$C_V=3.9\times10^2 Jkg^{-1}\deg^{-1}$, $\nu=0.3$

A dispersion curve is drawn between the non-dimensional axial wave number versus dimensionless frequency with and without thermal effect for the different thickness parameters t*=0.01,0.1,0.25,0.5 for the circumferential wave number $\delta=1$ is shown in Fig.2 and Fig.3 respectively. From the Figs.2 and 3, it is observed that the non-dimensional frequency increases rapidly to become linear for $t_L\le0.8$ and quite dispersive for $t_L>0.8$ for all value of t*.On comparison, the trends of variation of non dimensional frequency in thermo elastic shell is similar to that in elastic shell as can be observed from Fig.2 and Fig.3, but there is a significant modifications in their magnitude due to thermal effect at all values of t*with tL. when the thickness of the cylindrical panel is increased, the dimensionless frequency is decreases . This is the proper physical behavior of a cylindrical panel with respect to its thickness.

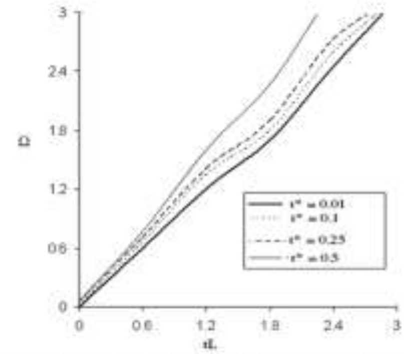

Fig.2. variation of non dimensinal frequency in elastic zinc cylindrical shell with tL.

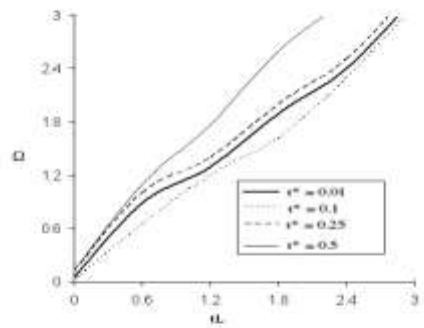

Fig 3. variation of non dimensional frequency in thermoelastic zinc cylindrical shell with tL

Conclusion

The three dimensional wave propagation analysis of a homogenous isotropic generalized thermo elastic cylindrical panel subjected to simply supported boundary conditions has been considered for this paper. For this problem, the governing equations of three dimensional linear elasticity have been employed and solved by modified Bessel function with complex argument. The effect of the axial wave number on the natural frequencies with and without thermal field of a closed Zinc cylindrical shell is investigated and the results are presented as dispersion curves.

References:

[1]. Nowacki. W., 1975,"Dynamical problems of thermo elasticity" , Noordhoff, Leyden, The Netherlands.

[2]. Lord and Y. Shulman., 1967, "A generalized dynamical theory of thermo elasticity", J. Mech. Phys. Solids.,**15**, pp. 299–309.

[3]. Green A.E. ,and K.A. Lindsay , 1972, "Thermo elasticity",. J. Elast., 2, pp. 1–7.

[4]. Wang.X., 2008,"Three dimensional analysis of multi layered functionally graded anisotropic cylindrical panel under thermo mechanical load", Mechanics of materials., 40,pp.235-254.

[5]. Hallam, C. B., Ollerton, E., 1973. "Thermal stresses in axially connected circular cylinders"., J. Strain Analysis., 8(3), pp.160-167.

[6]. Gao, C. F., and Noda, N, 2004. "Thermal-induced interfacial cracking of magneto electro elastic materials"., Int. J. Engg. Sci, 42, pp1347-1360.

[7] Chen, W. Q., Lim, C. W., and Ding, H. J., 2005, "Point temperature solution for a penny-shaped crack in an infinite transversely isotropic thermo-piezo-elastic medium"., Engng. Analysis with Boundary elements. 29, 524-532.
[8]. Banerjee D.K., and Y.H. Pao , 1974, "Thermo elastic waves in anisotropic solids"., *J.* Acoust. Soc. Am. 56 ,pp. 1444–1454.
[9] Dhaliwal R.S., and H.H. Sherief , 1980, "Generalized thermo elasticity for anisotropic media"., Quart.J. Appl. Math. 38 ,pp. 1–8.
[10] Chadwick. P., 2002, "Basic prosperities of plane harmonic waves in a pre stressed heat conducting elastic material", J.Therm.Stresses, 2,PP.193-214.
[11] Sharma J.N., and R.S.Sidhu,1986,"On the propagation of plane harmonic waves in anisotropic generalized thermoelasticity",Int.J.Eng.Sci,24,pp.1511-1516.
[12] Sharma J.N., 2002,"Free vibration analysis of homogenous transversly isotropic thermo elastic cylindrical panel",J.Thermal stress., 25,pp.169-182.
[13] Prevost J.H., and D. Tao , 1983, "Finite element analysis of dynamic coupled thermo elasticity problems with relaxation times", J. Appl. Mech. Trans. ASME ,50**,** pp. 817–822.
[14] Eslami M.R.,and H. Vahedi ,1989, "Coupled thermo elasticity beam problems". AIAA J,27 5,pp. 662–665.
[15] Huang, N.N., Tauchert, T.R., 1991,"Thermoelastic solution for cross-ply cylindrical panels", J. Thermal Stresses, 14, pp.227–237.