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# A differential equation model of Iodine cycle in the human body

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**ABSTRACT**

The paper mainly calculates and analyzes the model of Iodine cycle in the human body. The model can be divided into two cases: when the parameter is not certain, it could analyze the deviation of content of iodine in the body and normal; when the parameter is certain, it makes the general form of solution and the method is general and practical.



## Introduction

Iodine is a component constitutes the core of thyroid hormone, TSH stimulates carbohydrate, protein, fat and salt metabolism and promote protein synthesis and body growth and development, on the central nervous system, circulatory system, hematopoietic system, and muscle activity and so has a significant role. In addition, iodine also can regulate the body's metabolism of calcium and phosphorus and other elements. Therefore, the iodine on human health has a very important role. If people are long iodine deficiency in the diet, it will suffer from iodine deficiency disorders (IDD), which is thyroid disease, or "to a disease ". However, excessive intake of iodine in the body long term, the iodine would inhibit the peroxidation activity of thyroid cells, impeding the synthesis of TSH, thyroid follicular cavity and hinder the hydrolysis of thyroglobulin, resulting in lower blood PH level, under the action of the feedback. The secret too much pituitary TSH, the formation of goiter caused by the excess of iodine. If who once intake of excessive iodine (For example, blindly taking iodized calcium and iodized oil pills), Iodine poisoning will be happened.

## Model proposed

Iodine in the human body has three existing forms. The first comes from the food potassium iodide (KI) and sodium iodide (NaI) and other inorganic iodine. The intake of inorganic iodine in the human body through the role of the body turn into organic iodine. Organic iodine transformed into hormone iodine. Assuming every day the volume of iodine exposure is I0, inorganic iodine is, organic iodine is I2(t), hormone iodine is I3(t), literature[1] offered iodine in cycle of the human body differential equation model.

$$\begin{cases} \dfrac{dI_1}{dt} = \text{-}(k_1+k_2)I_1 + k_3 I_3 + I_0 \\ \dfrac{dI_2}{dt} = k_2 I_1 - k_4 I_2 \\ \dfrac{dI_3}{dt} = k_4 I_2 - (k_3+k_5) I_3 \end{cases}$$

Which, $k_1,k_2,k_3,k_4,k_5$ Î $Z^{+}$ is constant, data can be measured by the clinical trials. Then calculated.

Note,

$$A = \begin{bmatrix} -(k_1+k_2) & 0 & k_3 \\ k_2 & -k_4 & 0 \\ 0 & k_4 & -(k_3+k_5) \end{bmatrix}$$

, $x(t) = (I_1(t), I_2(t), I_3(t))^T$ , $f(t) = (I_0, 0, 0)^T$ .Then formula (1) can be written as $x(t) = Ax + f(t)$ the characteristic matrix of matrix A is

$$B = \begin{bmatrix} -(k_1+k_2)-\lambda & 0 & k_3 \\ k_2 & -k_4-\lambda & 0 \\ 0 & k_4 & -(k_3+k_5)-\lambda \end{bmatrix} \quad (2)$$

## Model Analysis

We can reference to such a model to analyzed mechanism of patient illness. If we do not have a convenient measurement find out specific values of $k_1,k_2,k_3,k_4,k_5$ .we can get help from Mathematical calculate the differential equation has three forms of the complex eigenvalue a complex form eigenvalue $\lambda_1,\lambda_2,\lambda_3$ .then we use $\hat{I}_1,\hat{I}_2,\hat{I}_3$ separately express the normal average of inorganic iodine, organic iodine, hormone iodine's content. We need to study the deviation between $I_1(t), I_2(t), I_3(t)$ and their normal values: $\tilde{I}_1 = I_1(t) - \hat{I}_1, \tilde{I}_2 = I_2(t) - \hat{I}_2, \tilde{I}_3 = I_3(t) - \hat{I}_3$ . Then

$$\begin{cases} \dfrac{d\tilde{I}_1}{dt} = F_1(\hat{I}_1+\tilde{I}_1, \hat{I}_2+\tilde{I}_2, \hat{I}_3+\tilde{I}_3) + I_0 \\ \dfrac{d\tilde{I}_2}{dt} = F_2(\hat{I}_1+\tilde{I}_1, \hat{I}_2+\tilde{I}_2, \hat{I}_3+\tilde{I}_3) \\ \dfrac{d\tilde{I}_3}{dt} = F_3(\hat{I}_1+\tilde{I}_1, \hat{I}_2+\tilde{I}_2, \hat{I}_3+\tilde{I}_3) \end{cases}$$

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Suppose $F_1, F_2, F_3$ are continuously differentiable ternary function. we can learn from the Brook Taylor expansion of multivariate function, $F_1(\hat{I}_1+\tilde{I}_1, \hat{I}_2+\tilde{I}_2, \hat{I}_3+\tilde{I}_3)$

$$= F_1(\hat{I}_1,\hat{I}_2,\hat{I}_3) + \frac{\partial F_1(\hat{I}_1,\hat{I}_2,\hat{I}_3)}{\partial I_1(t)}\tilde{I}_1 + \frac{\partial F_1(\hat{I}_1,\hat{I}_2,\hat{I}_3)}{\partial I_2(t)}\tilde{I}_2 + \frac{\partial F_1(\hat{I}_1,\hat{I}_2,\hat{I}_3)}{\partial I_3(t)}\tilde{I}_3 + o_1$$

$$F_2(\hat{I}_1+\tilde{I}_1, \hat{I}_2+\tilde{I}_2, \hat{I}_3+\tilde{I}_3)$$

$$= F_2(\hat{I}_1,\hat{I}_2,\hat{I}_3) + \frac{\partial F_2(\hat{I}_1,\hat{I}_2,\hat{I}_3)}{\partial I_1(t)}\tilde{I}_1 + \frac{\partial F_2(\hat{I}_1,\hat{I}_2,\hat{I}_3)}{\partial I_2(t)}\tilde{I}_2 + \frac{\partial F_2(\hat{I}_1,\hat{I}_2,\hat{I}_3)}{\partial I_3(t)}\tilde{I}_3 + o_2$$

$$F_3(\hat{I}_1+\tilde{I}_1, \hat{I}_2+\tilde{I}_2, \hat{I}_3+\tilde{I}_3)$$

$$= F_3(\hat{I}_1,\hat{I}_2,\hat{I}_3) + \frac{\partial F_3(\hat{I}_1,\hat{I}_2,\hat{I}_3)}{\partial I_1(t)}\tilde{I}_1 + \frac{\partial F_2(\hat{I}_1,\hat{I}_2,\hat{I}_3)}{\partial I_2(t)}\tilde{I}_2 + \frac{\partial F_3(\hat{I}_1,\hat{I}_2,\hat{I}_3)}{\partial I_3(t)}\tilde{I}_3 + o_3$$

Which $o_1, o_2, o_3$ are higher order small amount about $\tilde{I}_1, \tilde{I}_2, \tilde{I}_3$, when deviations between $I_1(t), I_2(t), I_3(t)$ and Normal of $\tilde{I}_1, \tilde{I}_2, \tilde{I}_3$ are not big, $o_1, o_2, o_3$ can be ignore. And because $\hat{I}_1, \hat{I}_2, \hat{I}_3$ are normal, should satisfy $F_1(\hat{I}_1,\hat{I}_2,\hat{I}_3) = F_2(\hat{I}_1,\hat{I}_2,\hat{I}_3) = F_3(\hat{I}_1,\hat{I}_2,\hat{I}_3) = 0$, so

$$\begin{cases} \frac{d\tilde{I}_1}{dt} = \frac{\partial F_1(\hat{I}_1,\hat{I}_2,\hat{I}_3)}{\partial I_1(t)}\tilde{I}_1 + \frac{\partial F_1(\hat{I}_1,\hat{I}_2,\hat{I}_3)}{\partial I_2(t)}\tilde{I}_2 + \frac{\partial F_1(\hat{I}_1,\hat{I}_2,\hat{I}_3)}{\partial I_3(t)}\tilde{I}_3 + I_0 \\ \frac{d\tilde{I}_2}{dt} = \frac{\partial F_2(\hat{I}_1,\hat{I}_2,\hat{I}_3)}{\partial I_1(t)}\tilde{I}_1 + \frac{\partial F_2(\hat{I}_1,\hat{I}_2,\hat{I}_3)}{\partial I_2(t)}\tilde{I}_2 + \frac{\partial F_2(\hat{I}_1,\hat{I}_2,\hat{I}_3)}{\partial I_3(t)}\tilde{I}_3 \\ \frac{d\tilde{I}_3}{dt} = \frac{\partial F_3(\hat{I}_1,\hat{I}_2,\hat{I}_3)}{\partial I_1(t)}\tilde{I}_1 + \frac{\partial F_2(\hat{I}_1,\hat{I}_2,\hat{I}_3)}{\partial I_2(t)}\tilde{I}_2 + \frac{\partial F_3(\hat{I}_1,\hat{I}_2,\hat{I}_3)}{\partial I_3(t)}\tilde{I}_3 \end{cases}$$

Even if the specific values of $k_1, k_2, k_3, k_4, k_5$ can be known, so the value of partial derivative of $F_1, F_2, F_3$ in the point of $(\hat{I}_1, \hat{I}_2, \hat{I}_3)$ cannot be determined, But we can analyze the symbol, to simulate the diagnosis of the patient's basic information, analyze the three levels of iodine in the human body is inadequate or excessive.

If we know the value of $k_1, k_2, k_3, k_4, k_5$, and matrix of initial conditions $C = [I_1(0), I_2(0), I_3(0)]^T$ .Then we solve the answer for model $(1)$ [2]. From $(2)$ we can know the three eigenvalues about $(1)$ : $\lambda_1, \lambda_2, \lambda_3$ (Because the expression is too complex form, so I do not give).If $\lambda_1, \lambda_2, \lambda_3$ are three different simple eigenvalue .

Order $r(l) = b_2 l^2 + b_1 l + b_0$

Then $\begin{cases} r(\lambda_1) = b_2\lambda_1^2 + b_1\lambda_1 + b_0 = e^{\lambda_1 t} \\ r(\lambda_1) = b_2\lambda_1^2 + b_1\lambda_1 + b_0 = e^{\lambda_2 t} \\ r(\lambda_1) = b_2\lambda_1^2 + b_1\lambda_1 + b_0 = e^{\lambda_3 t} \end{cases}$

Solved

$$\begin{cases} b_0 = \frac{e^{\lambda_1 t}\lambda_2\lambda_3(\lambda_2-\lambda_3) + e^{\lambda_2 t}\lambda_1(\lambda_1-\lambda_3)(2\lambda_3+\lambda_2) + e^{\lambda_3 t}\lambda_1\lambda_2(\lambda_1-\lambda_2)}{(\lambda_1-\lambda_2)(\lambda_2-\lambda_3)(\lambda_1-\lambda_3)} \\ b_1 = \frac{e^{\lambda_1 t}(\lambda_2-\lambda_3) - e^{\lambda_2 t}(\lambda_1-\lambda_3) + e^{\lambda_3 t}(\lambda_1-\lambda_2)}{(\lambda_1-\lambda_2)(\lambda_2-\lambda_3)(\lambda_1-\lambda_3)} \\ b_2 = \frac{e^{\lambda_1 t}(\lambda_3^2-\lambda_2^2) + e^{\lambda_2 t}(\lambda_1-\lambda_3)(\lambda_1-\lambda_2+2\lambda_3) - e^{\lambda_3 t}(\lambda_1^2-\lambda_2^2)}{(\lambda_1-\lambda_2)(\lambda_2-\lambda_3)(\lambda_1-\lambda_3)} \end{cases}$$

so, $e^{At} = b_2A^2 + b_1A + b_0I$ ,then can be drawn $e^{At}C$ .

But $\int_0^t e^{-A\tau} f(\tau)d\tau = \int_0^t \begin{bmatrix} I_0 e^{-\tau} \\ 0 \\ 0 \end{bmatrix} d\tau = \begin{bmatrix} I_0 - I_0 e^{-t} \\ 0 \\ 0 \end{bmatrix}$,

So solutions of model $(1)$ $x(t) = e^{A(t-t_0)}C + e^{At}\int_{t_0}^t e^{-A\tau} f(\tau)d\tau$ can be obtained.

Select time $t_1, t_2, \ldots, t_n$, Have observations $x_1, x_2, \ldots, x_n$, requires a variance

$\sum_{j=1}^{n}[x_j - x(t)]^2 = \min$, from minimum variance method we can solved $I_0$, Thus reasonable to diagnose the patient's condition.

In the absence of clinical data, this paper only gives a general solution, but the conclusion has scientific value, the method is also general.

## Conclusion

In this paper, the cycle of iodine in the body differential equation model to do a detailed analysis, presented a number of methods to solve the problem from mathematical. In this paper, the body has the general solution of iodine cycle, for the other model, you can use the same method. In short, differential equation and its solution in medicine, biology, economics and other fields has great prospect[3-5].

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