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K – Even Mean Labeling of $C_n \cup P_m$

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ABSTRACT

Mean labeling of graphs was discussed in [10] and the concept of odd mean labeling was introduced in [9]. k-odd mean labeling and (k, d) - odd mean labeling are introduced and discussed in [5], [6], [7]. In this paper, we introduce the concept of k-even mean labeling and investigate k-even mean labeling of $C_n \cup P_m$.

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Introduction

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of Harary [8]. The symbols $V(G)$ and $E(G)$ will denote the vertex set and edge set of a graph. Labeled graphs serve as useful models for a broad range of applications [1-3].

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices (or edges) then the labeling is called a vertex labeling (or an edge labeling).

Graph labeling were first introduced in the late 1960's. Many studies in graph labeling refer to Rosa's research in 1967 [11].

Labeled graphs serve as useful models for a broad range of applications such as X-ray crystallography, radar, coding theory, astronomy, circuit design and communication network addressing. Particularly interesting applications of graph labeling, can be found in [4].

Mean labeling of graphs was discussed in [10] and the concept of odd mean labeling was introduced in [9]. k-odd mean labeling and (k, d) – odd mean labeling are introduced and discussed in [5], [6], [7]. In this paper, we introduce the concept of k-even mean labeling and here we investigate the k-even mean labeling of $C_n \cup P_m$.

Throughout t0068is paper, k denote any positive integer ≥ 1. For brevity, we use k-EML for k-even mean labeling.

Main Results

Definition: *k*-even mean labeling

A (p, q) graph G is said to have a k-even mean labeling if there exists a injection $f : V \to \{0, 1, 2, ..., 2k + 2(q\text{-}1)\}$ such that the induced map $f^* : E(G) \to \{2k, 2k + 2, 2k + 4, ..., 2k + 2(q – 1)\}$ defined by is a bijection.

$$f^*(uv) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$$

A graph that admits a k – even mean labeling is called a k – even mean graph.

Theorem

The graph $C_n \cup P_m$ ($n \geq 4$ and $m \geq 2$) is a k-even mean graph for any k when $n \neq 6$.

Proof

Let $V(C_n \cup P_m) = \{v_i, 1 \leq i \leq n\} \cup \{u_j, 1 \leq j \leq m\}$ and $E(C_n \cup P_m) = \{e_i, 1 \leq i \leq n\} \cup \{e_j^{'}, 1 \leq j \leq m-1\}$ (see Fig. 2.1)

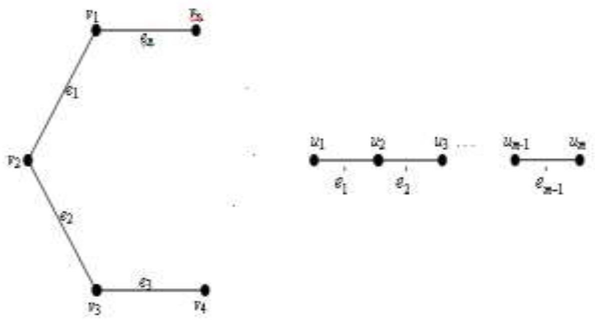

Fig. 2.1: Ordinary labeling of $C_n \cup P_m$

First we label the vertices of $C_n \cup P_m$ as follows:

Define

$f : V(C_n \cup P_m) \to \{0, 1, 2, \ldots, 2k + 2q - 2\}$ by

Case (i) $n \equiv 0 \pmod 4$

For $1 \leq i \leq \frac{n}{2}$,

$$f(v_i) = \begin{cases} 2k + 4i - 5 & \text{if } i \text{ is odd} \\ 2k + 4i - 7 & \text{if } i \text{ is even} \end{cases}$$

For $\frac{n+2}{2} \leq i \leq n$,,

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$$f(v_i) = \begin{cases} 2k+4n-4i+2 & \text{if } i \text{ is odd} \\ 2k+4n-4i+5 & \text{if } i \text{ is even} \end{cases}$$

The vertex labels of P_m are

For $1 \le j \le m-1$,

$$f(u_j) = 2k+2n+2j-3$$

$$f(u_m) = 2k+2(n+m-1)-2$$

Then the induced edge labels are

$$f^*(e_i) = \begin{cases} 2k+4i-4, & 1 \le i \le \frac{n}{2} \\ 2k+4n-4i+2, & \frac{n+2}{2} \le i \le n \end{cases}$$

For $1 \le j \le m-1$

$$f^*(e_j') = 2k+2n+2j-2$$

$$f^*(e_m') = 2k + 2(n + m - 1) - 2$$

2-EML of $C_4 \cup P_5$ is shown in Fig. 2.2.

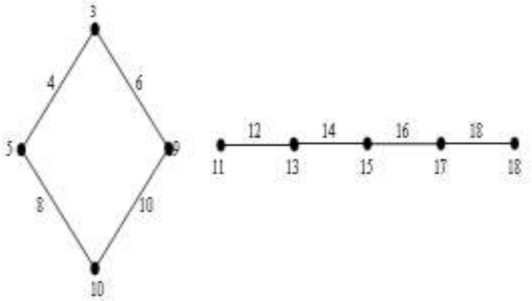

Fig. 2.2: 2-EML of $C_4 \cup P_5$

2-OML of $C_8 \cup P_8$ is shown in Fig. 2.3.

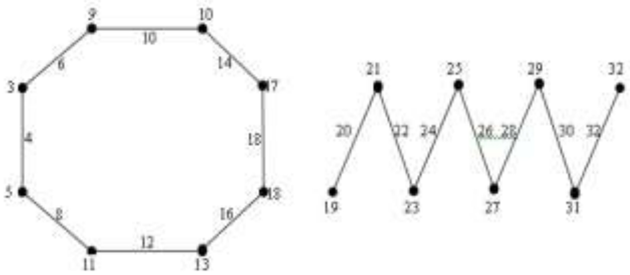

Fig. 2.3: 2-OML of $C_8 \cup P_8$

Case (ii) $n \equiv 1, 3 \pmod 4$

For $1 \le i \le \frac{n-3}{2}$,

$$f(v_i) = 2k+2i-3$$

$$f\left(v_{\frac{n-1}{2}}\right) = 2k+n-5$$

$$f\left(v_{\frac{n+1}{2}}\right) = 2k+n-1$$

For $\frac{n+3}{2} \le i \le n-1$,

$$f(v_i) = 2k+2i-1$$

$$f(v_n) = 2k+2n-2$$

For $1 \le j \le m-1$,

$$f(u_j) = 2k+2n+2j-3$$

$$f(u_m) = 2k+2(n+m-1)-2$$

Then the induced edge labels are

$$f^*(e_i) = \begin{cases} 2k+2i-2, & 1 \le i \le \frac{n-1}{2} \\ 2k+2i, & \frac{n+1}{2} \le i \le n-1 \end{cases}$$

$$f^*(e_n) = 2k+n-1$$

For $1 \le j \le m-1$

$$f^*(e_j') = 2k+2n+2j-2$$

$$f^*(e_m') = 2k + 2(m + n - 1) - 2$$

Therefore,

$$f^*(E(C_n \cup P_m) \to \{2k, 2k+2, 2k+4, \ldots, 2k+2q-2\}$$

So, f is a k-even mean labeling and hence, $C_n \cup P_m$, $m \ge 2$ and $n \ne 6$ is a k-even mean graph for any k. 3-EML of $C_5 \cup P_4$ is shown in Fig. 2.4.

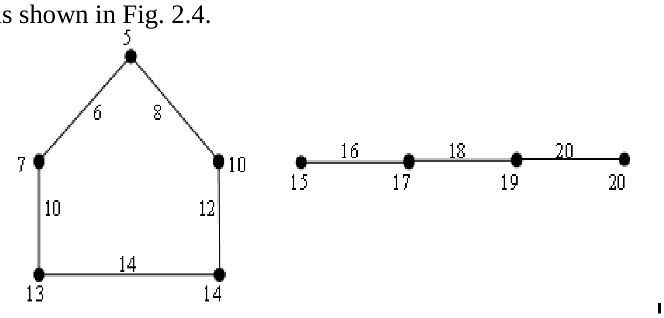

Fig. 2.4: 3-EML of $C_5 \cup P_4$

1-EML of $C_9 \cup P_3$ is shown in Fig. 2.5.

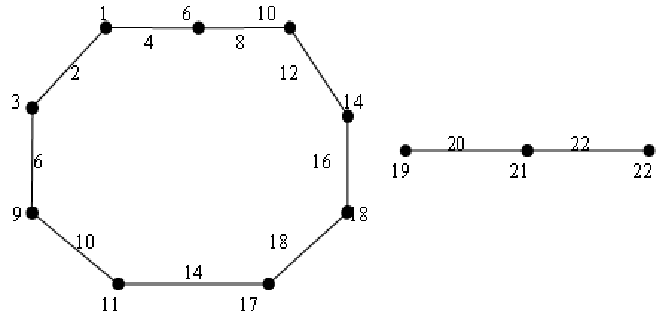

Fig. 2.5: 1-EML of $C_9 \cup P_3$

2-EML of $C_7 \cup P_6$ is shown in Fig. 2.6.

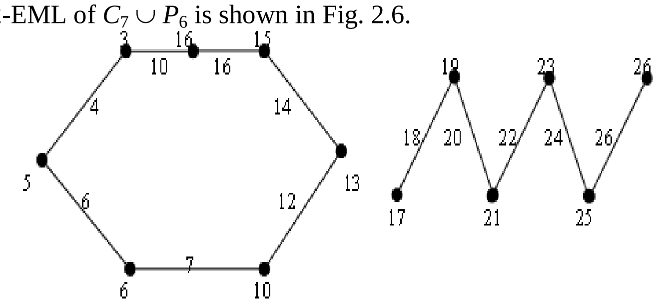

Fig. 2.6: 2-EML of $C_7 \cup P_6$

Case (iii) $n \equiv 2 \pmod 4$, $n > 6$

For $1 \le i \le \frac{n-4}{2}$,

$f(v_i) = 2k + 2i - 3$

$f\left(v_{\frac{n-2}{2}}\right) = 2k + n - 6$

$f\left(v_{\frac{n}{2}}\right) = 2k + n - 2$

For $\frac{n+2}{2} \le i \le n-3,$

$f(v_i) = 2k + 2i - 1$

$f(v_{n-2}) = 2k + 2n - 6$

$f(v_{n-1}) = 2k + 2n - 2$

$f(v_n) = 2k + 2n - 3$

For $1 \le j \le m - 1$,

$f(u_j) = 2k + 2n + 2j - 3$

$f(u_m) = 2k + 2(n + m - 1) - 2$

Then the induced edge labels are

$$f^*(e_i) = \begin{cases} 2k + 2i - 2, & 1 \le i \le \frac{n-2}{2} \\ 2k + 2i, & \frac{n}{2} \le i \le n-1 \end{cases}$$

$f^*(e_n) = 2k + n - 2$

For $1 \le j \le m-1$

$f^*(e_j^{'}) = 2k + 2n + 2j - 2$

$f^*(e_m^{'}) = 2k + 2(m + n - 1) - 2$

3-EML of $C_{14} \cup P_5$ is shown in Fig. 2.7.

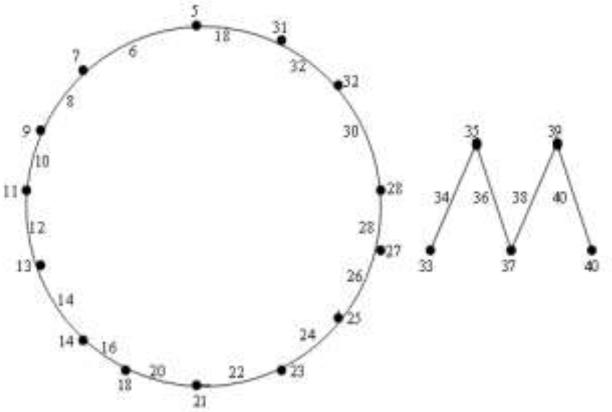

Fig. 2.7: 3-EML of $C_{14} \cup P_5$

Theorem

The graph $C_6 \cup P_m$, $m \ge 2$ is a k-even mean graph for any k.

Proof

Let $V(C_6 \cup P_m) = \{v_i, 1 \le i \le 6\} \cup \{u_j, 1 \le j \le m\}$ and $E(C_6 \cup P_m) = \{e_i, 1 \le i \le 5\} \cup \{e_n\} \cup \{e_j^{'}, 1 \le j \le m-1\}$ (see Fig. 2.1)

First we label the vertices of $C_6 \cup P_m$ as follows:

Define $f : V(C_6 \cup P_m) \to \{0, 1, 2, \ldots, 2k + 2q - 2\}$ by

For $1 \le i \le 4$,

$f(v_i) = 2k + 2i - 3$

$f(v_5) = 2k + 11$

$f(v_6) = 2k + 12$

Case (i) when $m = 2$

$f(u_1) = 2k + 9$

$f(u_2) = 2k + 10$

Case (ii) when $m > 2$

$f(u_1) = 2k + 7$

For $2 \le i \le m - 1$,

$f(u_i) = 2k + 2i + 9$

$f(u_m) = 2k + 2m + 8$

Then the induced edge labels are

$$f^*(e_i) = \begin{cases} 2k + 2i - 2, & 1 \le i \le 3 \\ 2k + 4i - 8, & 4 \le i \le 5 \end{cases}$$

$f^*(e_6) = 2k + 6$

$f^*(e_1^{'}) = 2k + 10$

For $2 \le j \le m - 1$

$f^*(e_j^{'}) = 2k + 2j + 10$

$f^*(e_m^{'}) = 2k + 2(m + n - 1) - 2$

Therefore,

$f^*(E(C_6 \cup P_m) \to \{2k, 2k+2, 2k+4, \ldots, 2k+2q-2\}$

So, f is a k-even mean labeling and hence $C_6 \cup P_m$, $m \ge 2$ is a k-even mean graph for any k.

5-EML of $C_6 \cup P_2$ is shown in Fig. 2.8.

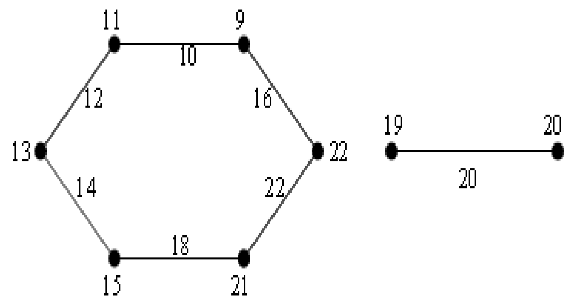

Fig. 2.8: 5-EML of $C_6 \cup P_2$

8-EML of $C_6 \cup P_7$ is shown in Fig. 2.9.

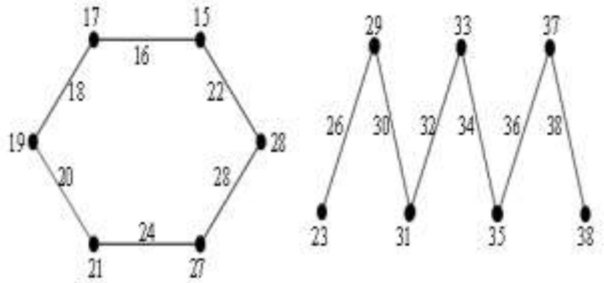

Fig. 2.9: 8-EML of $C_6 \cup P_7$

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