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Discrete Mathematics

Elixir Dis. Math. 36 (2011) 3094-3096

ISSN: 2229-712X

Theorem on n-triangular form of fuzzy context free grammar

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ARTICLE INFO

Article history:
Received: 6 May 2011;
Received in revised form:
20 June 2011;
Accepted: 29 June 2011;

Keywords
Fuzzy context free grammar,
Triangular form of fuzzy context free grammar,
N-triangular form of fuzzy context free grammar.

ABSTRACT
Every fuzzy context free language L(G) can be generated by N-triangular form of fuzzy context free grammar is Proved in this paper with illustrated examples.

Introduction

Fuzzy context free grammar is a natural choice to generate a fuzzy context free language. Similarly the N-triangular form of fuzzy context free grammar is used to generate more than one fuzzy context free languages. In this paper, equivalent to the given fuzzy context free grammar and N-triangular form of fuzzy context free grammar are discussed with illustrative example. Definitions of context free grammar and procedure for derivation of the string are discussed by Hopcroft.J.E, Ullman.J.D. [1]. Definitions of fuzzy grammar, fuzzy context free grammars and calculation of membership degree of the string are discussed by Blanco.A., Delgado.M., Pegalajar.M.C., John N. Mordeson and Davender S. Malik. [1], [4]. N-triangular form of fuzzy context free grammar is defined by Ismail Mohideen.S., and Satheesh.V. [3] with examples.

In section 2 of this paper basic definitions of triangular form of fuzzy context free grammar and N-triangular form of fuzzy context free grammar are given, For more details [4]. In section 3 it is proved that there exist an equivalent N-triangular form of fuzzy context free grammar G*'* for the given fuzzy context free grammar G and the language L(G*'*) derived from G*'* is the same as the language L(G) derived from G. Using this result it is easy to transform fuzzy context free grammar to N-triangular form of fuzzy context free grammar.

Preliminaries

Triangular form of Fuzzy Context Free Grammar

Fuzzy triangular form of context free grammar is a four-tuple (V_N, V_T, P, S) where

i) V_N is a non-empty set whose elements are called variables.

ii) V_T is a finite non empty set whose elements are called terminals.

$V_N \cap V_T = \varphi$

iii) $S = A_i \in \{A_1, A_2, \ldots A_n\}$ is a special variable called start symbol.

iv) The set of productions P of the form

$$P_0 : S_1 \xrightarrow{0} \Lambda,\ P_1 : A_1 \xrightarrow{c_1} \alpha_1,\ P_2 : A_2 \xrightarrow{c_2} \alpha_2 \ldots P_i : A_i \xrightarrow{c_i} \alpha_i$$

$$P_{i+1} : A_{i+1} \xrightarrow{c_{i+1}} \alpha_{i+1}, P_n : A_n \xrightarrow{c_n} \alpha_n, P_{n+1} : S_2 \xrightarrow{0} \Lambda$$

Where Λ is empty string, $\alpha_1, \alpha_2, \ldots, \alpha_n \in (V_N \cup V_T)^*$ and $|\alpha_1| < |\alpha_2| < \ldots |\alpha_i| > |\alpha_{i+1}| > |\alpha_{i+2}| > \ldots |\alpha_n|$

The number of alphabets in α_j is denoted by $|\alpha_j|$. Here c_j's, $j \in \{1, 2, 3, \ldots, n-1\}$ is used to denote the membership grade of the production P_j. $c_j, j \in \{2, 3, \ldots, n-1\}$ is calculated using $c_j = \frac{|\alpha_j|}{|\alpha_i|}$, where α_i is a string with maximum number of alphabets. c_1 is fixed as $c_1 < c_2$ *and* $c_1 \in (0, c_2)$. Similarly c_n is fixed as $c_n < c_{n-1}$ *and* $c_n \in (0,1)$.

N-Triangular form of Fuzzy context free grammar

The N-triangular form of fuzzy context free grammar is a four tuple (V_N, V_T, P, S). Here $V_N, V_T,$ S are defined as in Triangular form of fuzzy context free grammar. The set of Productions are in the following form.

$$P_0 : S_1 \xrightarrow{0} \Lambda \ \ P_1 : A_1 \xrightarrow{c_1} \alpha_1,\ P_2 : A_2 \xrightarrow{c_2} \alpha_2 \ldots P_i : A_i \xrightarrow{c_i} \alpha_i$$

$$P_{i+1} : A_{i+1} \xrightarrow{c_{i+1}} \alpha_{i+1}, P_n : A_n \xrightarrow{c_n} \alpha_n, P_{n+1} : S_2 \xrightarrow{0} \Lambda,$$

$$P_{n+2} : B_1 \xrightarrow{\theta_1} \beta_1, \quad P_{n+3} : B_2 \xrightarrow{\theta_2} \beta_2, \ldots, P_j : B_j \xrightarrow{\theta_j} \beta_j, \ldots,$$

$$P_{j+1} : B_{j+1} \xrightarrow{\theta_{j+1}} \beta_{j+1}, \ldots, P_r : B_r \xrightarrow{\theta_r} \beta_r, \ldots,$$

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$P_{r+1}: S_3 \overset{0}{\rightarrow} \Lambda$,…, $P_k : S_{N+1} \overset{0}{\rightarrow} \Lambda$

Here the productions P_0 to P_{n+1} forms a triangular form, where α_i has the maximum number of alphabets among $\alpha_1, \alpha_2, ..., \alpha_n$, also c_i' s (i = 1,2,..n) are calculated by the same way as section 2.1. Similarly the productions P_{n+1} to P_r forms another triangular form, β_j has the maximum number of alphabets among $\beta_1, \beta_2, ..., \beta_r$, also θ_j' s (j = 1,2,…,r) are calculated by the same way as in section 2.1

Theorem

Every fuzzy context free language L(G) can be generated by N-triangular form of fuzzy context free grammar.

Proof

It is enough to construct Equivalent N-triangular form of fuzzy context free grammar $G' = (N, T, P, A_1)$ for the fuzzy context free grammar $G = (V_N, V_T, P, S)$.

Step 1:

Consider a fuzzy context free language L(G) generated by the fuzzy context free grammar $G = (V_N, V_T, P, S)$. Let every string of the language L(G) is having single membership grade $\mu(x) = c$.

Step 2:

By definition

$$\mu(x) = \mu_G\left(S \overset{*}{\Rightarrow} x\right)$$
$$= \underset{S \overset{*}{\Rightarrow} x}{Max\,Min}\left(\mu(S \to \alpha_1), \mu(\alpha_1 \to \alpha_2), ..., \mu(\alpha_m \to x)\right)$$

Without loss of generality, the interval [0, 1] is divided into two intervals [0, l] and [l, 1], where $0.3 \le l \le 0.4$

If $c \in [0, l]$ we will get 1-triangular form of fuzzy context free grammar, where as *if* $c \in [l,1]$ we will get N-triangular form of fuzzy context free grammar.

Case(i) : $c \in [0, l]$

Let us consider $N = \{A_1, A_2, ..., A_n\}$ *and* $T = V_T$ the productions $P_0 : S_1 \overset{0}{\rightarrow} \Lambda,\ P_1 : A \overset{c}{\rightarrow} S$ *and* $P_f : S_2 \overset{0}{\rightarrow} \Lambda$ for the 1-triangular form of fuzzy context free grammar. where N is the set of non-terminals, T is the set of terminals, P_0 is the initial production, P_f is the final production, A_1 is the start symbol of 1-triangular form of fuzzy context free grammar and S is the start symbol of existing fuzzy context free grammar. Let $x \in L(G)\ \&\ \mu(x) = c, c \in [0, l]$.

We know that "c" is the membership degree of all strings in L(G).The construction $P_2, P_3, ...$ of 1-triangular form of fuzzy context free grammar from the existing fuzzy context free grammar based on the following procedure.

The Calculation of $c_2, c_3, ...$ are explained in [3].

The Selection of the production $A_\alpha \overset{c_\alpha = 1}{\longrightarrow} w_\alpha$ depends on the fixed membership degree c. Since "c" is the minimum of all membership degree's of all c_i, i=2,3,..n.

$P_0 : S_1 \overset{0}{\rightarrow} \Lambda$

$P_1 : A_1 \overset{c}{\rightarrow} S$

$P_2 : A_2 \overset{c_2}{\rightarrow} w_2\ ,\ c < c_2$

$P_3 : A_3 \overset{c_3}{\rightarrow} w_3\ ,\ c_2 < c_3,\ \&|w_2| < |w_3|$

...

$P_\alpha : A_\alpha \overset{c_\alpha = 1}{\rightarrow} w_\alpha \quad |w_\alpha|$ is max *imum among all* $|w_i|$

$P_{\alpha+1} : A_{\alpha+1} \overset{c_{\alpha+1}}{\rightarrow} w_{\alpha+1},\quad c_{\alpha+1} < c_\alpha, \&|w_{\alpha+1}| < |w_\alpha|$

...

$P_f : S_2 \overset{0}{\rightarrow} \Lambda$

Here $S = A_i \in \{A_2, A_3, ..., A_n\}$ *and* $w_2, w_3, ...(N \cup T)^*$

The Single nonterminals are considerd $A_2 = N_1 \in V_N, A_3 = N_2 \in V_N, ..., A_n = N_m \in V_N$

The selection w_α from the fuzzy context free grammar "G" has the following way

If $S \to x_1\beta_1, \beta_1 \to x_2\beta_2, ..., \beta_m \to x_m\beta, \beta \to x_{m+1}$

Hence Selection of w_α *becomes*

$w_\alpha = x_1 x_2 \beta_2$

(or)

$w_\alpha = x_1 x_2 x_3 \beta_3$

(or)

$w_\alpha = x_1 x_2 x_3 ... \beta_n$

Hence the given fuzzy context free grammar is transformed into a 1-triangular form of fuzzy context free grammar and it derives the language L(G')=L(G).

Case (ii) : $c \in (l,1]$

As in case(i) set of Non-terminals N and set of terminals T, the productions $P_0, P_1\ \&\ P_f$ are Considered for N-triangular form of fuzzy context free grammar above case. Now we can construct the N-triangular form from the fuzzy context free grammar as in case (i) with the following structure.

$$P_0 : S_1 \overset{0}{\rightarrow} \Lambda, P_1 : A_1 \overset{c}{\rightarrow} S, P_2 : A_2 \overset{1}{\rightarrow} w_2, P_3 : S_2 \overset{0}{\rightarrow} \Lambda, P_4 : A_3 \overset{1}{\rightarrow} w_3, P_5 : S_3 \overset{0}{\rightarrow} \Lambda, ..., P_f : S_n \overset{0}{\rightarrow} \Lambda$$

The membership degree c of the production P_1 is assumed to be minimum, and membership of all other productions has the single non terminals $A_2, A_3, ..., A_n \in N$ are assumed to be 1.

Case (iii) :

If the strings in L(G) has the different membership degree i.e., $\mu(x_1) = \theta_1, \mu(x_2) = \theta_2, ..., \mu(x_n) = \theta_n$ where $x_1, x_2, ..., x_n \in L(G)$, then collect the productions for the membership degree $\theta_1, \theta_2, ..., \theta_n$ from the fuzzy context free grammar and assume the membership degree 1 for the remaining productions except the production have Λ symbol and construct the N-triangular form of fuzzy context free grammar in the following structure.

$$P_0 : S_1 \overset{0}{\rightarrow} \Lambda, P_1 : A_1 \overset{\theta_1}{\rightarrow} w, P_2 : A_2 \overset{1}{\rightarrow} w_2, P_3 : S_2 \overset{0}{\rightarrow} \Lambda, P_4 : A_3 \overset{\theta_2}{\rightarrow} w_3, P_5 : A_4 \overset{1}{\rightarrow} w_4$$
$$,...\ P_f : S_n \overset{0}{\rightarrow} \Lambda$$

In all the cases the given fuzzy context free grammar is transformed into N-triangular form of fuzzy context free grammar and it derives the language L(G ')=L(G).

Example

Consider the fuzzy context free grammar $G=\left(V_N,V_T,P,S\right)$, where $V_N=\{S,A,B\},V_T=\{a,b\}$ and

$$P=\left\{A\xrightarrow{0.2}a,B\xrightarrow{0.4}b,S\xrightarrow{0.9}bA,S\xrightarrow{0.8}aB,A\xrightarrow{0.6}bSA,B\xrightarrow{0.7}aSB\right\}$$

The corresponding Language of G is L(G) = {(ab, 0.4), (b(a+b)*, 0.2), (a(a+b)*, 0.2) , (a(a+b)*,0.4)}

Now let us consider the productions $P_0:S_1\xrightarrow{0}\Lambda,\ P_1:A\xrightarrow{c}a\ and\ P_f:S_f\xrightarrow{0}\Lambda$ in N-triangular form. Here all the strings in L(G) has only two different membership degree 0.2 and 0.4. The derivation of the strings using the production $A\xrightarrow{0.2}a$ will yield strings with membership degree 0.2, otherwise membership degree of the strings will be 0.4.

The Structure of N-triangular form of fuzzy context free grammar is given below.

$$P_0:S_1\xrightarrow{0}\Lambda$$
$$P_1:A\xrightarrow{0.2}a$$
$$P_2:S\xrightarrow{1}bA$$
$$P_3:S_2\xrightarrow{0}\Lambda$$
$$P_4:B\xrightarrow{0.4}b$$
$$P_5:A\xrightarrow{1}bSA$$
$$P_6:S_3\xrightarrow{0}\Lambda$$
$$P_7:S\xrightarrow{0.66}b$$
$$P_8:B\xrightarrow{1}aSB$$
$$P_9:S_4\xrightarrow{0}\Lambda$$

Here 0.2 is fixed as the membership degree for P_1 and 0.4 is for P_4 using the definition of N-triangular form of fuzzy context free grammar. The membership degrees of other productions are calculated by the rules of N-triangular form of fuzzy context free grammar. Here $G'=\{N,T,P,S\}$ Where $N=\{A,S,B,S_1,S_2\},T=\{a,b\}$. The Language of G ' is

$$L(G')=\{(\mathrm{ab},0.4),(\mathrm{b(a+b)^*},0.2),(\mathrm{a(a+b)^*},0.2),(\mathrm{a(a+b)}$$

Hence L(G ')=L(G).

Conclusion:

A theorem called "Every fuzzy context free language L(G) can be generated by N-triangular form of fuzzy context free grammar" is stated with proof. Also it is illustrated with example.

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