



## Ion-acoustic soliton in nonthermal electron-positron-ion plasmas with dust particulates

Louis E. Akpabio, Akpan N. Ikot and Joseph G. Atat

Department of Physics, Theoretical Plasma Group, University of Uyo, Uyo – Nigeria.

### ARTICLE INFO

#### Article history:

Received: 20 April 2011;

Received in revised form:

16 June 2011;

Accepted: 25 June 2011;

### ABSTRACT

Propagation of large amplitude ion acoustic solitons are investigated in four component plasmas, whose constituents are inertial ions, nonthermal electrons, Boltzmannian positrons and dust particulates. The solitary waves as well as its properties are investigated through pseudopotential approach. It is observed that the presence of dust particulates considerably modify the potentials as well as width of the solitary wave.

© 2011 Elixir All rights reserved.

### Keywords

Ion-acoustic soliton,  
Nonthermal electron-positron-ion-dust.

### Introduction

Fully ionized gases consisting of electrons and positrons of equal masses and ion are usually characterized as electron-positron-ion plasma [1,2]. The presence of ions brings about the existence of several low frequency waves which otherwise do not propagate in electron – positron plasmas. There are electron-positron plasmas in astrophysical plasmas such as in magnetosphere of pulsars, in active galactic nuclei, in early universe and in the regions of the accretion disks surrounding the central black holes [3-8]. Due to impressive developments in plasma, there has been considerable interest in different types of linear and nonlinear wave structure such as solitons, double layers, vortices etc, in electron-positron plasmas [9-15] as well as in multi-component electron – positron – ion plasmas [16-18].

Most of the astrophysical plasmas usually contain highly charged (negative/positive) impurities or dust particulates in addition to the electrons, positrons and ions. However, the properties of wave motions-electron-positron-ion-dust plasma should be different from those in three component electron-position-ion plasmas [16-19]. As an example, the presence of highly charged dust particulates modifies the dispersion relation for ion acoustic wave by increasing its phase velocity-electron-ion plasma [20-24]. As well as modifying the nonlinear properties of ion acoustic wave [25-31]. Therefore, it is pertinent to study the collective behavior of nonlinear wave motions in nonthermal electron-positron-ion-dust plasmas.

In this paper, the nonlinear wave structure of ion acoustic solitons in nonthermal electron-positron-ion-dust plasmas is investigated which was not considered in the earlier investigation [19]. The paper is organized in the following manner. Formulation of the problem and the basic equations as well as the energy integral equation and Sagdeev potential are given in section 2. The nature of the nonlinear solitary structures and the numerical results are discussed in section 3. Section 4 deals with the conclusions.

### Formulation and Basic Equations

A four-component plasma consisting of nonthermal electrons distribution (e) with number density ( $N_e$ ),

Boltzmannian positrons (p) with number density ( $N_p$ ), singly charged cold positive ions (i) with number density ( $N_i$ ) and negatively fixed charged immobile dust grains (d) with number density  $N_d$  is considered. The quasi neutrality of the plasma at equilibrium is given as  $N_{eo} + Z_d N_{do} = N_{io} + N_{po}$  where  $N_{eo}$ ,  $N_{io}$ ,  $N_{po}$  and  $N_{do}$  are the unperturbed electron, ion, positron and dust densities respectively. The nonthermal electrons and positrons densities are given respectively as:

$$N_e = N_{eo} [1 - \beta\phi + \beta\phi^2] \exp(\phi) \quad (1)$$

Where  $\beta = \frac{4\alpha}{1 + 3\alpha}$  and  $\alpha$  represent a parameter defining

the population of nonthermal electrons [31].

$$N_p = N_{po} \exp(-\sigma\phi) \quad (2)$$

Where  $\phi$  is the electrostatic potential;  $\sigma = \frac{T_e}{T_p}$  and  $Z_d$  is the number of negative elementary charges on the dust grains.

The nonlinear propagation of low phase velocity (in comparison with the electron positron thermal velocities) ion acoustic waves are governed by the following normalized equations:

$$\frac{\partial n_i}{\partial t} + \frac{\partial n_i u_i}{\partial x} = 0 \quad (3)$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = -\frac{\partial \phi}{\partial x} \quad (4)$$

$$\frac{\partial^2 \phi}{\partial x^2} = (1 - \beta\phi + \beta\phi^2) \exp(\phi) - p \exp(-\sigma\phi) + \delta_d - n_i \quad (5)$$

where  $u_i$  is the ion fluid velocity normalized to the ion acoustic speed  $C_s = (T_e / m_i)^{1/2}$ ,  $\phi$  is the electrostatic potential normalized to  $\frac{e\phi}{T_e}$ , the time and space variables are

written in units of the plasma period  $W_p^{-1} = (4\pi e^2 N_o / m_i)^{1/2}$  and the effective Debye length

$$\lambda_D = (T_o / 4\pi e^2 N_o)^{1/2}, \delta_d = N_{do} / N_{eo} \text{ and } p = N_{po} / N_{eo}.$$

**Finite Amplitude linear Ion-acoustic Waves**

Linearizing equations (1-5) and assuming that the first order quantities of  $N_p, N_e, N_i, N_d, u_i$  and  $\phi$  are proportional to  $\exp[i(kx - \omega t)]$ , we obtain the dispersion relation of the ion-acoustic waves in nonthermal electron-ion-positron-dust plasma as:

$$\omega^2 = \frac{k^2 c_s^2 (1 + \delta_d - p)}{(1 - \beta) + p \frac{T_e}{T_p} + k^2 \lambda_D^2} \tag{6}$$

Where  $\frac{N_o}{T_p} = \frac{N_{po}}{T_p} + (1 - \beta) \frac{N_{eo}}{T_e}$ ,  $\omega$  is the frequency,  $K$  is the wave number,  $c_s$  the ion-sound velocity,  $N_o$  the effective number density and  $T_o$  the effective temperature. The limiting case of three component ( $e - p - i$ ) plasma can be obtained from equation (6) by inserting  $N_{do} = 0$  (in the absence of dust particulates) which gives:

$$\omega^2 = \frac{k^2 c_s^2 (1 - p)}{(1 - \beta) + p \frac{T_e}{T_p} + k^2 \lambda_D^2} \tag{7}$$

While, the linear dispersion relation of ion acoustic waves in plasma with two components, electrons and ions is gotten from equation (6) by setting  $N_{do} = 0, N_{po} = 0$  and  $\beta = 0$  to have:

$$\omega^2 = \frac{k^2 c_s^2}{1 + k^2 \lambda_D^2} \tag{8}$$

**Arbitrary Amplitude Solitary Structures**

We are interested in a localized stationary solution and therefore we define a moving coordinate as  $\xi = x - Mt$ , where  $M$  is the speed of the localized nonlinear structure in the moving frame called Mach number. For steady state condition, we have from equation (3-5) the density as:

$$n_i = \frac{MN_{io}}{\sqrt{M^2 - 2\phi}} \tag{9}$$

To obtain equation (9), we have imposed the boundary conditions for localized disturbances as  $u_i \rightarrow 0, n_i \rightarrow N_{io}$  at  $\xi \rightarrow \pm\infty$ . Using equation (9) in equation (5) and multiplying both sides of the resulting equation by  $\partial\phi/\partial\xi$ ; integrating once and taking into account the appropriate boundary conditions as  $\phi \rightarrow 0$  and  $\partial\phi/\partial\xi \rightarrow 0$  at  $\xi \rightarrow \pm\infty$ , we have the energy integral equation as follows:

$$\frac{1}{2} \left( \frac{d\phi}{d\xi} \right)^2 + V(\phi) = 0 \tag{10}$$

Where the Sagdeev potential is defined as

$$V(\phi) = 1 + 3\beta + \frac{p}{\sigma} - (1 + 3\beta - 3\beta\phi + \beta\phi^2) e^\phi - \frac{p}{\sigma} e^{-\sigma\phi} + \delta_d \phi + (1 + \delta_d - P)M \left( M - \sqrt{M^2 - 2\phi} \right) \tag{11}$$

The limiting case of three component (e-p-i) plasma can be obtained by substituting  $N_{do} = 0$  (i.e. in the absence of dust particulates) in equation (11), which then reduces to the case of [19] for the ion-acoustic solitons in electron-positron nonthermal plasma.

The condition for the existence of a localized solution of equation (10) requires that  $V(\phi) = dv(\phi)/d\phi = 0$  at  $\phi = 0$  and  $V(\phi) < 0$  for  $0 < |\phi| < |\phi_0|$  where  $|\phi_0|$  is the maximum amplitude of the solitons. To obtain the condition on the Mach number, we expand the Sagdeev potential  $V(\phi)$  around the origin to arrive at the critical Mach number;

$$M_l = \sqrt{(1 + \delta_d - P)/(1 + P\sigma - \beta)} \tag{12}$$

Also, at  $\phi_m = -M^2/2$  the Sagdeev potential should vanish i.e.

$$V(\phi_m) = 1 + 3\beta - \left( 1 + 3\beta + \frac{3\beta M^2}{2} + \frac{\beta M^4}{4} \right) \exp(-M^2/2) + \frac{P}{\sigma} \left[ 1 - \exp\left(\frac{\sigma M^2}{2}\right) \right] + \left[ 2(1 + \delta_d - P)(1 - \sqrt{2}) - \delta_d \right] \frac{M^2}{2} \geq 0 \tag{13}$$

**Numerical analysis and discussion**

In figures 1-3, we examine the dependence of the required Mach number M (which supports the existence of the solitary waves) on the ratio of electron and positron ( $T_e/T_p$ ) temperature  $\sigma$  or different values of  $\sigma$  and  $d$ . From figures 1 and 2, we find that the  $\sigma$  dependence of M is both nonlinear. It is also clear from both figures that the amplitude of the solitary wave increases with different values of  $\sigma$  and  $d$ , respectively. When the nonthermal parameter is increased to 1.0 as in Fig. 3; it is observed that the Mach number hump of the solitary wave increases with increasing up to 0.12 for different values of  $\sigma d$ , beyond which the amplitude are seen to decrease with increasing  $\sigma$ .

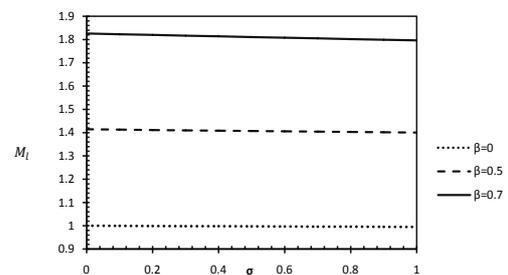


Fig 1. Variation of  $M_l$  against  $\sigma$  for different values of  $\beta$  (0, 0.5, 0.7) with  $p=0.01$  and  $\delta_d=0.01$

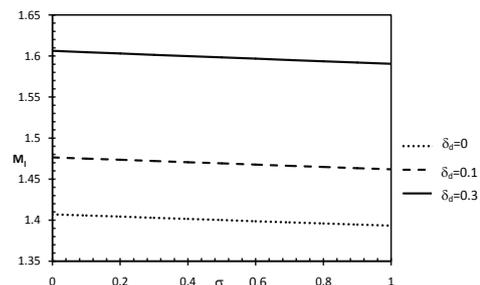


Fig 2. Variation of  $M_l$  against  $\sigma$  for different values of  $\delta_d$  (0, 0.1, 0.3) with  $p=0.01$  and  $\beta=0.5$

The behavior of Sagdeev potential  $V(\phi)$  versus  $\phi$  is displayed in figures 4.-6. The values of  $P = 0.01$ ,  $\sigma = 0.01$  and  $\beta = 0.01$  have been taken from [19] where they were used for three component e-p-i non thermal plasma. It can be seen from figure 4 that soliton exist for different values of positron concentration  $P$ . The potential well for the different values of  $\beta$  exist for  $0.2 < \phi < 1.04$ . It is observe that, the depth and width of the potential well increases with increase in the positron concentration  $P$ . From Fig. 5, we have numerically analysis the potential  $V(\phi)$  and found the range of values of  $M$  in which large amplitude stationary solitary solution exist. These are depicted in figure 5, which shows that there exists a potential well only for  $\phi < 1.24$  i.e. there exist solitary wave with  $0.2 < \phi < 1.24$  for  $1.35 < M < 1.57$ . Also, the figure illustrates the dependence of  $V(\phi)$  on  $\phi$  with different values of  $M$  for fixed values of  $\beta$ ,  $\sigma$ ,  $\delta_d$  and  $P$ . It is seen that both the depth and the width of the potential well increases with increasing values of  $M$ .

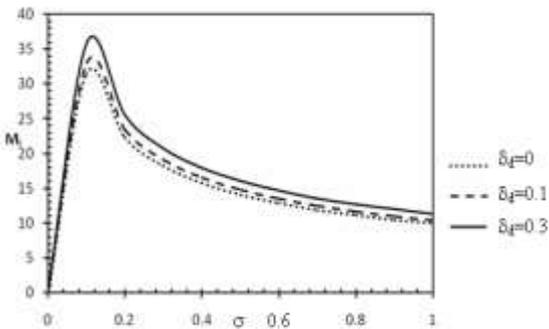


Fig 3. Variation of  $M$ , against  $\sigma$  for different values of  $\delta_d$  (0, 0.1, 0.3) with  $p=0.01$  and  $\beta=1.0$

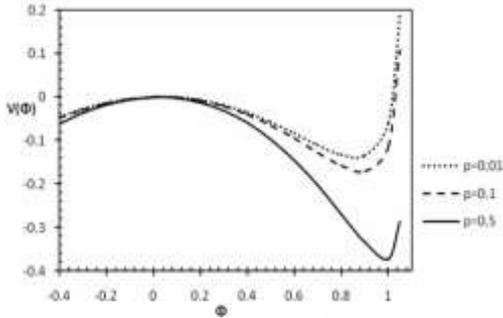


Fig 4. Variation of solitary waves profiles  $V(\Phi)$  against  $\Phi$  for different values of  $p$  (0.01, 0.1, 0.5) with  $\beta=0.01$ ,  $M=1.45$ ,  $\sigma=0.01$  and  $\delta_d=0.01$

On the other hand, the dependence of  $V(\phi)$  on  $\phi$  for different values of the dust particle concentration  $\delta_d$  with fixed values of  $\beta$ ,  $M$ ,  $\sigma$  and  $P$  is displayed in figure 6. It is obvious from this figure that the depth and width of the potential well decreases with increases in  $\delta_d$ . It is of interest to examine whether or not there is an upper limit of  $M$  for which solitary waves exist. This upper limit of  $M$  can be found by the condition  $V(\phi_m) \geq 0$ , where  $\phi_m = -M^2 / 2$  is the minimum value of  $\phi$  for which the ion acoustic density  $n_i$  is real. The dependence of  $V(\phi_m)$  is plotted in figures 7 – 8. From Fig. 7, it is clear that the amplitude of the solitary waves increases to a maximum and then drops as the Mach number increases for different values of  $\beta$ . Whereas; in Fig. 8, the amplitude of the

solitary wave decreases with increase in the Mach number for different values of the dust particle concentration.

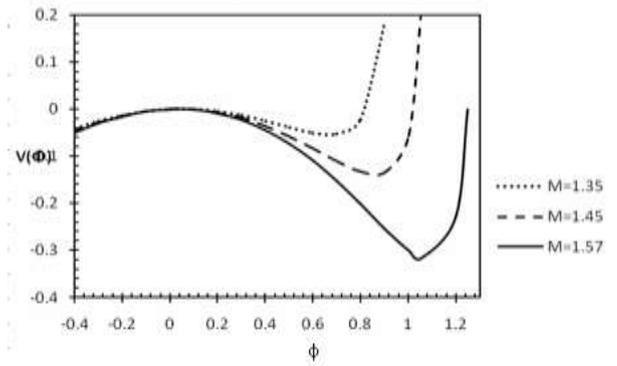


Fig 5. Solitary waves profiles  $V(\Phi)$  against  $\Phi$  for different values of  $M$  (1.35, 1.45, 1.57) with  $\beta=0.01$ ,  $\sigma=0.01$ ,  $\delta_d=0.01$  and  $p=0.01$

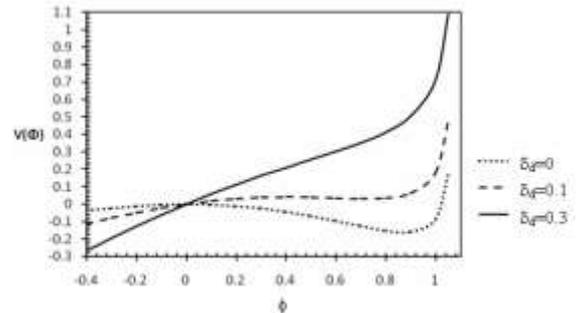


Fig 6. Solitary wave profiles  $V(\Phi)$  against  $\Phi$  for different values of  $\delta_d$  (0, 0.1, 0.3) with  $\beta=0.01$ ,  $M=1.45$ ,  $\sigma=0.01$  and  $p=0.01$

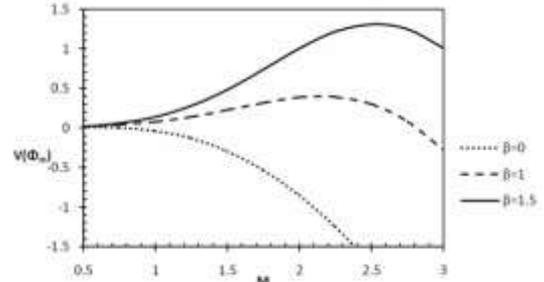


Fig 7. Variation of  $V(\Phi_m)$  against  $M$  for different values of  $\beta$  (0, 1, 1.5) with  $\delta_d=0.01$ ,  $\sigma=0.01$  and  $p=0.01$

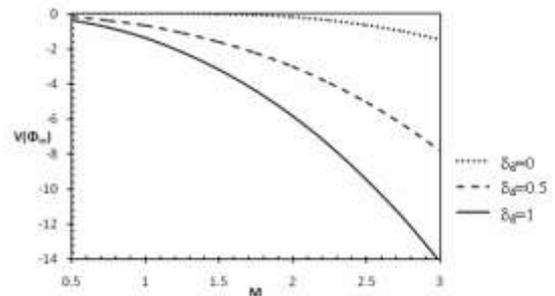


Fig 8. Variation of  $V(\Phi_m)$  against  $M$  for different values of  $\delta_d$  (0, 0.5, 1) with  $\beta=0.5$ ,  $\sigma=0.01$  and  $p=0.01$

**Conclusions**

In this paper, we have investigated the electrostatic ion-acoustic structures by employing pseudopotential approach in unmagnetized four-component plasma consisting of inertial ions, nonthermal electrons, Boltzmannian positron and dust particulates. It is shown that in these four component plasma, the speed of the ion-acoustic soliton of a given amplitude decreases as  $\beta$  and  $\delta_d$  increases respectively. Also, when a fixed value of the nonthermal parameter is considered as  $\beta = 1.0$ , the speed of the soliton produces a hump which increases with increase in the dust concentration  $\delta_d$ . There are potential wells

on the positive  $\phi$ -axis, resulting in the existence of compressive ( $\phi > 0$ ) ion-acoustic solitary waves. Numerical simulation reveals that the solitary solution exists in  $0.2 < \phi < 1.24$  for  $1.35 < M < 1.57$ . Finally, the presence of negatively charged dust particulates causes considerable modification of the potential as well as width of the solitary wave. We emphasize that; our theoretical results may be useful for the better understanding of nonlinear propagation of electrostatic structures associated with positrons and dust that may occur in space.

#### Acknowledgement

This work was supported by the Theoretical Physics Group under grant No. TPG.44-500.

#### References

- [1] Tandberg-Hansen, E., Emisle, A. G.: The Physics of Solar Flares. P. 124. Cambridge University Press. Cambridge (1988).
- [2] Ghosh, S and Bharuthram, R: Ion acoustic solitons and double layers in electron-positron-ion plasmas with dust particulates. *Astrophys. Space Sci.* 314, 121(2008).
- [3] Golderich, P., Julian, W. H.: Pulsar Electrodynamics. *Astrophys. J.* 157, 869 (1969).
- [4] Sturrock, P. A. A model of Pulsars. *Astrophys J.* 164, 529 (1971).
- [5] Misner, W., Thorne, K. S., Wheeler, J. A. *Gravitation*, P. 763. Freeman, San Francisco (1973).
- [6] Michael, F. C.: Theory of Pulsar Magnetospheres. *Rev. Mod. Phys.* 54, 1 (1982).
- [7] Rees, M. J., In: Gibbons, G. W., Hawking, S. W., Siklas, S. (eds.). *The Early Universe*. Cambridge University Press, Cambridge (1983).
- [8] Miller, H. R., Witta, P. J.: *Active Galactic Nuclei*, P. 202. Springer, Berlin (1987).
- [9] Shukla, P. K., Rao, N. N., Yu, M. Y., Tsintsadze, N. L.: Relativistic nonlinear effect in Plasmas. *Phys. Rep.* 138, 1 (1986).
- [10] Yu, M.Y., Shukla, P. K.; Stenflo, L: Alfvén vortices in strongly magnetized electron-positron plasma. *Astrophys. J.* 309, 63 (1986).
- [11] Tajima, T., Taniuti, T: Nonlinear interaction of Photons and Phonons in electron-positron plasma. *Phys. Rev.* A42, 3587 (1990).
- [12] Bharuthram, R.: Arbitrary amplitude double layers in a multi-species electron-positron plasmas. *Astrophys. Space Sci.* 189, 213 (1992).
- [13] Shukla, P. K., Tsintsadze, N. L., Tsintsadze, L. N.: Nonlinear interaction of Photons and Phonons in a relativistically hot electron-positron gas. *Phys. Fluids B5.* 233 (1993).
- [14] Shukla, P. K. and Stenflo, L.: Nonlinear Coupling between electromagnetic fields in a strongly magnetized electron-positron plasmas. *Astrophys. Space Sci.* 209, 323 (1993).
- [15] Shukla, P. K.: Nonlinear Coupling between electromagnetic and sound waves in relativistically hot electron-positron magnetoplasma. *Europhys. Lett.* 22, 695 (1993).
- [16] Rizzato, F. B.: Weak nonlinear electromagnetic waves and low frequency magnetic field generation in electron-positron-ion plasmas. *J. Plasma Phys.* 40, 289 (1988).
- [17] Berezhiani, V. I., El-Ashry, M. Y., Mofize U. A.: Theory of strong-electromagnetic-wave propagation in electron-positron-ion plasma. *Phys. Rev.* E50, 448 (1994).
- [18] Popel, S. I., Vladimirov, S. V., Shukla, P. K.: Ion acoustic solitons in electron-positron-ion plasmas. *Phys. Plasmas* 2, 716 (1995).
- [19] Bahamida, S., Annou, K., Annou, R; Ion-acoustic solitons in electron-positron nonthermal plasma. 34th EPS Conference on Plasma Phys. Warsaw, 2-6 July, 2007 ECA Vol. 31F, P-4.139 (2007).
- [20] D'Angelo, N: Low frequency electrostatic waves in dusty plasmas. *Planet. Space Sci.* 38, 1143 (1990).
- [21] Shukla, P. K. and Silin, V. P.: Dust-ion acoustic wave. *Phys. Scr.* 45, 508 (1992).
- [22] Barkan, A., D'Angelo, N., Merlino, R. L.: Experiments on ion-acoustic waves in dusty plasmas. *Planet. Space Sci.* 44, 239 (1996).
- [23] Nakamura, Y., Bailung, H., Shukla, P. K.: Observation of ion-acoustic shocks in a dusty plasma. *Phys. Rev. Lett.* 83, 1602 (1999).
- [24] Popel, S. I., Yu, M. Y., Tsytovich, V. N.: Shock waves in plasmas containing variable charge impurities. *Phys. Plasmas* 3, 4313 (1996).
- [25] Luo, Q. Z., D'Angelo, N., Merlino, R. L.: Experimental Study of Shock formation in a dusty plasma. *Phys. Plasmas* 6, 3455 (1999).
- [26] Nakamura, Y., Sharma, A.: Observation of ion-acoustic solitary waves in a dusty plasma. *Phys. Plasmas* 8, 3921 (2001).
- [27] Shukla, P. K.: Dust-ion acoustic shocks and holes. *Phys. Plasmas* 7, 1044 (2000).
- [28] Ghosh, S., Sarkar, S. Khan, M., Gupta, M. R.: Nonlinear properties of small amplitude dust ion-acoustic solitary waves. *Phys. Plasmas* 7, 3594 (2000).
- [29] Ghosh, S., Sarkar, S., Khan, M., Gupta, M. R.: Dust ion acoustic shock waves in a collisionless dusty plasma. *Phys. Lett. A* 274, 162 (2000).
- [30] Ghosh, S.: Formation of ion-acoustic weak double layers in a dusty plasma. *Eur. Phys. J. Appl. Phys.* 33, 199 (2006).
- [31] Cairns, R. A., Mamun, A. A., Bingham, R., Bostron, R., Dendy, R. O., Nairn, C. M. C. and Shukla, P. K.: Electrostatic Solitary Structures in nonthermal plasmas. *Geophys. Res. Lett.* 22, 2709 (1995).