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A summation formula allied with hypergeometric function

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ABSTRACT

The main objective of the present paper is to establish a summation formula based on half argument in connection with Hypergeometric function.

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Introduction

Generalized Gaussian Hypergeometric function of one variable

$${}_A F_B(a_1, a_2, \dots, a_A; b_1, b_2, \dots, b_B; z) = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \dots (a_A)_k z^k}{(b_1)_k (b_2)_k \dots (b_B)_k k!} \quad (1)$$

or

$${}_A F_B((a_A); (b_B); z) \equiv {}_A F_B((a_j)_{j=1}^A; (b_j)_{j=1}^B; z) = \sum_{k=0}^{\infty} \frac{((a_A))_k z^k}{((b_B))_k k!} \quad (2)$$

where the parameters $b_1, b_2, \dots, b_B$ are neither zero nor negative integers and A, B are non negative integers.

Contiguous Relations

[Andrews p.363(9.16), E.D. p.51(10)]

$$(a-b) {}_2 F_1(a, b; c; z) = a {}_2 F_1(a+1, b; c; z) - b {}_2 F_1(a, b+1; c; z) \quad (3)$$

Gauss second summation theorem is defined as

[Prudnikov p.491(7.3.7.3)]

$${}_2 F_1(a, b; \frac{a+b+1}{2}; \frac{1}{2}) = 2^{b-1} \frac{\Gamma(\frac{b}{2}) \Gamma(\frac{a+b+1}{2})}{\Gamma(b) \Gamma(\frac{a+1}{2})} \quad (4)$$

Recurrence relation

$$\Gamma(\zeta+1) = \zeta \Gamma(\zeta) \quad (5)$$

Main summation formula :

$${}_2 F_1(a, b; \frac{a+b+33}{2}; \frac{1}{2}) = 2^b \frac{\Gamma(\frac{a+b+33}{2})}{(a-b) \Gamma(b)} \left\{ \frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a+1}{2})} \right.$$

$$\left. \begin{aligned} & \frac{32768a(-6190283353629375 + 14459713484342175a)}{[\prod_{\sigma=1}^{15} \{a-b-(2\sigma-1)\}] [\prod_{\tau=1}^{16} \{a-b+(2\tau-1)\}]} \\ & + \frac{32768a(-13121113142970855a^2 + 6520139954828519a^3 - 2046215352864875a^4 + 437602985498315a^5)}{[\prod_{\sigma=1}^{15} \{a-b-(2\sigma-1)\}] [\prod_{\tau=1}^{16} \{a-b+(2\tau-1)\}]} \\ & + \frac{32768a(-66696220706115a^5 + 7442156684963a^7 - 617014151325a^8 + 38205040445a^9 - 1759562805a^{10})}{[\prod_{\sigma=1}^{15} \{a-b-(2\sigma-1)\}] [\prod_{\tau=1}^{16} \{a-b+(2\tau-1)\}]} \\ & + \frac{32768a(59394517a^{11} - 1426425a^{12} + 23065a^{13} - 225a^{14} + a^{15} + 24389502603617025b)}{[\prod_{\sigma=1}^{15} \{a-b-(2\sigma-1)\}] [\prod_{\tau=1}^{16} \{a-b+(2\tau-1)\}]} \\ & + \frac{32768a(-1785407765182130ab + 42573217123457739a^2b - 10333396242678612a^3b)}{[\prod_{\sigma=1}^{15} \{a-b-(2\sigma-1)\}] [\prod_{\tau=1}^{16} \{a-b+(2\tau-1)\}]} \\ & + \frac{32768a(6075644560374313a^4b - 747920977520110a^5b + 186562964963627a^6b - 12908512055576a^7b)}{[\prod_{\sigma=1}^{15} \{a-b-(2\sigma-1)\}] [\prod_{\tau=1}^{16} \{a-b+(2\tau-1)\}]} \end{aligned} \right.$$

$$\begin{aligned} & + \frac{32768a(1639119660219a^8b - 64396599150a^9b + 4424102969a^{10}b - 91805012a^{11}b + 3337243a^{12}b)}{[\prod_{\sigma=1}^{15} \{a-b-(2\sigma-1)\}] [\prod_{\tau=1}^{16} \{a-b+(2\tau-1)\}]} \\ & + \frac{32768a(-28210a^{15}b + 465a^{16}b + 2370529116251865b^2 + 58128564092927445ab^2 - 9155202435582722a^2b^2)}{[\prod_{\sigma=1}^{15} \{a-b-(2\sigma-1)\}] [\prod_{\tau=1}^{16} \{a-b+(2\tau-1)\}]} \\ & + \frac{32768a(2164770175674084a^2b^2 - 2129188265326509a^4b^2 + 1281597603735775a^5b^2 - 76112859912236a^6b^2)}{[\prod_{\sigma=1}^{15} \{a-b-(2\sigma-1)\}] [\prod_{\tau=1}^{16} \{a-b+(2\tau-1)\}]} \\ & + \frac{32768a(19475108139668a^7b^2 - 682473587217a^8b^2 + 87991013955a^9b^2 - 1669441202a^{10}b^2 + 114275486a^{11}b^2)}{[\prod_{\sigma=1}^{15} \{a-b-(2\sigma-1)\}] [\prod_{\tau=1}^{16} \{a-b+(2\tau-1)\}]} \\ & + \frac{32768a(-899899a^{12}b^2 + 31465a^{13}b^2 + 18159104671902873b^3 + 2480564200530348ab^3)}{[\prod_{\sigma=1}^{15} \{a-b-(2\sigma-1)\}] [\prod_{\tau=1}^{16} \{a-b+(2\tau-1)\}]} \\ & + \frac{32768a(27282903497429586a^2b^3 - 1537286046338244a^3b^3 + 3478735147718871a^4b^3)}{[\prod_{\sigma=1}^{15} \{a-b-(2\sigma-1)\}] [\prod_{\tau=1}^{16} \{a-b+(2\tau-1)\}]} \\ & + \frac{32768a(-162351526593960a^5b^3 + 95007438460284a^6b^3 - 2883492522792a^7b^3 + 718520065623a^8b^3)}{[\prod_{\sigma=1}^{15} \{a-b-(2\sigma-1)\}] [\prod_{\tau=1}^{16} \{a-b+(2\tau-1)\}]} \\ & + \frac{32768a(-12381441540a^9b^3 + 1541912658a^{10}b^3 - 11359764a^{11}b^3 + 736281a^{12}b^3 + 2255118016687893b^4)}{[\prod_{\sigma=1}^{15} \{a-b-(2\sigma-1)\}] [\prod_{\tau=1}^{16} \{a-b+(2\tau-1)\}]} \\ & + \frac{32768a(12557375628149751ab^4 + 612351776446611a^2b^4 + 4202344924969545a^3b^4 - 101613788986350a^4b^4)}{[\prod_{\sigma=1}^{15} \{a-b-(2\sigma-1)\}] [\prod_{\tau=1}^{16} \{a-b+(2\tau-1)\}]} \\ & + \frac{32768a(220832406672390a^5b^4 - 514888163770a^6b^4 + 2858620125330a^7b^4 - 42519216375a^8b^4)}{[\prod_{\sigma=1}^{15} \{a-b-(2\sigma-1)\}] [\prod_{\tau=1}^{16} \{a-b+(2\tau-1)\}]} \\ & + \frac{32768a(10040593875a^9b^4 - 67492425a^{10}b^4 + 7888725a^{11}b^4 + 1691350769714133b^5 + 76748800121678ab^5)}{[\prod_{\sigma=1}^{15} \{a-b-(2\sigma-1)\}] [\prod_{\tau=1}^{16} \{a-b+(2\tau-1)\}]} \\ & + \frac{32768a(2295152534656641a^2b^5 + 53204609549400a^3b^5 + 259918658081850a^4b^5 - 2898007679700a^5b^5)}{[\prod_{\sigma=1}^{15} \{a-b-(2\sigma-1)\}] [\prod_{\tau=1}^{16} \{a-b+(2\tau-1)\}]} \\ & + \frac{32768a(6018599335770a^6b^5 - 67328447400a^7b^5 + 34971527025a^8b^5 - 201600750a^9b^5 + 44352165a^{10}b^5)}{[\prod_{\sigma=1}^{15} \{a-b-(2\sigma-1)\}] [\prod_{\tau=1}^{16} \{a-b+(2\tau-1)\}]} \\ & + \frac{32768a(134430637526525b^6 + 524283494489845ab^6 + 75410486941140a^2b^6 + 156372445185540a^3b^6)}{[\prod_{\sigma=1}^{15} \{a-b-(2\sigma-1)\}] [\prod_{\tau=1}^{16} \{a-b+(2\tau-1)\}]} \\ & + \frac{32768a(1867545606150a^4b^6 + 696247417350a^5b^6 - 34832887740a^6b^6 + 68858131860a^7b^6)}{[\prod_{\sigma=1}^{15} \{a-b-(2\sigma-1)\}] [\prod_{\tau=1}^{16} \{a-b+(2\tau-1)\}]} \\ & + \frac{32768a(-293096475a^8b^6 + 141120525a^9b^6 + 37988267624125b^7 + 22719594504424ab^7)}{[\prod_{\sigma=1}^{15} \{a-b-(2\sigma-1)\}] [\prod_{\tau=1}^{16} \{a-b+(2\tau-1)\}]} \\ & + \frac{32768a(46275398753004a^2b^7 + 2809945073880a^3b^7 + 4453828901550a^4b^7 + 26521974360a^5b^7)}{[\prod_{\sigma=1}^{15} \{a-b-(2\sigma-1)\}] [\prod_{\tau=1}^{16} \{a-b+(2\tau-1)\}]} \end{aligned}$$

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$$\begin{aligned}
& 32768a(78706173420a^6b^7 - 141430680a^7b^7 + 265182525a^8b^7 + 1868941834339b^8 + 6029531658533ab^9) \\
& + \frac{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))}{32768b(1107757039023a^2b^3 + 1539609426505a^3b^3 + 41187033225a^4b^3 + 52477295535a^5b^3 + 123751845a^6b^3)} \\
& + \frac{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))}{32768a(300540195a^7b^3 + 255962204579b^9 + 164785065618ab^9 + 270590627613a^2b^9 + 1909805500a^3b^9)} \\
& + \frac{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))}{32768a(20090290125a^4b^5 + 195397650a^5b^5 + 206253075a^6b^5 + 7509769995b^{10} + 21224351655ab^{10})} \\
& + \frac{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))}{32768a(3853554510a^2b^{10} + 4240627950a^3b^{10} + 100800375a^4b^{10} + 84672315a^5b^{10} + 538269771b^{11})} \\
& + \frac{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))}{32768a(324477868ab^{11} + 456424098a^{11}b^{11} + 24542700a^{11}b^{11} + 20160075a^{11}b^{11} + 8535943b^{12} + 21539141ab^{12})} \\
& + \frac{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))}{32768a(2839941a^2b^{12} + 2629575a^3b^{12} + 312263b^{13} + 138446ab^{13} + 169911a^3b^{13} + 2015b^{14} + 4495ab^{14} + 31b^{15})} \\
& + \frac{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))}{32768b(-6190283353629375 + 24389502603617025a + 2370529116251865a^2 + 18159104671902873a^3)} \\
& + \frac{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))}{32768b(2255118016687893a^4 + 1691350769714133a^4 + 134430637526525a^5 + 37988267624125a^6)} \\
& + \frac{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))}{32768b(1868941834339a^8 + 255962204579a^9 + 7509769995a^{10} + 538269771a^{11} + 8535943a^{12} + 312263a^{13})} \\
& + \frac{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))}{32768b(2015a^{14} + 31a^{15} + 14459713484342175b - 1785407765182130ab + 58128564092927445a^2b)} \\
& + \frac{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))}{32768b(2480564200530348a^3b + 12557375628149751a^4b + 767488001216786a^5b + 52428349448945a^6b)} \\
& + \frac{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))}{32768b(22719594504424a^2b + 6029531658533a^3b + 164785065618a^4b + 21224351655a^{10}b + 324477868a^{11}b)} \\
& + \frac{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))}{32768b(21539141a^{12}b + 138446a^{13}b + 4495a^{14}b - 1312113142970855b^2 + 42573217123457739ab^2)} \\
& + \frac{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))}{32768b(-9255202435582722a^2b^2 + 27282903497429586a^3b^2 + 612351776446611a^4b^2)} \\
& + \frac{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))}{32768b(2295152534656641a^5b^3 + 75410486941140a^6b^3 + 46275398753004a^7b^3 + 1107757039023a^8b^3)} \\
& + \frac{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))}{32768b(270590627613a^9b^3 + 3853554510a^{10}b^3 + 456424098a^{11}b^3 + 2839941a^{12}b^3 + 169911a^{13}b^3)} \\
& + \frac{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))}{32768b(6520139954328519b^5 + 10333396242678612ab^5 + 21647701756740846a^2b^5 - 153786046338244a^3b^5)} \\
& + \frac{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))}{32768b(4202344924969545a^4b^5 + 53204609549400a^5b^5 + 156372445185540a^6b^5 + 2809945073880a^7b^5)} \\
& + \frac{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))}{32768b(1539689426505a^8b^5 + 1909805500a^9b^5 + 4240627950a^{10}b^5 + 24542700a^{11}b^5 + 2629575a^{12}b^5)} \\
& + \frac{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))}{32768b(-2046225352864075b^4 + 6075644560374313ab^4 - 212928826536509a^2b^4 + 3478735147718871a^3b^4)} \\
& + \frac{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))}{32768b(-10161378896350a^4b^4 + 259918658081850a^5b^4 + 1867545606150a^6b^4 + 4453828901550a^7b^4)} \\
& + \frac{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))}{32768b(41187033225a^8b^4 + 20090290125a^9b^4 + 100800375a^{10}b^4 + 20160075a^{11}b^4 + 437602985498315b^5)} \\
& + \frac{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))}{32768b(-747920977520110ab^5 + 1281597603735775a^2b^5 - 162351526593960a^3b^5 + 220832406672390a^4b^5)} \\
& + \frac{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))}{32768b(-2898007679700a^5b^5 + 6962474197350a^6b^5 + 26521974360a^7b^5 + 52477295535a^8b^5 + 195397650a^9b^5)} \\
& + \frac{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))}{32768b(84672315a^{10}b^5 - 66696220706115b^6 + 186562964963627ab^6 - 76112859912236a^2b^6)} \\
& + \frac{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))}{32768b(95007438460284a^3b^6 - 5148881563770a^4b^6 + 6018599335770a^5b^6 - 34832887740a^6b^6)} \\
& + \frac{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))}{32768b(78706173420a^7b^6 + 123751845a^8b^6 + 206253075a^9b^6 + 7442156684963b^7 - 12908512055576ab^7)} \\
& + \frac{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))}{32768b(19475108139668a^2b^7 - 2883492522792a^3b^7 + 2858620125330a^4b^7 - 67328447400a^5b^7)} \\
& + \frac{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))}{32768b(68858131860a^6b^7 - 141430680a^7b^7 + 300540195a^8b^7 - 617014151325b^8 + 1639119660219ab^8)}
\end{aligned}$$

$$\begin{aligned}
& + \frac{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))}{32768b(-682473587217a^3b^8 + 718520065623a^4b^8 - 42519216375a^5b^8 + 34971527025a^6b^8 - 293096475a^7b^8)} \\
& + \frac{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))}{32768b(265182525a^7b^8 + 38205040445b^9 - 64396599150ab^9 + 87991013955a^2b^9 - 12381441540a^3b^9)} \\
& + \frac{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))}{32768b(10040593875a^4b^9 + 201600750a^5b^9 + 141120525a^6b^9 - 1759562805b^{10} + 4424102969ab^{10})} \\
& + \frac{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))}{32768b(-1669441202a^2b^{10} + 1541912658a^3b^{10} - 67492425a^4b^{10} + 44352165a^5b^{10} + 59394517b^{11})} \\
& + \frac{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))}{32768b(-91805012ab^{11} + 114275486a^2b^{11} - 11359764a^3b^{11} + 7888725a^4b^{11} - 1426425b^{12} + 3337243ab^{12})} \\
& + \frac{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))}{32768b(-899899a^3b^{12} + 736281a^4b^{12} + 23065b^{13} - 28210ab^{13} + 31465a^2b^{13} - 225b^{14} + 465ab^{14} + b^{15})} \\
& + \frac{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))}{\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{b}{2})} \{ \frac{65536(6190283353629375 + 24389502603617025a - 2370529116251865a^2 + 18159104671902873a^3)}{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))} } \\
& + \frac{65536(-2255118016687893a^4 + 1691350769714133a^4 - 134430637526525a^5 + 37988267624125a^6)}{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))} \\
& + \frac{65536(-1868941834339a^8 + 255962204579a^9 + 7509769995a^{10} + 538269771a^{11} + 8535943a^{12} + 312263a^{13})}{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))} \\
& + \frac{65536(-2015a^{14} + 31a^{15} + 14459713484342175b - 1785407765182130ab + 58128564092927445a^2b)}{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))} \\
& + \frac{65536(-2480564200530348a^3b + 12557375628149751a^4b + 767488001216786a^5b + 52428349448945a^6b)}{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))} \\
& + \frac{65536(-22719594504424a^2b + 6029531658533a^3b + 164785065618a^4b + 21224351655a^{10}b - 324477868a^{11}b)}{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))} \\
& + \frac{65536(21539141a^{12}b - 138446a^{13}b + 4495a^{14}b + 1312113142970855b^2 + 42573217123457739ab^2)}{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))} \\
& + \frac{65536(9255202435582722a^2b^2 + 27282903497429586a^3b^2 - 612351776446611a^4b^2)}{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))} \\
& + \frac{65536(2295152534656641a^5b^3 - 75410486941140a^6b^3 + 46275398753004a^7b^3 - 1107757039023a^8b^3)}{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))} \\
& + \frac{65536(270590627613a^9b^3 - 3853554510a^{10}b^3 + 456424098a^{11}b^3 - 2839941a^{12}b^3 + 169911a^{13}b^3)}{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))} \\
& + \frac{65536(6520139954328519b^5 + 10333396242678612ab^5 + 21647701756740846a^2b^5 + 153786046338244a^3b^5)}{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))} \\
& + \frac{65536(4202344924969545a^4b^5 + 53204609549400a^5b^5 + 156372445185540a^6b^5 - 2809945073880a^7b^5)}{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))} \\
& + \frac{65536(1539689426505a^8b^5 + 1909805500a^9b^5 + 4240627950a^{10}b^5 - 24542700a^{11}b^5 + 2629575a^{12}b^5)}{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))} \\
& + \frac{65536(2046225352864075b^4 + 6075644560374313ab^4 + 212928826536509a^2b^4 + 3478735147718871a^3b^4)}{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))} \\
& + \frac{65536(10161378896350a^4b^4 + 259918658081850a^5b^4 - 1867545606150a^6b^4 + 4453828901550a^7b^4)}{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))} \\
& + \frac{65536(-41187033225a^8b^4 + 20090290125a^9b^4 - 100800375a^{10}b^4 + 20160075a^{11}b^4 + 437602985498315b^5)}{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))} \\
& + \frac{65536(747920977520110ab^5 + 1281597603735775a^2b^5 + 162351526593960a^3b^5 + 220832406672390a^4b^5)}{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))} \\
& + \frac{65536(2898007679700a^5b^5 + 6962474197350a^6b^5 - 26521974360a^7b^5 + 52477295535a^8b^5 - 195397650a^9b^5)}{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))} \\
& + \frac{65536(84672315a^{10}b^5 + 66696220706115b^6 + 186562964963627ab^6 + 76112859912236a^2b^6)}{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))} \\
& + \frac{65536(95007438460284a^3b^6 + 5148881563770a^4b^6 + 6018599335770a^5b^6 + 34832887740a^6b^6)}{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))} \\
& + \frac{65536(78706173420a^7b^6 + 123751845a^8b^6 + 206253075a^9b^6 + 7442156684963b^7 + 12908512055576ab^7)}{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))} \\
& + \frac{65536(19475108139668a^2b^7 + 2883492522792a^3b^7 + 2858620125330a^4b^7 + 67328447400a^5b^7)}{(\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))) (\prod_{\tau=1}^{16}(a-b+(2\tau-1)))}
\end{aligned}$$

$$\begin{aligned}
& 65536(19475108139668a^2b^7 + 2883492522792a^3b^7 + 2858620125330a^4b^7 + 67328447400a^5b^7) \\
& + \frac{[\prod_{\sigma=1}^{16}(a-b-(2\sigma-1))][\prod_{\omega=1}^{16}(a-b+(2\omega-1))]}{65536(68858131860a^6b^7 + 141480680a^7b^7 + 300540195a^8b^7 + 617014151325b^8 + 1639119660219ab^8)} \\
& + \frac{[\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))][\prod_{\omega=1}^{16}(a-b+(2\omega-1))]}{65536(680473587217a^2b^8 + 718520065623a^3b^8 + 42519216375a^4b^8 + 34971527025a^5b^8 + 293096475a^6b^8)} \\
& + \frac{[\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))][\prod_{\omega=1}^{16}(a-b+(2\omega-1))]}{65536(265182525a^7b^8 + 38205040445b^9 + 64396599150ab^9 + 87991013955a^2b^9 + 12381441540a^3b^9)} \\
& + \frac{[\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))][\prod_{\omega=1}^{16}(a-b+(2\omega-1))]}{65536(10040593875a^4b^9 + 201600750a^5b^9 + 141120525a^6b^9 + 1759562805b^{10} + 4424102969ab^{10})} \\
& + \frac{[\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))][\prod_{\omega=1}^{16}(a-b+(2\omega-1))]}{65536(1669441202a^2b^{10} + 1541912658a^3b^{10} + 67492425a^4b^{10} + 44352165a^5b^{10} + 59394517b^{11})} \\
& + \frac{[\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))][\prod_{\omega=1}^{16}(a-b+(2\omega-1))]}{65536(91805012ab^{11} + 114275406a^2b^{11} + 11539764a^3b^{11} + 7888725a^4b^{11} + 1426425b^{12} + 3337243ab^{12})} \\
& + \frac{[\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))][\prod_{\omega=1}^{16}(a-b+(2\omega-1))]}{65536(899899a^3b^{12} + 736281a^4b^{12} + 23065b^{13} + 28210ab^{13} + 31465a^2b^{13} + 225b^{14} + 465ab^{14} + b^{15})} \\
& + \frac{[\prod_{\sigma=1}^{15}(a-b-(2\sigma-1))][\prod_{\omega=1}^{16}(a-b+(2\omega-1))]}{65536(6190283353629375 + 1445971348424175a + 13121113142970855a^2 + 6520139954328519a^3)} \\
& + \frac{[\prod_{\sigma=1}^{16}(a-b-(2\sigma-1))][\prod_{\omega=1}^{15}(a-b+(2\omega-1))]}{65536(2046225352864875a^4 + 437602985498315a^5 + 66696220706115a^6 + 7442156684963a^7)} \\
& + \frac{[\prod_{\sigma=1}^{16}(a-b-(2\sigma-1))][\prod_{\omega=1}^{15}(a-b+(2\omega-1))]}{65536(617014151325a^8 + 38205040445a^9 + 1759562805a^{10} + 59394517a^{11} + 1426425a^{12} + 23065a^{13})} \\
& + \frac{[\prod_{\sigma=1}^{16}(a-b-(2\sigma-1))][\prod_{\omega=1}^{15}(a-b+(2\omega-1))]}{65536(225a^{14} + a^{15} + 24389502603617025b + 17854077765182130ab + 42573217123457739a^2b)} \\
& + \frac{[\prod_{\sigma=1}^{16}(a-b-(2\sigma-1))][\prod_{\omega=1}^{15}(a-b+(2\omega-1))]}{65536(10333396242678612a^3b + 6075644560374313a^4b + 747920977520110a^5b + 186562964963627a^6b)} \\
& + \frac{[\prod_{\sigma=1}^{16}(a-b-(2\sigma-1))][\prod_{\omega=1}^{15}(a-b+(2\omega-1))]}{65536(12908512055576a^7b + 1639119660219a^8b + 64396599150a^9b + 4424102969a^{10}b + 91805012a^{11}b)} \\
& + \frac{[\prod_{\sigma=1}^{16}(a-b-(2\sigma-1))][\prod_{\omega=1}^{15}(a-b+(2\omega-1))]}{65536(3337243a^{12}b + 28210a^{13}b + 465a^{14}b - 2370529116251865b^2 + 58128564092927445ab^2)} \\
& + \frac{[\prod_{\sigma=1}^{16}(a-b-(2\sigma-1))][\prod_{\omega=1}^{15}(a-b+(2\omega-1))]}{65536(9255202435582722a^2b^2 + 21647701756740846a^3b^2 + 2129288265326509a^4b^2)} \\
& + \frac{[\prod_{\sigma=1}^{16}(a-b-(2\sigma-1))][\prod_{\omega=1}^{15}(a-b+(2\omega-1))]}{65536(1281597603735775a^5b^2 + 76112859912236a^6b^2 + 19475108139668a^7b^2 + 682473587217a^8b^2)} \\
& + \frac{[\prod_{\sigma=1}^{16}(a-b-(2\sigma-1))][\prod_{\omega=1}^{15}(a-b+(2\omega-1))]}{65536(87991013955a^9b^2 + 1669441202a^{10}b^2 + 114275486a^{11}b^2 + 899899a^{12}b^2 + 31465a^{13}b^2)} \\
& + \frac{[\prod_{\sigma=1}^{16}(a-b-(2\sigma-1))][\prod_{\omega=1}^{15}(a-b+(2\omega-1))]}{65536(18159104671902873b^3 - 2480564200530348ab^3 + 27282903497429586a^2b^3 + 1537286046338244a^3b^3)} \\
& + \frac{[\prod_{\sigma=1}^{16}(a-b-(2\sigma-1))][\prod_{\omega=1}^{15}(a-b+(2\omega-1))]}{65536(3478735147718871a^4b^3 + 162351526593960a^5b^3 + 95007438460284a^6b^3 + 2883492522792a^7b^3)} \\
& + \frac{[\prod_{\sigma=1}^{16}(a-b-(2\sigma-1))][\prod_{\omega=1}^{15}(a-b+(2\omega-1))]}{65536(718520065623a^8b^3 + 12381441540a^9b^3 + 1541912658a^{10}b^3 + 11359764a^{11}b^3 + 736281a^{12}b^3)} \\
& + \frac{[\prod_{\sigma=1}^{16}(a-b-(2\sigma-1))][\prod_{\omega=1}^{15}(a-b+(2\omega-1))]}{65536(-2255118016687893b^4 + 12557375628149751ab^4 - 612351776446611a^2b^4 + 4202344924969545a^3b^4)} \\
& + \frac{[\prod_{\sigma=1}^{16}(a-b-(2\sigma-1))][\prod_{\omega=1}^{15}(a-b+(2\omega-1))]}{65536(101613788986350a^4b^4 + 220832406672390a^5b^4 + 5148881563770a^6b^4 + 2858620125330a^7b^4)} \\
& + \frac{[\prod_{\sigma=1}^{16}(a-b-(2\sigma-1))][\prod_{\omega=1}^{15}(a-b+(2\omega-1))]}{65536(42519216375a^8b^4 + 10040593875a^9b^4 + 67492425a^{10}b^4 + 7888725a^{11}b^4 + 1691350769714133b^5)} \\
& + \frac{[\prod_{\sigma=1}^{16}(a-b-(2\sigma-1))][\prod_{\omega=1}^{15}(a-b+(2\omega-1))]}{65536(-767488001216786ab^5 + 229515353465641a^2b^5 - 53204609549400a^3b^5 + 259918658081850a^4b^5)} \\
& + \frac{[\prod_{\sigma=1}^{16}(a-b-(2\sigma-1))][\prod_{\omega=1}^{15}(a-b+(2\omega-1))]}{
\end{aligned}$$

$$\begin{aligned}
& 65536(2898007679700a^5b^5 + 6018599335770a^6b^5 + 67328447400a^7b^5 + 34971527025a^8b^5 + 201600750a^9b^5) \\
& + \frac{[\prod_{\sigma=1}^{16}(a-b-(2\sigma-1))][\prod_{\omega=1}^{15}(a-b+(2\omega-1))]}{65536(44352165a^{10}b^5 - 134430637526525b^6 + 524283494489845ab^6 - 75410486941140a^2b^6)} \\
& + \frac{[\prod_{\sigma=1}^{16}(a-b-(2\sigma-1))][\prod_{\omega=1}^{15}(a-b+(2\omega-1))]}{65536(156372445185540a^3b^6 - 1867545606150a^4b^6 + 6962474197350a^5b^6 + 34832887740a^6b^6)} \\
& + \frac{[\prod_{\sigma=1}^{16}(a-b-(2\sigma-1))][\prod_{\omega=1}^{15}(a-b+(2\omega-1))]}{65536(68858131860a^7b^6 + 293096475a^8b^6 + 141120525a^9b^6 + 37988267624125b^7 - 22719594504424ab^7)} \\
& + \frac{[\prod_{\sigma=1}^{16}(a-b-(2\sigma-1))][\prod_{\omega=1}^{15}(a-b+(2\omega-1))]}{65536(46275398753004a^2b^7 - 2809945073880a^3b^7 + 4453828901550a^4b^7 - 26521974360a^5b^7)} \\
& + \frac{[\prod_{\sigma=1}^{16}(a-b-(2\sigma-1))][\prod_{\omega=1}^{15}(a-b+(2\omega-1))]}{65536(78706173420a^6b^7 + 141430680a^7b^7 + 265182525a^8b^7 - 186894183439b^8 + 602953165853ab^8)} \\
& + \frac{[\prod_{\sigma=1}^{16}(a-b-(2\sigma-1))][\prod_{\omega=1}^{15}(a-b+(2\omega-1))]}{65536(-1107757039023a^2b^8 + 1539689426505a^3b^8 - 41187033225a^4b^8 + 52477295535a^5b^8)} \\
& + \frac{[\prod_{\sigma=1}^{16}(a-b-(2\sigma-1))][\prod_{\omega=1}^{15}(a-b+(2\omega-1))]}{65536(-123751845a^6b^8 + 300540195a^7b^8 + 25592204579b^9 - 164785065618ab^9 + 270590627613a^2b^9)} \\
& + \frac{[\prod_{\sigma=1}^{16}(a-b-(2\sigma-1))][\prod_{\omega=1}^{15}(a-b+(2\omega-1))]}{65536(-1908805500a^8b^9 + 20090290125a^9b^9 - 195397650a^5b^9 + 206253075a^6b^9 - 7509769995b^{10})} \\
& + \frac{[\prod_{\sigma=1}^{16}(a-b-(2\sigma-1))][\prod_{\omega=1}^{15}(a-b+(2\omega-1))]}{65536(212243515655ab^{10} - 3853554510a^2b^{10} + 4240627950a^3b^{10} - 100800375a^4b^{10} + 84672315a^5b^{10})} \\
& + \frac{[\prod_{\sigma=1}^{16}(a-b-(2\sigma-1))][\prod_{\omega=1}^{15}(a-b+(2\omega-1))]}{65536(538269771b^{11} - 324477868ab^{11} + 456424098a^2b^{11} - 24542700a^3b^{11} + 20160075a^4b^{11} - 8535943b^{12})} \\
& + \frac{[\prod_{\sigma=1}^{16}(a-b-(2\sigma-1))][\prod_{\omega=1}^{15}(a-b+(2\omega-1))]}{65536(21539141ab^{12} - 2839941a^2b^{12} + 2629575a^3b^{12} + 312263b^{13} - 138446ab^{13} + 169911a^2b^{13})} \\
& + \frac{[\prod_{\sigma=1}^{16}(a-b-(2\sigma-1))][\prod_{\omega=1}^{15}(a-b+(2\omega-1))]}{65536(-2015b^{14} + 4495ab^{14} + 31b^{15})} \\
& + \frac{[\prod_{\sigma=1}^{16}(a-b-(2\sigma-1))][\prod_{\omega=1}^{15}(a-b+(2\omega-1))]}{
\end{aligned}$$

(6)

Derivation of main formula:

Replacing $c = \frac{a+b+33}{2}$ and $z = \frac{1}{2}$ in equation (3), we get

$$(a-b) {}_2F_1(a, b; \frac{a+b+33}{2}; \frac{1}{2}) = a {}_2F_1(a+1, b; \frac{a+b+33}{2}; \frac{1}{2}) - b {}_2F_1(a, b+1; \frac{a+b+33}{2}; \frac{1}{2})$$

Now preceding the same parallel method of Ref [4], the main formula is derived.

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