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Two summation formulae of half argument involving hypergeometric function

Salahuddin
P.D.M College of Engineering, Bahadurgarh , Haryana , India.

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ABSTRACT
The aim of the present paper is to obtain two summation formulae based on half argument involving hypergeometric function. The results derived in this paper are of general character and are believed to be new.
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Introduction

Generalized Gaussian Hypergeometric function of one variable

$${}_AF_B(a_1,a_2,\ldots,a_A;b_1,b_2,\ldots b_B;z) = \sum_{k=0}^{\infty}\frac{(a_1)_k(a_2)_k\ldots(a_A)_k\, z^k}{(b_1)_k(b_2)_k\ldots(b_B)_k\, k!}$$

or

$${}_AF_B((a_A);(b_B);z) \equiv {}_AF_B((a_j)_{j=1}^{A};(b_j)_{j=1}^{B};z) = \sum_{k=0}^{\infty}\frac{((a_A))_k\, z^k}{((b_B))_k\, k!} \quad (2)$$

where the parameters b_1 , b_2 , ….,b_B are neither zero nor negative integers and A , B are non negative integers.

Contiguous Relations

[Andrews p.363(9.16) , E.D. p.51(10), H.T.F.I. p.103(32)]

$$(a-b)\ {}_2F_1(a, b ; c; z) = a\ {}_2F_1(a+1,b; c; z) - b\ {}_2F_1(a,b+1; c; z) \quad (3)$$

[Abramowitz p.558(15.2.19)]

$$(a-b)\ (1-z)\ {}_2F_1(a, b ; c; z) = (c-b)\ {}_2F_1(a,b-1; c; z) + (a-c)\ {}_2F_1(a-1,b; c; z) \quad (4)$$

Gauss second summation theorem is defined as [Prud .,491(7.3,7.5)]

$${}_2F_1\left(a, b ; \frac{a+b+1}{2}; \frac{1}{2}\right) = 2^{b-1}\frac{\Gamma(\frac{b}{2})\,\Gamma(\frac{a+b+1}{2})}{\Gamma(b)\,\Gamma(\frac{a+1}{2})} \quad (5)$$

Recurrence relation

$$\Gamma(\zeta+1) = \zeta\,\Gamma(\zeta) \quad (6)$$

Main Results of Summation Formulae

$${}_2F_1\left(a, b ; \frac{a+b+19}{2}; \frac{1}{2}\right) = 2^{b}\frac{\Gamma(\frac{a+b+19}{2})}{(a-b)\Gamma(b)}\Bigg[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a+1}{2})}\Bigg\{\frac{256a(2027025-4098240a+2924172a^2-1038016a^3)}{[\prod_{\zeta=1}^{8}\{a-b-(2\zeta-1)\}]\,[\prod_{\omega=1}^{9}\{a-b-(2\omega-1)\}]}$$

$$+\frac{256a(208054a^4-24640a^5+1708a^6-64a^7+a^8-1040400b+12903000ab-3491664a^2b+2808536a^3b)}{[\prod_{\zeta=1}^{8}\{a-b-(2\zeta-1)\}]\,[\prod_{\omega=1}^{9}\{a-b-(2\omega-1)\}]}$$

$$+\frac{256a(-270640a^4b+59976a^5b-1904a^6b+136a^7b+7410300b^2-511904ab^2+7452052a^2b^2-538560a^3b^2)}{[\prod_{\zeta=1}^{8}\{a-b-(2\zeta-1)\}]\,[\prod_{\omega=1}^{9}\{a-b-(2\omega-1)\}]}$$

$$+\frac{256a(415140a^4b^2-11424a^5b^2+2380a^6b^2+984368b^3+5298424ab^3-35360a^2b^3+901680a^3b^3-17680a^4b^3)}{[\prod_{\zeta=1}^{8}\{a-b-(2\zeta-1)\}]\,[\prod_{\omega=1}^{9}\{a-b-(2\omega-1)\}]}$$

$$+\frac{256a(12376a^5b^3+872678b^4+250240ab^4+693940a^2b^4+24310a^4b^4+55760b^5+180744ab^5+10608a^2b^5)}{[\prod_{\zeta=1}^{8}\{a-b-(2\zeta-1)\}]\,[\prod_{\omega=1}^{9}\{a-b-(2\omega-1)\}]}$$

$$+\frac{256a(19448a^3b^5+11900b^6+3808ab^6+6188a^2b^6+272b^7+680ab^7+17b^8)}{[\prod_{\zeta=1}^{8}\{a-b-(2\zeta-1)\}]\,[\prod_{\omega=1}^{9}\{a-b-(2\omega-1)\}]} \quad (1)$$

$$+\frac{256b(2027025-1040400a+7410300a^2+984368a^3+872678a^4+55760a^5+11900a^6+272a^7+17a^8-4098240b)}{[\prod_{\mu=1}^{9}\{a-b-(2\mu-1)\}]\,[\prod_{\lambda=1}^{8}\{a-b-(2\lambda-1)\}]}$$

$$+\frac{256b(12903000ab-511904a^2b+5298424a^3b+250240a^4b+180744a^5b+3808a^6b+680a^7b+2924172b^2)}{[\prod_{\mu=1}^{9}\{a-b-(2\mu-1)\}]\,[\prod_{\lambda=1}^{8}\{a-b-(2\lambda-1)\}]}$$

$$+\frac{256b(-3491664ab^2+7452052a^2b^2-35360a^3b^2+693940a^4b^2+10608a^5b^2+6188a^6b^2-1038016b^3)}{[\prod_{\mu=1}^{9}\{a-b-(2\mu-1)\}]\,[\prod_{\lambda=1}^{8}\{a-b-(2\lambda-1)\}]}$$

$$+\frac{256b(2808536ab^3-538560a^2b^3+901680a^3b^3+19448a^5b^3+208054b^4-270640ab^4+415140a^2b^4)}{[\prod_{\mu=1}^{9}\{a-b-(2\mu-1)\}]\,[\prod_{\lambda=1}^{8}\{a-b-(2\lambda-1)\}]}$$

$$+\frac{256b(-17680a^3b^4+24310a^4b^4-24640b^5+59976ab^5-11424a^2b^5+12376a^3b^5+1708b^6-1904ab^6)}{[\prod_{\mu=1}^{9}\{a-b-(2\mu-1)\}]\,[\prod_{\lambda=1}^{8}\{a-b-(2\lambda-1)\}]}$$

$$+\frac{256b(2380a^2b^6-64b^7+136ab^7+b^8)}{[\prod_{\mu=1}^{9}\{a-b-(2\mu-1)\}]\,[\prod_{\lambda=1}^{8}\{a-b-(2\lambda-1)\}]}\Bigg\}-\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a}{2})}\Bigg\{\frac{512(2027025+1040400a+7410300a^2)}{[\prod_{\zeta=1}^{8}\{a-b-(2\zeta-1)\}]\,[\prod_{\omega=1}^{9}\{a-b-(2\omega-1)\}]}$$

$$+\frac{512(-984368a^3+872678a^4-55760a^5+11900a^6-272a^7+17a^8+4098240b+12903000ab+511904a^2b)}{[\prod_{\zeta=1}^{8}\{a-b-(2\zeta-1)\}]\,[\prod_{\omega=1}^{9}\{a-b-(2\omega-1)\}]}$$

$$+\frac{512(5298424a^3b-250240a^4b+180744a^5b-3808a^6b+680a^7b+2924172b^2+3491664ab^2+7452052a^2b^2)}{[\prod_{\zeta=1}^{8}\{a-b-(2\zeta-1)\}]\,[\prod_{\omega=1}^{9}\{a-b-(2\omega-1)\}]}$$

$$+\frac{512(35360a^3b^2+693940a^4b^2-10608a^5b^2+6188a^6b^2+1038016b^3+2808536ab^3+538560a^2b^3)}{[\prod_{\zeta=1}^{8}\{a-b-(2\zeta-1)\}]\,[\prod_{\omega=1}^{9}\{a-b-(2\omega-1)\}]}$$

$$+\frac{512(901680a^3b^3+19448a^5b^3+208054b^4+270640ab^4+415140a^2b^4+17680a^3b^4+24310a^4b^4+24640b^5)}{[\prod_{\zeta=1}^{8}\{a-b-(2\zeta-1)\}]\,[\prod_{\omega=1}^{9}\{a-b-(2\omega-1)\}]}$$

Tele:
E-mail addresses: sludn@yahoo.com, vsludn@gmail.com

$$+\frac{512(59976ab^5+11424a^2b^5+12376a^3b^5+1708b^6+1904ab^6+2380a^2b^6+64b^7+136ab^7+b^8)}{[\prod_{\zeta=1}^{8}\{a-b-(2\zeta-1)\}][\prod_{\omega=1}^{9}\{a-b-(2\omega-1)\}]}$$

$$+\frac{512(2027025+4098240a+2924172a^2+1038016a^3+208054a^4+24640a^5+1708a^6+64a^7+a^8+1040400b)}{[\prod_{\mu=1}^{9}\{a-b-(2\mu-1)\}][\prod_{\lambda=1}^{8}\{a-b-(2\lambda-1)\}]}$$

$$+\frac{512(12903000ab+3491664a^2b+2808536a^3b+270640a^4b+59976a^5b+1904a^6b+136a^7b+7410300b^2)}{[\prod_{\mu=1}^{9}\{a-b-(2\mu-1)\}][\prod_{\lambda=1}^{8}\{a-b-(2\lambda-1)\}]}$$

$$+\frac{512(511904ab^2+7452052a^2b^2+538560a^3b^2+415140a^4b^2+11424a^5b^2+2380a^6b^2-984368b^3+5298424ab^3)}{[\prod_{\mu=1}^{9}\{a-b-(2\mu-1)\}][\prod_{\lambda=1}^{8}\{a-b-(2\lambda-1)\}]}$$

$$+\frac{512(35360a^2b^3+901680a^3b^3+17680a^4b^3+12376a^5b^3+872678b^4-250240ab^4+693940a^2b^4+24310a^4b^4)}{[\prod_{\mu=1}^{9}\{a-b-(2\mu-1)\}][\prod_{\lambda=1}^{8}\{a-b-(2\lambda-1)\}]}$$

$$+\frac{512(-55760b^5+180744ab^5-10608a^2b^5+19448a^3b^5+11900b^6-3808ab^6+6188a^2b^6-272b^7+680ab^7+17b^8)}{[\prod_{\mu=1}^{9}\{a-b-(2\mu-1)\}][\prod_{\lambda=1}^{8}\{a-b-(2\lambda-1)\}]}\}] \quad (7)$$

$${}_2F_1\left(a, b; \frac{a+b+20}{2}; \frac{1}{2}\right) = 2^b\frac{\Gamma(\frac{a+b+20}{2})}{(a-b)\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a}{2})}\{\frac{512(10321920a-14026752a^2+7559936a^3-2153088a^4)}{[\prod_{\xi=0}^{8}\{a-b-2\xi\}][\prod_{\delta=1}^{9}\{a-b-2\delta\}]}\right.$$

$$+\frac{512(359184a^5-36288a^6+2184a^7-72a^8+a^9+10321920b+46048512a^2b-8891136a^3b+5874384a^4b)}{[\prod_{\xi=0}^{8}\{a-b-2\xi\}][\prod_{\delta=1}^{9}\{a-b-2\delta\}]}$$

$$+\frac{512(-470016a^5b+88536a^6b-2448a^7b+153a^8b+14026752b^2+46048512ab^2+19915296a^3b^2-1126080a^4b^2)}{[\prod_{\xi=0}^{8}\{a-b-2\xi\}][\prod_{\delta=1}^{9}\{a-b-2\delta\}]}$$

$$+\frac{512(725832a^5b^2-17136a^6b^2+3060a^7b^2+7559936b^3+8891136ab^3+19915296a^2b^3+1935960a^4b^3-31824a^5b^3)}{[\prod_{\xi=0}^{8}\{a-b-2\xi\}][\prod_{\delta=1}^{9}\{a-b-2\delta\}]}$$

$$+\frac{512(18564a^6b^3+2153088b^4+5874384ab^4+1126080a^2b^4+1935960a^3b^4+43758a^5b^4+359184b^5+470016ab^5)}{[\prod_{\xi=0}^{8}\{a-b-2\xi\}][\prod_{\delta=1}^{9}\{a-b-2\delta\}]}$$

$$+\frac{512(725832a^2b^5+31824a^3b^5+43758a^4b^5+36288b^6+88536ab^6+17136a^2b^6+18564a^3b^6+2184b^7+2448ab^7)}{[\prod_{\xi=0}^{8}\{a-b-2\xi\}][\prod_{\delta=1}^{9}\{a-b-2\delta\}]}$$

$$+\frac{512(3060a^2b^7+72b^8+153ab^8+b^9)}{[\prod_{\xi=0}^{8}\{a-b-2\xi\}][\prod_{\delta=1}^{9}\{a-b-2\delta\}]}+\frac{1024b(10321920+3274752a+11950848a^2+1564032a^3+849936a^4)}{[\prod_{\sigma=0}^{9}\{a-b-2\sigma\}][\prod_{\tau=1}^{8}\{a-b-2\tau\}]}$$

$$+\frac{1024b(48768a^5+8232a^6+168a^7+9a^8-3274752b+29706752ab+2295680a^2b+6370240a^3b+315520a^4b)}{[\prod_{\sigma=0}^{9}\{a-b-2\sigma\}][\prod_{\tau=1}^{8}\{a-b-2\tau\}]}$$

$$+\frac{1024b(146064a^5b+2856a^6b+408a^7b+11950848b^2-2295680ab^2+11708512a^2b^2+304640a^3b^2)}{[\prod_{\sigma=0}^{9}\{a-b-2\sigma\}][\prod_{\tau=1}^{8}\{a-b-2\tau\}]}$$

$$+\frac{1024b(673880a^4b^2+11016a^5b^2+4284a^6b^2-1564032b^3+6370240ab^3-304640a^2b^3+1096160a^3b^3)}{[\prod_{\sigma=0}^{9}\{a-b-2\sigma\}][\prod_{\tau=1}^{8}\{a-b-2\tau\}]}$$

$$+\frac{1024b(8840a^4b^3+15912a^5b^3+849936b^4-315520ab^4+673880a^2b^4-8840a^3b^4+24310a^4b^4-48768b^5)}{[\prod_{\sigma=0}^{9}\{a-b-2\sigma\}][\prod_{\tau=1}^{8}\{a-b-2\tau\}]}$$

$$+\frac{1024b(146064ab^5-11016a^2b^5+15912a^3b^5+8232b^6-2856ab^6+4284a^2b^6-168b^7+408ab^7+9b^8)}{[\prod_{\sigma=0}^{9}\{a-b-2\sigma\}][\prod_{\tau=1}^{8}\{a-b-2\tau\}]}\}-\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a+1}{2})}$$

$$\{\frac{1024a(10321920-3274752a+11950848a^2-1564032a^3+849936a^4-48768a^5+8232a^6-168a^7)}{[\prod_{\xi=0}^{8}\{a-b-2\xi\}][\prod_{\delta=1}^{9}\{a-b-2\delta\}]}$$

$$+\frac{1024a(9a^8+3274752b+29706752ab-2295680a^2b+6370240a^3b-315520a^4b+146064a^5b-2856a^6b+408a^7b)}{[\prod_{\xi=0}^{8}\{a-b-2\xi\}][\prod_{\delta=1}^{9}\{a-b-2\delta\}]}$$

$$+\frac{1024a(11950848b^2+2295680ab^2+11708512a^2b^2-304640a^3b^2+673880a^4b^2-11016a^5b^2+4284a^6b^2)}{[\prod_{\xi=0}^{8}\{a-b-2\xi\}][\prod_{\delta=1}^{9}\{a-b-2\delta\}]}$$

$$+\frac{1024a(1564032b^3+6370240ab^3+30464a^2b^3+1096160a^3b^3-8840a^4b^3+15912a^5b^3+849936b^4+315520ab^4)}{[\prod_{\xi=0}^{8}\{a-b-2\xi\}][\prod_{\delta=1}^{9}\{a-b-2\delta\}]}$$

$$+\frac{1024a(673880a^2b^4+8840a^3b^4+24310a^4b^4+48768b^5+146064ab^5+11016a^2b^5+15912a^3b^5+8232b^6)}{[\prod_{\xi=0}^{8}\{a-b-2\xi\}][\prod_{\delta=1}^{9}\{a-b-2\delta\}]}$$

$$+\frac{1024a(2856ab^6+4284a^2b^6+168b^7+408ab^7+9b^8)}{[\prod_{\xi=0}^{8}\{a-b-2\xi\}][\prod_{\delta=1}^{9}\{a-b-2\delta\}]}+\frac{512(10321920a+14026752a^2+7559936a^3)}{[\prod_{\sigma=0}^{9}\{a-b-2\sigma\}][\prod_{\tau=1}^{8}\{a-b-2\tau\}]}$$

$$+\frac{512(2153088a^4+359184a^5+36288a^6+2184a^7+72a^8+a^9+10321920b+46048512a^2b+8891136a^3b)}{[\prod_{\sigma=0}^{9}\{a-b-2\sigma\}][\prod_{\tau=1}^{8}\{a-b-2\tau\}]}$$

$$+\frac{512(5874384a^4b+470016a^5b+88536a^6b+2448a^7b+153a^8b-14026752b^2+46048512ab^2+19915296a^3b^2)}{[\prod_{\sigma=0}^{9}\{a-b-2\sigma\}][\prod_{\tau=1}^{8}\{a-b-2\tau\}]}$$

$$+\frac{512(1126080a^4b^2+725832a^5b^2+17136a^6b^2+3060a^7b^2+7559936b^3-8891136ab^3+19915296a^2b^3)}{[\prod_{\sigma=0}^{9}\{a-b-2\sigma\}][\prod_{\tau=1}^{8}\{a-b-2\tau\}]}$$

$$+\frac{512(1935960a^4b^3+31824a^5b^3+18564a^6b^3-2153088b^4+5874384ab^4-1126080a^2b^4+1935960a^3b^4)}{[\prod_{\sigma=0}^{9}\{a-b-2\sigma\}][\prod_{\tau=1}^{8}\{a-b-2\tau\}]}$$

$$+\frac{512(43758a^5b^4+359184b^5-470016ab^5+725832a^2b^5-31824a^3b^5+43758a^4b^5-36288b^6+88536ab^6)}{[\prod_{\sigma=0}^{9}\{a-b-2\sigma\}][\prod_{\tau=1}^{8}\{a-b-2\tau\}]}$$

$$+\frac{512(-17136a^2b^6+18564a^3b^6+2184b^7-2448ab^7+3060a^2b^7-72b^8+153ab^8+b^9)}{[\prod_{\sigma=0}^{9}\{a-b-2\sigma\}][\prod_{\tau=1}^{8}\{a-b-2\tau\}]}\}] \quad (8)$$

Derivations of summation formulae (7) to (8):

Derivation of (7): Replacing $c = \frac{a+b+19}{2}$ and $z = \frac{1}{2}$ in equation (3) , we get

$$(a-b)\ {}_2F_1\left(a, b; \frac{a+b+19}{2}; \frac{1}{2}\right) = a\ {}_2F_1\left(a+1, b; \frac{a+b+19}{2}; \frac{1}{2}\right) - b\ {}_2F_1\left(a, b+1; \frac{a+b+19}{2}; \frac{1}{2}\right)$$

Now with the help of the derived result from Gauss second summation theorem, we get

$$\text{L.H.S} = a\,2^b\frac{\Gamma(\frac{a+b+19}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a+1}{2})}\{\frac{256(2027025-4098240a+2924172a^2-1038016a^3+208054a^4-24640a^5)}{[\prod_{\zeta=1}^{8}\{a-b-(2\zeta-1)\}][\prod_{\omega=1}^{9}\{a-b-(2\omega-1)\}]}\right.$$

$$+\frac{256(1708a^6-64a^7+a^8-1040400b+12903000ab-3491664a^2b+2808536a^3b-270640a^4b+59976a^5b-1904a^6b)}{[\prod_{\zeta=1}^{8}\{a-b-(2\zeta-1)\}][\prod_{\omega=1}^{9}\{a-b-(2\omega-1)\}]}$$

$$+\frac{256(136a^7b+7410300b^2-511904ab^2+7452052a^2b^2-538560a^3b^2+415140a^4b^2-11424a^5b^2+2380a^6b^2)}{[\prod_{\zeta=1}^{8}\{a-b-(2\zeta-1)\}][\prod_{\omega=1}^{9}\{a-b-(2\omega-1)\}]}$$

$$+\frac{256(984368b^3+5298424ab^3-35360a^2b^3+901680a^3b^3-17680a^4b^3+12376a^5b^3+872678b^4+250240ab^4)}{[\prod_{\zeta=1}^{8}\{a-b-(2\zeta-1)\}][\prod_{\omega=1}^{9}\{a-b-(2\omega-1)\}]}$$

$$+\frac{256(693940a^2b^4+24310a^4b^4+55760b^5+180744ab^5+10608a^2b^5+19448a^3b^5+11900b^6+3808ab^6)}{[\prod_{\zeta=1}^{8}\{a-b-(2\zeta-1)\}][\prod_{\omega=1}^{9}\{a-b-(2\omega-1)\}]}$$

$$+\frac{256(6188a^2b^6+272b^7+680ab^7+17b^8)}{[\prod_{\zeta=1}^{8}\{a-b-(2\zeta-1)\}][\prod_{\omega=1}^{9}\{a-b-(2\omega-1)\}]}\}-\frac{\Gamma(\frac{b-1}{2})}{\Gamma(\frac{a+1}{2})}\{\frac{256(2027025+1040400a+7410300a^2-984368a^3)}{[\prod_{\zeta=1}^{8}\{a-b-(2\zeta-1)\}][\prod_{\omega=1}^{9}\{a-b-(2\omega-1)\}]}$$

$$+\frac{256(872678a^4-55760a^5+11900a^6-272a^7+17a^8+4098240b+12903000ab+511904a^2b+5298424a^3b)}{[\prod_{\zeta=1}^{8}\{a-b-(2\zeta-1)\}][\prod_{\omega=1}^{9}\{a-b-(2\omega-1)\}]}$$

$$+\frac{256(-250240a^4b+180744a^5b-3808a^6b+680a^7b+2924172b^2+3491664ab^2+7452052a^2b^2+35360a^3b^2)}{[\prod_{\zeta=1}^{8}\{a-b-(2\zeta-1)\}][\prod_{\omega=1}^{9}\{a-b-(2\omega-1)\}]}$$

$$+\frac{256(693940a^4b^2-10608a^5b^2+6188a^6b^2+1038016b^3+280536ab^3+538560a^2b^3+901680a^3b^3+19448a^5b^3)}{[\prod_{\zeta=1}^{8}\{a-b-(2\zeta-1)\}][\prod_{\omega=1}^{9}\{a-b-(2\omega-1)\}]}$$

$$+\frac{256(208054b^4+270640ab^4+415140a^2b^4+17680a^3b^4+24310a^4b^4+24640b^5+59976ab^5+11424a^2b^5)}{[\prod_{\zeta=1}^{8}\{a-b-(2\zeta-1)\}][\prod_{\omega=1}^{9}\{a-b-(2\omega-1)\}]}$$

$$+\frac{256(12376a^3b^5+1708b^6+1904ab^6+2380a^2b^6+64b^7+136ab^7+b^8)}{[\prod_{\zeta=1}^{8}\{a-b-(2\zeta-1)\}][\prod_{\omega=1}^{9}\{a-b-(2\omega-1)\}]}\}]-b\,2^{b+1}$$

$$\frac{\Gamma(\frac{a+b+19}{2})}{\Gamma(b+1)}[\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a}{2})}\{\frac{256(2027025+4098240a+2924172a^2+1038016a^3+208054a^4+24640a^5+1708a^6)}{[\prod_{\mu=1}^{9}\{a-b-(2\mu-1)\}][\prod_{\lambda=1}^{8}\{a-b-(2\lambda-1)\}]}$$

$$+\frac{256(64a^7+a^8+1040400b+12903000ab+3491664a^2b+2808536a^3b+270640a^4b+59976a^5b+1904a^6b+136a^7b)}{[\prod_{\mu=1}^{9}\{a-b-(2\mu-1)\}][\prod_{\lambda=1}^{8}\{a-b-(2\lambda-1)\}]}$$

$$+\frac{256(7410300b^2+511904ab^2+7452052a^2b^2+538560a^3b^2+415140a^4b^2+11424a^5b^2+2380a^6b^2-984368b^3)}{[\prod_{\mu=1}^{9}\{a-b-(2\mu-1)\}][\prod_{\lambda=1}^{8}\{a-b-(2\lambda-1)\}]}$$

$$+\frac{256(5298424ab^3+35360a^2b^3+901680a^3b^3+17680a^4b^3+12376a^5b^3+872678b^4-250240ab^4+693940a^2b^4)}{[\prod_{\mu=1}^{9}\{a-b-(2\mu-1)\}][\prod_{\lambda=1}^{8}\{a-b-(2\lambda-1)\}]}$$

$$+\frac{256(+24310a^4b^4-55760b^5+180744ab^5-10608a^2b^5+19448a^3b^5+11900b^6-3808ab^6+6188a^2b^6)}{[\prod_{\mu=1}^{9}\{a-b-(2\mu-1)\}][\prod_{\lambda=1}^{8}\{a-b-(2\lambda-1)\}]}$$

$$+\frac{256(-272b^7+680ab^7+17b^8)}{[\prod_{\mu=1}^{9}\{a-b-(2\mu-1)\}][\prod_{\lambda=1}^{8}\{a-b-(2\lambda-1)\}]}\}-$$

$$\frac{\Gamma(\frac{b+2}{2})}{\Gamma(\frac{a+1}{2})}\{\frac{256(2027025-1040400a+7410300a62+984368a^3)}{[\prod_{\mu=1}^{9}\{a-b-(2\mu-1)\}][\prod_{\lambda=1}^{8}\{a-b-(2\lambda-1)\}]}$$

$$+\frac{256(872678a^4+55760a^5+11900a^6+272a^7+17a^8-4098240b+12903000ab-511904a^2b+250240a^4b)}{[\prod_{\mu=1}^{9}\{a-b-(2\mu-1)\}][\prod_{\lambda=1}^{8}\{a-b-(2\lambda-1)\}]}$$

$$+\frac{256(180744a^5b+3808a^6b+680a^7b+2924172b^2-3491664ab^2+7452052a^2b^2-35360a^3b^2+693940a^4b^2)}{[\prod_{\mu=1}^{9}\{a-b-(2\mu-1)\}][\prod_{\lambda=1}^{8}\{a-b-(2\lambda-1)\}]}$$

$$+\frac{256(10608a^5b^2+6188a^6b^2-1038016b^3+2808536ab^3-538560a^2b^3+901680a^3b^3+19448a^5b^3+208054b^4)}{[\prod_{\mu=1}^{9}\{a-b-(2\mu-1)\}][\prod_{\lambda=1}^{8}\{a-b-(2\lambda-1)\}]}$$

$$+\frac{256(-270640ab^4+415140a^2b^4-17680a^3b^4+24310a^4b^4-24640b^5+59976ab^5-11424a^2b^5+12376a^3b^5)}{[\prod_{\mu=1}^{9}\{a-b-(2\mu-1)\}][\prod_{\lambda=1}^{8}\{a-b-(2\lambda-1)\}]}$$

$$+\frac{256(1708b^6-1904ab^6+2380a^2b^6-64b^7+136ab^7+b^8)}{[\prod_{\mu=1}^{9}\{a-b-(2\mu-1)\}][\prod_{\lambda=1}^{8}\{a-b-(2\lambda-1)\}]}\}]$$

On simplification we get the result (7)

Similarly , we can prove the formula (8).

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