



Stopping power of carbon, aluminium and silicon for electrons and positrons

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ABSTRACT

In this paper we present a relation for total stopping power of Carbon, Aluminium and Silicon for electrons and positrons in terms of total energy from 30 keV to 3000 keV and have been fitted by an exponential inverse power approximation with two parameters. These parameters depend upon the atomic weight (A) and atomic number (Z) of the absorber. It has been found that the exponential potential function gives better agreement with the available experimental data as compared to the inverse power form.

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Introduction

Energy deposition models of electrons passing through matter are necessary for several radiological and dosimetric applications. For example, models of electron transport through toluene are important in understanding the energy transfer in some radiation detectors such as scintillating counters. It has been shown recently [1] that low-energy electrons play an important role in these mechanisms. A significant part of these electrons are produced by energy degradation of high-energy electrons by means of successive collisions. Therefore parameters involved in the energy loss procedure are needed over a broad energy range. The simplest energy deposition models for electrons are based on the stopping power (mean energy loss per unit path length).

These parameters have been calculated in the frame work of the first Born approximation by means of the Bethe formula [2] and are available for some molecules in the NIST databases (<http://physics.nist.gov/PhysRefData/Star/Text/ESTAR.html>). However, as it is well known, this approximation is only valid for energies above 10 keV, in the case of toluene, and therefore more accurate techniques should be used for energies below this limit. In this paper, we propose a method to obtain the stopping power for electrons and positrons at intermediate energy (30–3000 keV) in terms of exponential power law. Results obtained by this procedure are compared with the available intermediate-energy data, above 30 keV, derived from the Born–Bethe approximation.

Previous empirical relations for stopping power

The stopping power has been used in Monte Carlo simulations of electron transport relevant to electron probe microanalysis [3-5]. The Bethe stopping power equation [6-8] has been used extensively for energies where it is expected to be valid, but there is a scarcity of data at lower energies. The classical Bethe theory on the interaction of electrons with matter has been based on the Born approximation. Stopping powers

calculated from the Bethe equation are available from a NIST database for electron energies of 10 keV and above [9].

Several empirical relations are given in literature to simplify the expressions for stopping power. One of them is due to Sargent [10]. His expression for the rate of change of velocity of low energy (<100 keV) electrons in Aluminum is

$$-\frac{d\beta}{dx} = \frac{2.02}{\beta^3}$$

β is the ratio of velocity of electron to velocity of light and thickness x is expressed in cms. This relation is limited to low energies and valid for Aluminum only. Further it cannot differentiate between electron and positron.

Heitler [11] established a relation for collision stopping power which is valid for somewhat higher energies also. This is given by

$$-\left(\frac{dE}{dx}\right)_{ion} = \xi \ln(\eta W) \quad (2)$$

W is the total energy expressed in MeV and ξ and η are the constants which differ from material to materials. Their values for Aluminum and Copper are,

Al	$\xi=0.22 \text{ MeV cm}^2/\text{gm}$	$\eta=316\text{MeV}^{-1}$
Cu	$\xi=0.21 \text{ MeV cm}^2/\text{gm}$	$\eta=187\text{MeV}^{-1}$

This expression does not take into account bremsstrahlung losses. Also it cannot differentiate between electron and positron.

According to Batra and Sehgal [12-14] the total stopping power of electron and positron may be represented by product of two functions. These functions must depend on the kinetic energy (T) of electron or positrons and the atomic number (Z) of the material. These equations take account of bremsstrahlung losses along with collision energy losses. According to them these equations were valid for energies up to 5.0 MeV for materials of atomic number up to 92.

For $T \leq 0.5 \text{ MeV}$

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$$\left(-\frac{1}{\rho} \frac{dE}{dx}\right)_{Tot}^{\pm} = (M_1 Z + C_1) F^{\pm}(\gamma) \quad (3)$$

$$F^+(\gamma) = \frac{\gamma^{2.4}}{\gamma^{(1.9-1)}}$$

$$F^-(\gamma) = \frac{\gamma^{2.56}}{\gamma^2 - 1}$$

Where ρ is the density of target material and γ is the total energy of electron or positron in electron mass unit. M_1 and C_1 are constants.

For $0.25 \text{ meV} \leq T \leq 5.0 \text{ MeV}$

$$\left(-\frac{1}{\rho} \frac{dE}{dx}\right)_{Tot}^{\pm} = (M_2 Z + C_2) \left[\frac{\gamma^2}{\gamma^{a^{\pm}z+b^{\pm}} - 1} \right] \quad (4)$$

Superscripts \pm represents for positron and electron respectively. M_2 , C_2 , a^{\pm} and b^{\pm} are constants.

Pal et al [15] developed a similar formula on the lines of Batra and Sehgal [12-14] but with different set of constants,

$$\left(-\frac{1}{\rho} \frac{dE}{dx}\right)_{Tot}^{\pm} = (M_2 Z + C_2) (P_0^{\pm} + P_1^{\pm} \gamma) \quad (5)$$

$P_n^{\pm} = A_n^{\pm} + B_n^{\pm} Z + C_n^{\pm} Z^2$, where A, B and C are constants and $n = 0$ & 1. This relation holds well from 5 to 1000 meV.

Recently, Tanuma et al [16] has calculated electron stopping power for 31 elemental solids. These stopping powers are determined with an algorithm previously used for the calculation of electron inelastic mean free paths and from energy loss functions derived from experimental optical data. The stopping power calculations are valid for electron energies between 100 eV and 30keV. Through exponential fitting, we have been able to find out a single empirical relation for total stopping power from 30 keV to 3000 keV. It is inferred that the total stopping power of electrons as well as positrons depends not only upon the incident kinetic energies of these particles, but also on the nature of the material through which they traverse. It was noticed that the dependence of total stopping power on incident kinetic energy could only be met through the use of some suitable power function.

Proposed relation for stopping power

In general the exponential fitting is a simple method for searching out any empirical relation. However, the relation for total stopping power should be simple enough to get easily integrable. One can infer from equations (1 to 5) that the total stopping powers of electrons as well as that of positrons depend not only upon the incident kinetic energy of these particles, but also on the nature of the material through which they traverse. It was noticed that the dependence of total stopping power on incident kinetic energy could only be met through the use of some suitable power function. Tan et al [17] have proposed a simple empirical relation for csda ranges for electrons with energies between 25 to 200 keV by the following relation,

$$R_{csda}^- = 190 \times 10^{-6} \left(\frac{A}{Z}\right)^{2.5} E^{1.6} \text{ g/cm}^2 \quad (6)$$

where A, Z and E are atomic weight, atomic number and energy respectively. This relation is valid for atomic numbers 30 to 92 with error comprising 2 to 10 %. The drawback of the relation is that it is valid for very small energy range and does not give any information for lower atomic number. We have

plotted graphs between available stopping power values vs $\left(\frac{A}{Z}\right)E^{0.25}$ and data are decreasing exponentially. Which are presented in following figures 1 to 6.

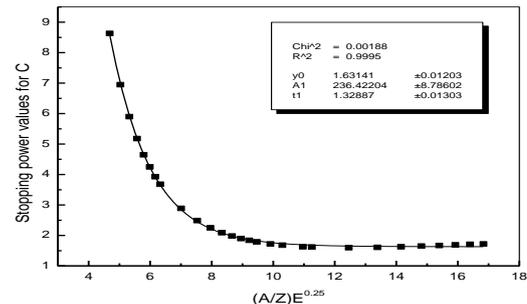


Figure 1. In the plot of stopping power (for electrons) and $(A/Z)E^{0.25}$ of Carbon lie on a decreasing exponentially line. In this figure all values are taken from Ref. [9]

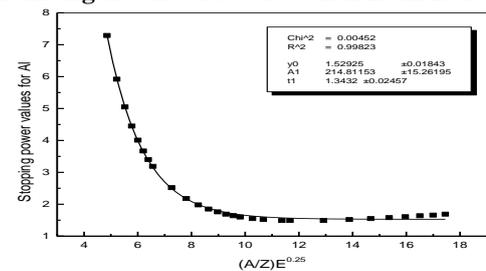


Figure 2. In the plot of stopping power (for electrons) and $(A/Z)E^{0.25}$ of Aluminium lie on a decreasing exponentially line. In this figure all values are taken from Ref. [9]

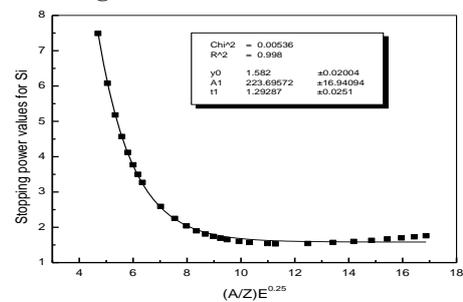


Figure 3. In the plot of stopping power (for electrons) and $(A/Z)E^{0.25}$ of Silicon lie on a decreasing exponentially line. In this figure all values are taken from Ref. [9]

Using this idea we have been able to find analytically convenient and simple empirical relation for total stopping power of electrons and positrons in intermediate energy regions 30 to 3000 keV by the following relation,

$$\left(-\frac{1}{\rho} \frac{dE}{ds}\right)_{Total}^{\pm} = y_0 + A_1 e^{\left(\frac{-x}{t_1}\right)} \quad (7)$$

where y_0 , A_1 and t_1 are constants. The value of 'x' = $(A/Z)E^{0.25}$, depends on atomic weight (A), atomic number (Z) and energy (E). In equation (7) superscript (+) and (-) stands for positron and electron respectively. The values of constants are presented in table 1 (for electrons) and 2 (for positrons).

Comparison between proposed and reported values

Tan [17] relation has been verified by proposed stopping power relation (7). This relation has been used to calculate total stopping power of electrons and positrons in different elements for various energies in their prescribed energy regions [9]. The calculated values for electrons thus obtained were compared with the standard values due to Berger and Seltzer [9]. In order

to compare evaluated values of total stopping power of positrons, following method has been adopted. The collision stopping power of the positrons has been extracted from some of known [9] ratios of this stopping power for positrons and electrons. Due to lack of any experimental data for bremsstrahlung losses of positrons, the collision stopping power of positrons was then added with the same percentage of bremsstrahlung losses as that given for the corresponding energy electrons [9]. The evaluated values of total stopping power have been presented in the tables 3 and 4. We note that the values of stopping power evaluated by proposed relation are in close agreement with the reported data as compared to the values reported by previous researchers so far. This relation is valid for lower atomic numbers together with error comprising 0.1 to 5.8 %.

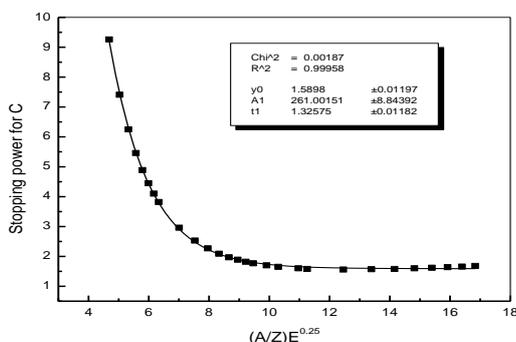


Figure 4. In the plot of stopping power (for positrons) and $(A/Z)E^{0.25}$ of Carbon lie on a decreasing exponentially line. In this figure all values are taken from Ref. [9]

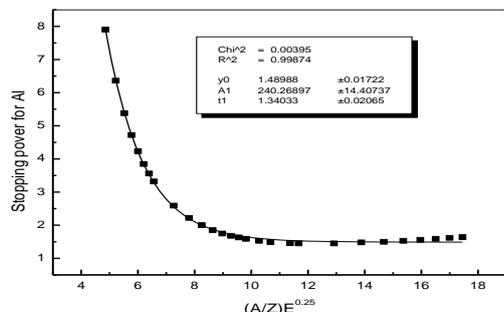


Figure 5. In the plot of stopping power (for positrons) and $(A/Z)E^{0.25}$ of Aluminium lie on a decreasing exponentially line. In this figure all values are taken from Ref. [9]

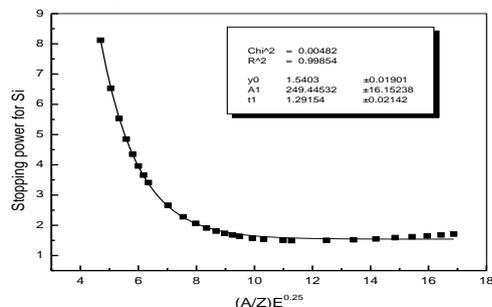


Figure 6. In the plot of stopping power (for positrons) and $(A/Z)E^{0.25}$ of Silicon lie on a decreasing exponentially line. In this figure all values are taken from Ref. [9]

Summary and Conclusions

From the above results obtained using the proposed empirical relation (7), it is quite obvious that the stopping power of materials can be expressed in terms of energy and atomic number of the material. We come to the conclusion that energy of the material is key parameter for the calculation of stopping power. It is also noteworthy that proposed empirical relation is simpler, widely applicable and values obtained are in better agreement with the experimental and theoretical data as compared to the empirical relations proposed by previous researchers [9, 12-15].

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Table 1. Values of constants for electrons

Materials	y_0	A_1	t_1	Chi^2	R^2
C	1.63141 ± 0.01203	236.42204 ± 8.78602	1.32887 ± 0.01303	0.00188	0.9995
Al	1.52925 ± 0.01843	214.81153 ± 15.26195	1.3432 ± 0.02457	0.00452	0.99823
Si	1.582 ± 0.02004	223.69572 ± 16.94094	1.29287 ± 0.0251	0.00536	0.998

Table 2. Values of constants for positrons

Materials	y_0	A_1	t_1	Chi^2	R^2
C	1.5898 ± 0.01197	261.00151 ± 8.84392	1.32575 ± 0.01182	0.00187	0.99958
Al	1.48988 ± 0.01722	240.26897 ± 14.40737	1.34033 ± 0.02065	0.00395	0.99874
Si	1.5403 ± 0.01901	249.44532 ± 16.15238	1.29154 ± 0.02142	0.00482	0.99854

Table 3. Values of stopping power for electrons of Carbon, Aluminium and Silicon

E keV	C this work	C [9]	% error	Al this work	Al [9]	% error	Si this work	Si [9]	% error
30	8.59	8.63	0.5	7.30	7.29	0.2	7.50	7.49	0.2
40	6.98	6.95	0.5	5.94	5.92	0.3	6.10	6.08	0.3
50	5.94	5.9	0.6	5.06	5.05	0.2	5.19	5.18	0.3
60	5.20	5.18	0.4	4.44	4.45	0.2	4.56	4.57	0.2
70	4.66	4.65	0.2	3.99	4.01	0.5	4.10	4.12	0.6
80	4.24	4.25	0.1	3.64	3.67	0.7	3.74	3.77	0.8
90	3.91	3.93	0.4	3.37	3.4	0.9	3.46	3.5	1.1
100	3.65	3.68	0.8	3.15	3.19	1.2	3.24	3.27	1.0
150	2.85	2.89	1.6	2.49	2.52	1.1	2.56	2.59	1.1
200	2.45	2.49	1.6	2.17	2.18	0.3	2.24	2.25	0.6
250	2.22	2.25	1.2	1.99	1.98	0.5	2.05	2.04	0.5
300	2.08	2.09	0.5	1.88	1.85	1.4	1.93	1.9	1.7
350	1.98	1.98	0.1	1.80	1.76	2.2	1.85	1.81	2.4
400	1.91	1.9	0.6	1.74	1.69	3.2	1.80	1.74	3.4
450	1.86	1.84	1.1	1.70	1.64	3.9	1.76	1.69	4.0
500	1.82	1.79	1.8	1.67	1.6	4.6	1.73	1.65	4.7
600	1.77	1.72	2.8	1.63	1.55	5.3	1.69	1.6	5.3
700	1.73	1.68	3.2	1.61	1.52	5.6	1.66	1.57	5.6
900	1.69	1.63	3.9	1.57	1.49	5.7	1.63	1.54	5.7
1000	1.68	1.62	3.8	1.57	1.49	5.1	1.62	1.53	5.8
1500	1.65	1.6	3.2	1.54	1.49	3.6	1.60	1.54	3.7
2000	1.64	1.61	1.9	1.54	1.52	1.1	1.59	1.57	1.2
2500	1.64	1.63	0.4	1.53	1.55	1.1	1.59	1.6	0.9
3000	1.63	1.65	0.9	1.53	1.58	3.1	1.58	1.63	2.8

Table 4. Values of stopping power for positrons of Carbon, Aluminium and Silicon

E keV	C this work	C [9]	% error	Al this work	Al [9]	% error	Si this work	Si [9]	% error
30	9.21	9.26	0.5	7.90	7.9	0.0	8.12	8.12	0.0
40	7.44	7.41	0.5	6.38	6.36	0.3	6.56	6.53	0.4
50	6.30	6.25	0.8	5.40	5.38	0.4	5.55	5.53	0.4
60	5.49	5.46	0.6	4.72	4.72	0.0	4.85	4.85	0.0
70	4.90	4.89	0.2	4.22	4.23	0.3	4.33	4.35	0.4
80	4.44	4.45	0.1	3.83	3.85	0.5	3.94	3.96	0.6
90	4.08	4.1	0.4	3.53	3.56	0.9	3.63	3.66	0.9
100	3.79	3.82	0.7	3.28	3.32	1.1	3.38	3.41	1.0
150	2.91	2.96	1.6	2.55	2.59	1.4	2.63	2.66	1.2
200	2.48	2.53	1.9	2.20	2.22	0.9	2.27	2.28	0.7
250	2.23	2.27	1.6	2.00	2	0.1	2.06	2.06	0.1
300	2.08	2.09	0.6	1.87	1.85	1.2	1.93	1.91	1.0
350	1.97	1.97	0.0	1.79	1.75	2.1	1.84	1.81	1.8
400	1.89	1.89	0.2	1.73	1.68	2.7	1.78	1.73	2.9
450	1.84	1.82	1.0	1.68	1.63	3.2	1.74	1.68	3.3
500	1.80	1.77	1.5	1.65	1.59	3.7	1.70	1.63	4.4
600	1.74	1.7	2.2	1.60	1.53	4.8	1.65	1.57	5.4
700	1.70	1.65	3.1	1.57	1.49	5.6	1.62	1.54	5.5
900	1.66	1.6	3.5	1.54	1.46	5.5	1.59	1.5	6.0
1000	1.64	1.58	4.0	1.53	1.45	5.5	1.58	1.49	6.1
1500	1.61	1.56	3.3	1.51	1.45	3.8	1.56	1.5	3.7
2000	1.60	1.57	1.9	1.50	1.48	1.2	1.55	1.52	1.8
2500	1.60	1.58	1.0	1.49	1.5	0.4	1.54	1.55	0.4
3000	1.59	1.6	0.4	1.49	1.53	2.5	1.54	1.59	3.0