Available online at www.elixirpublishers.com (Elixir International Journal)

Applied Mathematics

Elixir Appl. Math. 37 (2011) 3848-3851

ISSN: 2229-712X

A new genetic algorithm for solving multi peak optimization problems based on the trust region with memory

Jinghui Gao, Rui Shan, Lingling Li and Donglian Gao

College of Science, Yanshan University, Qinhuangdao066004, Hebei, China.

ARTICLE INFO

Article history:

Received: 12 June 2011;
Received in revised form: 19 July 2011;
Accepted: 29 July 2011;

Keywords

Trust Region Method with Memory,
The steepest descent direction
Effluent,
Conjugate direction,
Genetic algorithm,
Multi peak optimization problems.

ABSTRACT

This paper presented a trust region method with memory, which retained the individuals with secondary high fitness. What's more, we brought in the most decline direction and conjugate direction to keep the method going on and even joined it in genetic algorithm for enhancing search efficiency in the middle-late time. We find the trust region with memory had higher search ability than trust region comparing results and the algorithm was an effective method on solving multi-peak problems proved by examples.

Introduction

Genetic algorithm proposed by Professor John Holland in 1962. Now genetic algorithm has a great advantage in practical science. But its global search weaken the local search [1].The trust region method have pertinence and rapid convergence, even better local search, but weak search on the optimal solutions of multi-peak function problems[2]. So there are a number of researcher try to combine two methods above for prefect results. Shounan Z. proposed a genetic trust region method for answering multi-peak function problems [5] in 2001. Zhang presented a trust region genetic algorithm, which avoids the shortcoming of trust region method to solute secondary optimal problems [6]. In 2008, a global optimal hybrid algorithm for multi-peak value problems raised by Zhang in [8], which combines genetic algorithm with regional algorithm to deal with the shortcoming of regional algorithm in answering multi-peak optimal problems. Those algorithms above offer a good idea to answer multi-peak optimal problems, however, they don't improve the weakness obviously on itself performance. So we give genetic algorithm based on the trust region with memory in this paper, which mixes the trust region with memory into genetic algorithm according to the specialty of genetic algorithm for solving slow speed of genetic algorithm in the late search time. The examples show that the algorithm in this paper has better efficiency on complex multi-peak optimal problems.

Trust Region Method

For trust region method，we define $q^{(k)}$ as follows

$$q^{(k)}(d) = f(x^{(k)}) + (g^{(k)})^T d + \frac{1}{2} d^T G^{(k)} d \qquad (1)$$

where $g^{(k)} = \nabla f(x^{(k)})$ is the gradient of $f(x^{(k)})$ at the point $x^{(k)}$, $G^{(k)} = \nabla^2 f(x^{(k)})$. d is search direction at the point $x^{(k)}$, which demands HESSE matrix of objective function $f(x^{(k)})$ is positive and immobile. At the neighboring region of the point $x^{(k)}$, the problem(1) has already changed into secondary function minimum problem(2) in the condition of restriction d and $\Delta^{(k)} > 0$.

$$\min q^{(k)}(d), s.t. \|d\|_2 \le \Delta^{(k)} \qquad (2)$$

$\Delta^{(k)}$ is trust region radius, The ratio between these two reductions is

$$\beta^{(k)} = \frac{Ared^{(k)}}{pred^{(k)}}$$

when $pred^{(k)} = q^{(k)}(0) - q^{(k)}(d^{(k)})$ is called the actual reduction of the objection function ; $Ared^{(k)} = f(x^{(k)}) - f(x^{(k)} + d^{(k)})$ is the predictive reduction.

Trust Region Method with Memory (MTR)

We change the problem (1) into the form of Inner product as follow for bringing in trust region memory model

$$\min(g^{(k)}, d) + \frac{1}{2}(d, G^{(k)} d) \qquad (3)$$

$s.t. \|d\|_2 \le \Delta^{(k)}$

Sign current point model as

$Q^{(k)}(x) = f(x^{(k)}) + (g^{(k)}, x - x^{(k)}) + \frac{1}{2}(x - x^{(k)}, G^{(k)}(x - x^{(k)}))$, so found the model as follows

$$Q_M^{(k)}(x) = 1 - \mu^{(k)} Q^{(k)}(x) + \mu^{(k)} Q_M^{k-1}(x) \qquad (4)$$

where $\mu^{(k)} \in [0,1)$ is a parameter. In this paper, we choose $\mu^{(k)} = \min\{\overline{\mu}, \Delta^{(k)}\}$, $\overline{\mu} \in (0,1)$,

Tele:
E-mail addresses: gjhlove123@163.com

$$g_M^{(k)} = \nabla Q_M^{(k)}(x^{(k)}),$$
$$G_M^{(k)} = \nabla_{xx}^2 Q_M^{(k)}(x^{(k)}).$$

Consider the trust region problem with memory model

$$\min(g_M^{(k)}, d) + \frac{1}{2}(d, G_M^{(k)} d) \quad s.t.\|d\|_2 \le \Delta^{(k)} \qquad (5)$$

d_k is the solution of problem (4),which shows the decline direction of $Q_M^{(k)}(x)$ and might not be the decline direction of $f(x)$.

We pick secondary penalty function as value function ($\varphi > 0$ is penalty parameter) to test that whether the solution d_k of problem (3) is accepted.

$$V(x,\varphi) = f(x) + \varphi\|C_i(x)\|^2 \qquad (6)$$

$$Ared^{(k)} = f(x^{(k)}) - f(x^{(k)} + d^{(k)}) + \varphi^{(k)}(\|C_i(x^{(k)})\|^2 - \|C_i(x^{(k)} + d^{(k)})\|^2) \qquad (7)$$

$$Pred^{(k)} = 1 - \mu^{(k)}Q^{(k)}(x) + \mu^{(k)}Q_M^{k-1}(x) + \phi^{(k)}(\|C_i(x^{(k)})\|^2 - \|C_i(x^{(k)} + d^{(k)})\|^2) \qquad (8)$$

Penalty function can reduce the amount of infeasible solutions in iteration process only. So we consider the steepest descent direction as initial search direction in this paper, and then update infeasible solutions with conjugate gradient direction till $x^{(k)}$.

Define initial search direction (the steepest descent direction) as $d_{k-j} = -\nabla f(x^{(k-j)})$, we obtain $x^{(k-j)}$ in step length θ; conjugate gradient direction d_{k-j+1}, Λ, d_k, $d_{k-j+1} = \nabla f(x^{(k-j+1)})$, $d_k = \nabla f(x^{(k)})$, then obtain $x^{(k-j+1)} = x^{(k-j)} + \theta d_{k-j}$ in step length θ.

Then give the steps of trust region method:

Step 1 Give $0 < \alpha_1 < \alpha_2 < 1$, $0 < \beta_1 < 1 < \beta_2$ initial point $x^{(0)} \in R^n$, initial trust region radius $\Delta^{(0)}$ and its up-boundary $\bar{\Delta}$, $\Delta^{(0)} \in (0, \bar{\Delta})$, $\varepsilon > 0$, set $k := 0$.

Step 2 If $\|g^{(k)}\| \le \varepsilon$,stop. Else, go step3.

Step 3 Consider $d^{(k)}$ within $x^{(k)}$. If $d^{(k)} = 0$, stop; else, go step 4;

Step 4 Calculate $f(x^{(k-j)} + d^{(k-j)})$ and $\beta^{(k)} = \frac{Ared^{(k)}}{Pred^{(k)}}$.

If $\beta^{(k)} \ge \alpha_1$, select the most speed decline direction $d^{(k-j)}$ to keep search in step θ, $x^{(k-j+1)} = x^{(k-j)} + \theta d^{(k-j)}$. Else, order $x^{(k-j+1)} = x^{(k-j)}$. Then replace the direction with conjugate direction $d^{(k-j+1)}, \Lambda, d^{(k)}$ to update $f(x^{(k)}) = x^{(k-j)} + d^{(k-j)}$ in $(\Delta^{(0)}, \bar{\Delta})$ continuously till $f(x^{(k)} + d^{(k)})$.

Step 5. Adjust trust region radius: If $r^{(k)} < \alpha_1$, set $\Delta^{(k+1)} = \beta_1\Delta^{(k)}$,if $\alpha_1 < \beta^{(k)} < \alpha_2$,set $\Delta^{(k+1)} = \Delta^{(k)}$,if $\beta^{(k)} > \alpha_2$, set $\Delta^{(k+1)} = \beta_2\Delta^{(k)}$.

Step 6. Update $G^{(k+1)}$ with BFGS formula[7], $k = k+1$, return step2.

Trust Region Genetic Algorithm with Memory (MTR-GA)

1). Real encode have high precision, easily coalescing other optimal algorithm. In this paper, $X = \{x_1, x_2, ..., x_n\}^T$, chromosomes are $x_1 x_2 ... x_n$.

2). Fitness function: define $F(X) = -f(X)$.

3) Select operation: keep excellent individuals in sir generations, for the individuals with secondary higher, the select probability is

$$P_s^i = \frac{F(X_i)+1}{\sum_{j=1}^{n}[F(X_j)]+1}.$$

4) Arithmetic crossover operation: $x' = \alpha x + (1-\alpha)y$, $y' = (1-\alpha)x + \alpha y$.

5) Weak-distributed mutation: start Weak-distributed mutation on each same-position gene in mutation probability P_m for $x_1 x_2 ... x_n$.

$$P_m = \frac{0.9(Maxgen - gen)}{Maxgen}$$

6)Termination condition: maximum generation termination.

Give genetic algorithm based on trust region with memory (MTR-GA) as follows

Step 1 Initialize parameter of algorithm: population scale M, maximum generation number $Maxgen$, select probability P_S, crossover probability P_C, mutation probability P_m, set current generation $gen = 0$.

Step 2 Constitute initial populations by $x_1 x_2 ... x_n$ randomly.

Step 3 If $gen > Maxgen$, stop and output current optimal solutions. Else, go step 4.

Step 4 Calculate individuals' fitness values and select the reserved sir generation individuals.

Step 5 "Memory" those sir generation individuals which have secondary high fitness value with memory model (6) and start trust region method on basic of selecting a point near to "memory point" as initial point of iteration.

Step 6 start genetic operations with the reserved sir generation and $x^{(k)}$ from step5.

Step 7 If $gen < Maxgen$, $gen := gen + 1$, turn to step 4. Else, then stop algorithm and output current solutions.

Numerical Emulation

We take classic multi-peak function to test algorithm's availability.

Function 1:

$$f_1(x,y) = -(x^2 + 2y^2 - 0.4\cos(3\pi x) - 0.6\cos(4\pi y)), -5 \le x, y \le 5$$

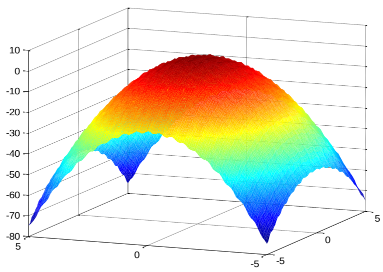

Fig.1 The three-dimension figure of function 1

Function 2:

$$f_2(x,y)=(4-2.1x^2+\frac{1}{3}y^4)x^2+xy+(-4+4y^2)y^2, -10\le x,y\le 10$$

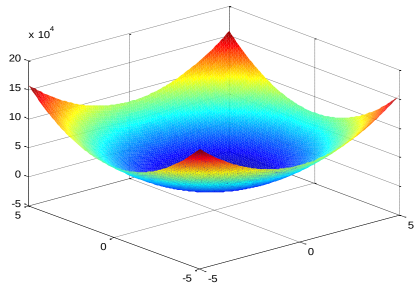

Fig.2 The three-dimension figure of function 2

Function 3: $f_3(x,y)=(x^2+y^2)^{0.25}[\sin(50(x^2+y^2)^{0.1})+1.0]$, $-10\le x,y\le 10$

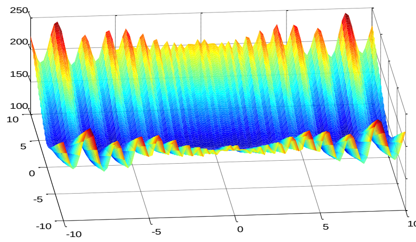

Fig.3 The three-dimension figure of function 3

Set parameters as follows to compare trust region method with MTR: $\alpha_1=0.5$, $\alpha_2=2$, $\beta_1=0.25$, $\beta_2=1.75$, $\mu^{(0)}=0$, $\Delta_0=0.1$, $\bar{\Delta}=10$, $\varepsilon=10^{-6}$.

We know that the number of finding optimization solutions with MTR algorithm is more than it with trust region from table 1 in the condition of the same initial points, which shows MTR algorithm effective.

Give the results of functions above with MTR-GA in C program (Table 2). Set MTR parameters as follows: $\mu^{(0)}=0$, $\alpha_1=0.01$, $\alpha_2=2$, $\beta_1=0.25$, $\beta_2=1.75$, $\theta^{(0)}=0.5$, $\Delta_0=0.1$, $\bar{\Delta}=10$, $\varepsilon=10^{-6}$.

The parameters of GA and the results of MTR-GA are given as follows: population scale: $M=100$, maximum generation: $Maxgen=100$; select probability: $P_s=1.0$, crossover probability: $P_c=0.99$.

Two algorithms all have high efficiency and good results from table 2. However, MTR-GA algorithm takes less time than the algorithm in [8] in achieving the optimal solutions.

Convergences

The Convergence of genetic algorithm usually depends on iteration population created by algorithm converges to a steady state, that is to say, the max fitness value or the average fitness value of algorithm tends to problem's optimization. The trust region method with memory was proved in [7] and the convergence of genetic algorithm was proved in [6].

Conclusions

We find the trust region with memory had higher search ability than trust region comparing results because of high efficiency and well-precision of trust region method with memory. There is some effect to wipe off infeasible solutions with MTR algorithm. The coalescence of two algorithms improves search efficiency in late time and has superiority on solving multi-peak optimal problems.

References

[1] Guoliang Chen, X.F Wang, Xiongfa Wang, Zhenquan Zhuang etc. Genetic Algorithm Theory and Its Application[M]. The people post press, 2006.6

[2] Schippers C A. Multi-parent scanning crossover and genetic drif [M].Theoretical Aspects of Evolutionary Computing, Berlin Springer, 2007: 307-330.

[3] Wang Yan, Lin K J. Rrputation-oriented Trustworthy Computing In E-commerce Environments [J]. IEEE Internet Computing, 2008, 12(4): 55-59.

[4] Hong L, Jiaoyong C, Li Z. etc. A Hybrid and New Genetic Algorithm for Solving Global Optimal Problems[J]. Control Theory and Application, 2007, 24(3): 343-348.

[5] Shounan Z, Fei G, Changming J. A Genetic Trust Region Algorithm[J]. Mathematical Journal, 2001, 21(4): 468-472.

[6] Xiaowei Z, Sanyang L. A genetic algorithm based on trust region [J]. system engineering and electronic technique, 2007, 29(8): 1377-1380.

[7] Zhensheng Y, Xincheng L. The Convergence Analysis of Trust Region with Memory[J]. Operations Research and management, 2004, 13(2): 16-19.

[8] Wenqiang L, Xiaowei Z. A Hybrid Algorithm for Solving Multi-peak Function[J]. Computer Engineering and Application, 2008, 44(33): 43-49.

[9] Xiaojun S.Jianmin J, Junhong H. A genetic accelerated trust region algorithm for solving optimization problems[J]. Anhui university journal (natural and science), 2008, 32(3):23-26.

Table1 Comparing results with two trust region algorithms

Method	Initial Point	The Numbers of Finding Optimization Solutions
Trust Region Method	(0,0)	16
MTR		29
Trust Region Method	(1,1)	14
MTR		27

Table 2 Comparing the results and efficiency of two algorithms

Function	Method	Variable x	Variable y	Global solution	Numbers of achievement	Time (s)
f_1	IN [8]	$4.69474e-03$	$2.97995e-03$	0.99914807	55	0.375
	MTR-GA	$-6.30722e-04$	$-1.18882e-03$	0.99992276	56	0.328125
f_2	IN [8]	-0.0932705	0.708274	-1.03141132	48	0.500
	MTR-GA	0.0874167	-0.713773	-1.03159258	54	0.375
f_3	IN [8]	$-1.04673e-04$	$-2.11421e-04$	-0.01536355	100	0.703125
	MTR-GA	$-2.63890e-11$	$4.71689e-11$	0.00000870	100	0.671875