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# Global asymptotic stability of uncertain stochastic neural networks with mixed time-delays

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**ABSTRACT**

This paper is concerned with the global asymptotic stability analysis problem for a general class of uncertain stochastic neural networks with mixed time-delays. The mixed time-delays under consideration comprise both the discrete time-varying delays and unbounded distributed delays. The main purpose of this paper is by using Lyapunov-Krasovskii functional, the well-known Leibniz-Newton formula and the linear matrix inequality (LMI) approach, and then to establish easy to test sufficient stability conditions under which the addressed neural network is globally, robustly, asymptotically stable in the mean square for all admissible parameter uncertainties. The proposed criteria can be checked effectively by the Matlab LMI toolbox. Finally a simple example is provided to demonstrate the effectiveness of the proposed criteria.



## Introduction

In the past two decades, the dynamics of neural networks have received a great deal of interest due to their fruitful applications in numerous areas such as associative memory, pattern recognition and combinatorial optimization [1-5]. Due to the finite speed of information processing, the existence of time-delays frequently causes oscillation, divergence, or even instability in neural networks, which may be harmful to successful applications of neural networks, however the achieved applications heavily depend on the dynamic behaviors of the equilibrium point of neural networks. That is, the stability analysis problems of delayed neural networks have gained considerable attention, and a large amount of results have appeared in the literature [7-16]. Various types of time-delays have been investigated, including constant or time-varying delays, discrete and distributed delays, and the corresponding stability criteria can be classified as delay-dependent or delay-independent conditions.

Generally speaking, there are two kinds of disturbances to be considered when one model neural networks. They are stochastic perturbations and parameter uncertainties, which are unavoidable in practice. For the stochastic perturbations, there has been a great deal of robust stability criteria proposed by many researchers, for example [8-10]. For the uncertainties, there has also been a great deal of robust stability criteria proposed, for example [11]. [14-16] have considered both stochastic perturbations and parameter uncertainties, however, the discrete delays are not time-varying and the distributed delays are bounded.

Motivated by the aforementioned discussion, this paper focuses on the global asymptotic stability of uncertain stochastic neural networks with discrete time-varying delays and unbounded distributed delays, the parameter uncertainties are norm-bounded, and the neural networks are subjected to stochastic disturbances described in terms of a Brownian motion. By using Lyapunov-Krasovskii functional, the well-known Leibniz-Newton formula and the linear matrix inequality (LMI) approach, and then to establish easy to test sufficient stability conditions under which the addressed neural network is globally, robustly, asymptotically stable in the mean square for all admissible parameter uncertainties. The LMIs can be easily solved by using the Matlab LMI toolbox, and no tuning of parameters is required. Ultimately, a simple example is provided to demonstrate the effectiveness of the proposed criteria.

*Notations.* The notations are quite standard. Throughout this letter, $R^n$ and $R^{n\times n}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n\times m$ real matrices. The superscript "T" denotes matrix transposition and the notation $X \ge Y$ (respectively, $X > Y$) where $X$ and $Y$ are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). $I$ is the identity matrix of appropriate dimension. $|\bullet|$ is the Euclidean norm in $R^n$. If $A$ is a matrix, denote by $\|A\|$ its operator norm, i.e.,

$$\|A\| = \sup\{|Ax| : |x| = 1\} = \sqrt{\lambda_{\max}(A^T A)}$$

where $\lambda_{\max}(\bullet)$ (respectively, $\lambda_{\min}(\bullet)$) means the largest (respectively, smallest) eigenvalue of $A$. $l_2[0,\infty)$ is the space of square integral vector. Moreover, let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\ge 0}, P)$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\ge 0}$ satisfying the usual conditions. Denote by $L^p_{\mathcal{F}_0}([-h,0];R^n)$ the family of all $\mathcal{F}_0$-measurable $C([-h,0];R^n)$-valued random variable $\xi=\{\xi(\theta): -h\le\theta\le 0\}$ such that

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$\sup_{-h\le\theta\le 0} E\left|\xi(\theta)\right|^p < \infty$

where $E\{\bullet\}$ stand for the mathematical expectation operator with respect to the given probability measure $P$. The symmetric terms in a symmetric matrix are denoted by *. Sometimes, the arguments of a function or a matrix will be omitted in the analysis when no confusion can arise.

**Problem formulation**

In this section, we consider the following uncertain stochastic neural networks with time-delays:

$$dx(t) = [-A(t)x(t) + B(t)f_1(x(t)) + C(t)f_2(x(t-\tau(t))) + D(t)\int_{-\infty}^{t} K(t-s)f_3(x(s))ds]dt + \sigma(t, x(t), x(t-\tau(t)))dw(t) \quad (1)$$

Where $x(t) = \left[x_1(t), x_2(t), \ldots, x_n(t)\right]^T \in R^n$ is the neural state vector, the matrix are the time-varying parameters uncertainties.

$$A(t) = A + \Delta A(t), B(t) = B + \Delta B(t), C(t) = C + \Delta C(t), D(t) = D + \Delta D(t),$$
$$\Delta A(t), \Delta B(t), \Delta C(t), \Delta D(t)$$

Where $A = diag\{a_1, a_2, \ldots a_n\}$ is a diagonal matrix, where $a_i > 0, i = 1,2,\ldots, n$, $K = (k_j), j = 1,2,\ldots n$ $B \in R^{n\times n}, C \in R^{n\times n}, D \in R^{n\times n}$ are the connection weight matrix, the discrete time-varying delays connection weight matrix, and the unbounded distributed delays connection weight matrix, respectively.

$f_i(x(t)) = \left[f_{i1}(x_1), f_{i2}(x_2), \ldots f_{in}(x_n)\right]^T, (i = 1,2,3)$ denotes the neuron activation function with $f_i(0) = 0$. $\tau(t)$ represents the discrete time-varying delay with $0 \le \tau(t) \le \tau, \dot{\tau}(t) \le u < 1$ and the delay kernel $k_j$ is a real valued continuous function defined on $[0, +\infty)$ and satisfies $\int_0^{\infty} k_j(s)ds = 1$.

$\omega(t) = \left[\omega_1(t), \omega_2(t), \ldots \omega_m(t)\right]^T \in R^m$ is a m-dimensional Brownian motion defined on a complete probability space $(\Omega, F, P)$. $\sigma(t, x(t), x(t-\tau(t))) : R^+ \times R \times R \to R^{n\times m}$ locally Lipschitz continuous and satisfies the linear growth condition as well.

Remark 1. The motivation for considering the system (1) containing parameter uncertainties and stochastic perturbations, which are unavoidable owing to the complexity of such systems, environmental noises in practice, etc. Indeed, it is more reasonable and practical than the model of the controlled system considered in [21].

Remark 2. It should be pointed out that, if we let $f_1(x(t)) = f_2(x(t)) = f_3(x(t)) = f(x(t))$, the model (1) is the one investigated in [12], however its delays are interval, that is $0 \le h_1 \le \tau(t) \le h_2$.

Remark 3. In system (1), the stochastic disturbance term, $\sigma(t, x(t), x(t-\tau(t)))dw(t)$ can be viewed as stochastic perturbations on the neuron states and delayed neuron states. It has been used in recent papers dealing with stochastic neural networks, see for example [8,12,15].

In order to obtain our main results, the following assumptions are made throughout this paper.

Assumption 1. The activation function $f_i(x), (i = 1,2,3)$ is bounded and satisfies the following Lipschitz condition:

$$\left|f_i(x)\right| \le \left|L_i x\right| \quad (2)$$

where $L_i = diag\{l_{i1}, l_{i2}, \ldots l_{in}\}$ are known constant matrices.

Assumption 2. The admissible parameter uncertainties are assumed to be of the following form:

$$\left[\Delta A(t), \Delta B(t), \Delta C(t), \Delta D(t)\right] = \left[M_1F(t)N_1, M_2F(t)N_2, M_3F(t)N_3, M_4F(t)N_4\right] \quad (3)$$

in which $M_i, N_i (i = 1,2,3,4)$ are known real constant matrices with appropriate dimensions. The uncertain matrix $F(t)$ satisfy:

$$F(t)^T F(t) \le I \quad \text{for } \forall t \in R. \quad (4)$$

Assumption 3. Assume that the noise intensity matrix $\sigma(t, x(t), x(t-\tau(t)))$ could be estimated by $trace\left[\sigma^T(t, x(t), x(t-\tau(t)))(P + ML_1)\sigma(t, x(t), x(t-\tau(t)))\right] \le x^T(t)P_1x(t) + x^T(t-\tau(t))P_2x(t-\tau(t))$

in which $P, P_1, P_2 \ge 0, M = diag\{m_1, m_2 \ldots m_n\}$ and $L_1$ is defined in (2).

Now, we give the following lemmas that are useful in deriving our LMI-based stability criteria.

Lemma 1 ([23]). For given matrices $D, E$ and $F$ with $F^T F \le I$ and scalar $\varepsilon > 0$, the following inequality holds: $DFE + E^T F^T D^T \le \varepsilon DD^T + \varepsilon^{-1} E^T E$.

Lemma 2 ([22]). Assume that $a \in R^{n_a}, b \in R^{n_b}$, and $N \in R^{n_a \times n_b}$ are defined, then for any matrices $X \in R^{n_a\times n_b}, Y \in R^{n_a\times n_b}$ and $Z \in R^{n_a\times n_b}$, the following holds:

$$-2a^T Nb \le \begin{bmatrix} a \\ b \end{bmatrix}^T \begin{bmatrix} X & Y-N \\ * & Z \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}, \qquad \begin{bmatrix} X & Y \\ * & Z \end{bmatrix} \ge 0$$

Lemma 3 ([17] Schur complement). Given constant matrices $\Sigma_1, \Sigma_2, \Sigma_3$ where $\Sigma_1 = \Sigma_1^T$ and $0 < \Sigma_2 = \Sigma_2^T$, then $\Sigma_1 + \Sigma_3^T \Sigma_2^{-1} \Sigma_3 < 0$ if and only if $\begin{bmatrix} \Sigma_1 & \Sigma_3^T \\ * & -\Sigma_2 \end{bmatrix} < 0$, $\begin{bmatrix} -\Sigma_2 & \Sigma_3 \\ * & \Sigma_1 \end{bmatrix} < 0$

Lemma 4 ([12]). For any real matrices $\Omega_1, \Omega_2, \Omega_3$ of appropriate dimensions and a positive scalar $\varepsilon$, such that $0 < \Omega_3 = \Omega_3^T$. Then the following inequality holds

$$2\Omega_1^T \Omega_2 \le \varepsilon \Omega_1^T \Omega_3 \Omega_1 + \varepsilon^{-1} \Omega_2^T \Omega_3^{-1} \Omega_2$$

Lemma 5 ([19]). For any constant matrix $M \in R^{n\times n}, M = M^T > 0$, a scalar $\rho > 0$, vector function $\omega : [0, \rho] \longrightarrow R^n$ such that the integrations are well defined, the following inequality holds:

$$\left[\int_0^{\rho} \omega(s)ds\right]^T M \left[\int_0^{\rho} \omega(s)ds\right] \le \rho \int_0^{\rho} \omega^T(s) M \omega(s) ds$$

**Main results and proofs**

For the sake of presentation simplicity, we denote:

$$\Pi_1 = diag\left\{\varepsilon_1 N_1^T N_1, \varepsilon_2 N_2^T N_2, 0, \varepsilon_3 N_3^T N_3, \varepsilon_4 N_4^T N_4, 0\right\}$$

$$\Pi_2 = diag\left\{\varepsilon_5 N_1^T N_1, \varepsilon_6 N_2^T N_2, 0, \varepsilon_7 N_3^T N_3, \varepsilon_8 N_4^T N_4, 0\right\}$$

$$\Omega_1 = \left[M_1^T P^T, 0,0,0,0,0\right]^T, \Omega_5 = \left[-N_1, 0,0,0,0,0\right]^T, \Omega_9 = \left[0, M_1^T M^T, 0,0,0,0\right]^T$$

$$\Omega_2 = \left[M_2^T P^T, 0,0,0,0,0\right]^T, \Omega_6 = \left[0, N_2, 0,0,0,0\right]^T, \Omega_{10} = \left[0, M_2^T M^T, 0,0,0,0\right]^T$$

$$\Omega_3 = \left[M_3^T P^T, 0,0,0,0,0\right]^T, \Omega_7 = \left[0,0,0,N_3,0,0\right]^T, \Omega_{11} = \left[0, M_3^T M^T, 0,0,0,0\right]^T$$

$$\Omega_4 = \left[M_4^T P^T, 0,0,0,0,0\right]^T, \Omega_8 = \left[0,0,0,0,N_4,0\right]^T, \Omega_{12} = \left[0, M_4^T M^T, 0,0,0,0\right]^T$$

$$\Sigma = \begin{bmatrix} (1,1) & PB & 0 & PC+Y^T & PD & -\tau A^T Z \\ * & 2MB-D_1 & 0 & MC & MD & \tau B^T Z \\ * & * & (u-1)Q+P_2+L_2D_2L_2 & Y^T & 0 & 0 \\ * & * & * & (u-1)R+\tau X-D_2 & 0 & \tau C^T Z \\ * & * & * & * & -E & \tau D^T Z \\ * & * & * & * & * & -\tau Z \end{bmatrix},$$

where

$$(1,1) = -P^T A^T - AP + L_2^T R L_2 + L_3^T E L_3 + P_1 + Q - 2L_1^T MA + L_1 D_1 L_1$$

Now, we are in the position to give the main results, which can be expressed as the feasibility of two linear matrix inequalities.

Theorem 1. For given $L_i = diag\{l_{i1}, l_{i2}, \ldots l_{in}\}, (i=1,2,3)$, the neural network (1) is globally, robustly, asymptotically stable in the mean square if there exist positive scalars $\varepsilon_i > 0 (i=1,\ldots,8)$ and positive definite matrices $P, P_1, P_2, Q, R, X, Y, Z,$ positive diagonal matrices $E, M, D_1, D_2$ such that the following two linear matrix inequalities hold:

$$\Xi_1 = \begin{bmatrix} \Sigma+\Pi_1+\Pi_2 & \Omega_1 & \Omega_2 & \Omega_3 & \Omega_4 & \Omega_9 & \Omega_{10} & \Omega_{11} & \Omega_{12} \\ * & -\varepsilon_1 I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon_2 I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\varepsilon_3 I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_4 I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon_5 I & 0 & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_6 I & 0 & 0 \\ * & * & * & * & * & * & * & -\varepsilon_7 I & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon_8 I \end{bmatrix} < 0 \quad (5)$$

$$\Xi_2 = \begin{bmatrix} X & Y \\ * & Z \end{bmatrix} \geq 0, \quad (6)$$

Proof. At first, we consider the following system

$$dx(t) = [-Ax(t) + Bf_1(x(t)) + Cf_2(x(t-\tau(t))) + D\int_{-\infty}^{t} K(t-s) f_3(x(s))ds]dt + \sigma(t, x(t), x(t-\tau(t)))dw(t) \quad (7)$$

The Lyapunov functional of system (7) is defined by:

$$V(t, x(t)) = V_1 + V_2 + V_3 + V_4 + V_5 + V_6$$

$$V_1 = x^T(t) P x(t) \qquad V_2 = 2\sum_{i=1}^{n} m_i \int_0^{x_i(t)} f_{1i}(s) ds$$

$$V_3 = \int_{t-\tau(t)}^{t} x^T(s) Q x(s) ds \qquad V_4 = \int_{t-\tau(t)}^{t} f_2^T(x(s)) R f_2(x(s)) ds$$

$$V_5 = \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{x}^T(s) Z \dot{x}(s) ds d\theta \qquad V_6 = \sum_{j=1}^{n} e_j \int_0^{\infty} k_j(\theta) \int_{t-\theta}^{t} f_{3j}^2(x(s)) ds d\theta$$

By $It\hat{o}'s$ differential formula, the stochastic derivative of $V(t, x(t))$ along (7) can be obtained as follows:

$$dV_1 = 2x^T(t) P\left(-Ax(t) + Bf_1(x(t)) + Cf_2(x(t-\tau(t))) + D\int_{-\infty}^{t} K(t-s) f_3(x(s))ds\right) + trace(\sigma^T P \sigma)$$

$$dV_2 = 2 f_1^T(x(t)) M\left(-Ax(t) + Bf_1(x(t)) + Cf_2(x(t-\tau(t))) + D\int_{-\infty}^{t} K(t-s) f_3(x(s))ds\right) + trace(\sigma^T M L_1 \sigma)$$

$$dV_3 \leq x^T(t) Q x(t) - (1-u) x^T(t-\tau(t)) Q x(t-\tau(t)) \quad (8)$$

$$dV_4 \leq f_2^T(x(t)) R f_2(x(t)) - (1-u) f_2^T(x(t-\tau(t))) R f_2(x(t-\tau(t)))$$

$$dV_5 \leq \tau \dot{x}^T(t) Z \dot{x}(t) - \int_{t-\tau(t)}^{t} \dot{x}^T(t) Z \dot{x}(t) ds$$

$$dV_6 \leq f_3^T(x(t)) E f_3(x(t)) - \left(\int_{-\infty}^{t} K(t-s) f_3(x(s))ds\right)^T E\left(\int_{-\infty}^{t} K(t-s) f_3(x(s))ds\right)$$

By well-known Leibniz-Newton formula, the following equation satisfies

$$2 f_2(x(t-\tau(t)))\left(x(t) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t} \dot{x}(s) ds\right) = 0 \quad (9)$$

and because

$$x^T(t) L_1 D_1 L_1 x(t) - f_1^T(x(t)) D_1 f_1(x(t)) + x^T(t-\tau(t)) L_2 D_2 L_2 x(t-\tau(t)) - f_2^T(x(t-\tau(t))) D_2 f_2(x(t-\tau(t))) > 0 \quad (10)$$

Using (2), (8),(9),(10) we have

$$dV(t, x(t)) \leq \zeta^T \begin{bmatrix} (1,1) & PB & 0 & PC & PD \\ * & 2MB-D_1 & 0 & MC & MD \\ * & * & (u-1)Q+P_2+L_2D_2L_2 & 0 & 0 \\ * & * & * & (u-1)R-D_2 & 0 \\ * & * & * & * & -E \end{bmatrix} \zeta + \zeta^T \Gamma^T (\tau Z)^{-1} \Gamma \zeta \quad (11)$$

$$-\int_{t-\tau(t)}^{t} \dot{x}^T(t) Z \dot{x}(t) ds + 2 f_2(x(t-\tau(t)))\left(x(t) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t} \dot{x}(s) ds\right)$$

where

$$\zeta(t) = \left[x^T(t), f_1^T(t), x^T(t-\tau(t)), f_2^T(t-\tau(t)), \left(\int_{-\infty}^{t} K(t-s) f_3(x(s))ds\right)^T\right]^T$$

$$\Gamma = \left[-\tau ZA, \tau ZB, 0, \tau ZC, \tau ZD\right]^T$$

By applying Lemma (2) to a term in (11), we have the following relationship:

$$-2\int_{t-\tau(t)}^{t} f_2(x(t-\tau(t))) \dot{x}(s) ds \leq \tau f_2^T(x(t-\tau(t))) X f_2(x(t-\tau(t))) + 2 f_2^T(x(t-\tau(t)))(Y-I)(x(t) - x(t-\tau(t))) + \int_{t-\tau(t)}^{t} \dot{x}^T(t) Z \dot{x}(t) ds \quad (12)$$

where $X, Y, Z$ satisfy the LMI condition (6).

substituting (12) to (11) and using Lemma (3), we know that

$$dV(t, x(t)) \leq \zeta^T \Sigma \zeta \quad (13)$$

If the matrix $\Sigma < 0$, then there exist a positive scalar $\gamma > 0$ such that

$$\Sigma + \begin{bmatrix} \gamma I & 0 & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & 0 & 0 \end{bmatrix} < 0$$

Taking the mathematical expectation of both sides of (13), we have

$$\frac{dEV(t, x(t))}{dt} \leq E\left(\zeta^T \Sigma \zeta\right) \leq -\gamma E\left|x(t)\right|^2.$$

Which indicates from the Lyapunov stability theory that the neural network (7) is globally, robustly, asymptotically stable in the mean square.

Then using Lemma (3) again, the system (1) is globally, robustly, asymptotically stable in the mean square if the following inequality holds:

$$\Sigma+2\Omega_1F(t)\Omega_5^T+2\Omega_2F(t)\Omega_6^T+2\Omega_3F(t)\Omega_7^T+2\Omega_4F(t)\Omega_8^T +2\Omega_9F(t)\Omega_5^T+2\Omega_{10}F(t)\Omega_6^T+2\Omega_{11}F(t)\Omega_7^T+2\Omega_{12}F(t)\Omega_8^T<0 \quad (14)$$

By inequality (4) and lemma 4, inequality (14) holds if the following inequality satisfies:

$$\begin{aligned}&\Sigma+\varepsilon_1^{-1}\Omega_1\Omega_1^T+\varepsilon_1\Omega_5\Omega_5^T+\varepsilon_2^{-1}\Omega_2\Omega_2^T+\varepsilon_2\Omega_6\Omega_6^T+\varepsilon_3^{-1}\Omega_3\Omega_3^T+\varepsilon_3\Omega_7\Omega_7^T+\varepsilon_4^{-1}\Omega_4\Omega_4^T+\varepsilon_4\Omega_8\Omega_8^T\\&+\varepsilon_5^{-1}\Omega_9\Omega_9^T+\varepsilon_5\Omega_5\Omega_5^T+\varepsilon_6^{-1}\Omega_{10}\Omega_{10}^T+\varepsilon_6\Omega_6\Omega_6^T+\varepsilon_7^{-1}\Omega_{11}\Omega_{11}^T+\varepsilon_7\Omega_7\Omega_7^T+\varepsilon_8^{-1}\Omega_{12}\Omega_{12}^T+\varepsilon_8\Omega_8\Omega_8^T\\&=\Sigma+\Pi_1+\Pi_2+\varepsilon_1^{-1}\Omega_1\Omega_1^T+\varepsilon_2^{-1}\Omega_2\Omega_2^T+\varepsilon_3^{-1}\Omega_3\Omega_3^T+\varepsilon_4^{-1}\Omega_4\Omega_4^T\\&\quad+\varepsilon_5^{-1}\Omega_9\Omega_9^T+\varepsilon_6^{-1}\Omega_{10}\Omega_{10}^T+\varepsilon_7^{-1}\Omega_{11}\Omega_{11}^T+\varepsilon_8^{-1}\Omega_{12}\Omega_{12}^T<0\end{aligned} \quad (15)$$

Then using Lemma (3) again, the inequality (14) is equivalent to the LMI (5). Thus it can now be concluded that if the LMIs given (5) and (6) hold, the neural network (1) is globally, robustly, asymptotically stable in the mean square. This completes the proof of Theorem 1.

Remark 4. In [15, 16], the authors studied the global stability results for neural networks with discrete delays and distributed delays, however, the discrete delays are constant and the distributed delays is bounded. In [10], exponential stability a criteria of neural networks with continuously distributed delays was derived, however, the stochastic term and parameter uncertainties were not taken into account in the models. Therefore, our model considered in this paper is more general than those reported in [10, 15, and 16].

## Numerical examples

Example. Let us consider a third-order delayed neural network (1). The network date are given as follows:

$$A=\begin{bmatrix}2.5&0&0\\0&3.9&0\\0&0&3.7\end{bmatrix} B=\begin{bmatrix}0.3&-1.7&0.6\\-1.0&1.6&1.1\\0.6&0.4&-0.2\end{bmatrix} C=\begin{bmatrix}-0.7&0.2&0.1\\-0.2&-0.5&0.5\\0.9&1.1&-1.3\end{bmatrix} D=\begin{bmatrix}0.4&-0.2&0.1\\0.3&0.7&-0.2\\1.2&1.1&-0.5\end{bmatrix}$$

$$P_1=P_2=diag\{0.1,0,3,0,.1\},\ \mathrm{L}_1=0.1I,\ \mathrm{L}_2=0.2I,\ \mathrm{L}_3=0.3I,\ \text{u=0.6}$$

$$M_1=M_2=M_3=M_4=0.2I,\ N_1=N_2=N_3=N_4=0.3I,\quad \tau=1.2$$

Then, solving the LMIs (5) and (6) by the Matlab's LMI Toolbox, then we found that the LMIs given in the Theorem 1 are feasible and we obtained

$$\varepsilon_1=925.7140,\quad \varepsilon_2=947.3097,\quad \varepsilon_3=926.7531,\quad \varepsilon_4=959.5230,$$

$$\varepsilon_5=901.4975,\quad \varepsilon_6=923.0604,\quad \varepsilon_7=902.5132,\quad \varepsilon_8=935.2631,$$

$$P=\begin{bmatrix}810.0292&112.3832&-0.3050\\112.3832&641.0574&-16.3042\\-0.3050&-16.3042&539.4389\end{bmatrix},Q=\begin{bmatrix}1.3395&0.2447&-0.0263\\0.2447&1.5294&-0.0771\\-0.0263&-0.0771&1.3633\end{bmatrix}\times10^3,R=\begin{bmatrix}1.5322&0.1219&-0.1925\\0.1219&1.4808&-0.3130\\-0.1925&-0.3130&1.5022\end{bmatrix}\times10^3,$$

$$X=\begin{bmatrix}671.0250&-9.8821&41.0062\\-9.8821&735.0114&76.6749\\41.0062&76.6749&746.2823\end{bmatrix},Y=\begin{bmatrix}164.0198&5.7990&0.6568\\5.7990&137.3637&-1.6511\\0.6568&-1.6511&135.8104\end{bmatrix},Z=\begin{bmatrix}195.9817&25.8394&1.9818\\25.8394&111.7199&-1.2315\\1.9818&-1.2315&96.0086\end{bmatrix},$$

$$E=\begin{bmatrix}1.4739&0&0\\0&1.5410&0\\0&0&1.1395\end{bmatrix}\times10^3,M=\begin{bmatrix}159.6858&0&0\\0&135.7029&0\\0&0&151.3877\end{bmatrix},$$

$$D_1=\begin{bmatrix}1.7370&0&0\\0&3.0396&0\\0&0&1.4173\end{bmatrix}\times10^3,D_2=\begin{bmatrix}1.3314&0&0\\0&1.4340&0\\0&0&1.3679\end{bmatrix}\times10^3$$

which indicates that the delayed stochastic neural network (1) is globally, robustly, asymptotically stable in the mean square.

## Conclusions

In this paper, we have performed a global asymptotic stability analysis problem for a general class of uncertain stochastic neural networks with mixed time-delays. We have removed the traditional monotonic and smoothness assumptions on the activation function. The stability criteria have been given in terms of liner matrix inequality (LMI), which can be checked easily by using the Matlab LMI toolbox. A simple example has been used to demonstrate the usefulness of the main results. In the future research, we can extend the present results to more general cases, for example, the case that the delays are multiple and the case that exponential stability is investigated.

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