



Comparison of bootstrap survival function with the Kaplan-Meier survival function under the influence of different percentage of censoring

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ABSTRACT

If survival data follows a specific survival distribution (weibull, exponential), survival probabilities can be estimated by using the specific survival function. Alternate easy and assumptions free methods for computing the survival probabilities are Kaplan-Meier and Bootstrap survival functions. The aim of present study is to compare the two non-parametric methods under the influence of different percentage of censoring presented in four different data sets "Leukaemia data set, Stanford Heart Transplant data set, Thalassaemia data set and Lung cancer data set". Results of analysis show that the performance of Kaplan-Meier survival function is better than the Bootstrap survival function. This is due to the easy concept, less time, less time, easily availability as well as in terms of survival probabilities.

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Introduction

Survival analysis is considered as the backbone of medical research and is based on the observed time of event.

The event of interest may for instance be death or recovery from treatment. Survival analysis is based on three techniques i) parametric ii) Semi-parametric and iii) Non-parametric.

Non-parametric technique is more commonly used technique (Fleming & Harrington, 1984), due to relax conditions about the assumptions of distributions.

Kaplan-Meier survival function (1958) is the most famous and commonly used non-parametric technique of survival analysis. To define the function we proceed as:

Let the survival times $X_1, X_2, \dots, X_n$ be independently identically distributed according to the distribution function $F(x)$.

Similarly, let $Y_1, Y_2, \dots, Y_n$ be independently identically distributed censoring times according to $G(y)$. In addition, the survival times X_i and censoring times Y_i are assumed to be independent (Miller and Rupert, 1983).

The observable random variables are $T_i = \min \{X_i, Y_i\}$ and $\delta_i = I(X_i \leq Y_i)$ indicates whether the survival time is uncensored or censored. Let $T_1 < \dots < T_n$ denote the ordered observed survival times, and let $\delta_1, \dots, \delta_n$ be their corresponding (unordered) indicator values.

Let the number of individuals who are alive just before time t_i , including those who are about to die at this time, be denoted by r_i and e_i denotes the number who die at this time.

Using these notations, the Kaplan-Meier estimator is defined as

$$\hat{S}_{KM}(t) = \prod_{i=1}^n \left(\frac{r_i - e_i}{r_i} \right)$$

Another non-parametric similar approach for calculating the survival function is the Bootstrap survival function.

A brief introduction of the function is defined below:

Bootstrap Method

The Bootstrap (Efron, 1979) is a method for calculating the approximated biases, standard deviations, confidence intervals etc. Except these applications, Bootstrap also doing a reasonable job under a variety of situations. e.g. in survival analysis, one can use the Bootstrap to estimate the survival probabilities (as by Kaplan-Meier, 1958).

Sampling scheme for Bootstrapping censored data was introduced by Efron for 'Bootstrapping individual' k , sample a survival time and censoring indicator pair (t_i, δ_i) with replacement from the data set $\{(t_i, \delta_i), i=1, \dots, n\}$. If n Bootstrap individuals are generated, a Bootstrap replicate can be found by applying the Kaplan-Meier estimator to the Bootstrap data set. We take M of these replicates and order them as

$$\hat{S}_{[1]}^B(t^*) \leq \dots \leq \hat{S}_{[M]}^B(t^*)$$

The procedure is describe as

Draw Bootstrap sample of the pair (t_i, δ_i) , and note $m_i = \#$ times (t_i, δ_i) appears in the Bootstrap sample, so $m = (m_1, m_2, \dots, m_n)$ is an n -category multinomial, n draws, probability $1/n$ for each category: $m \sim \text{mult}(n, 1/n)$

Define

$$M_j = \sum_{i=j}^n m_i, \quad j = 1, 2, \dots, n$$

Where

$M_1 = n, M_2 = n - m_1$ and so forth.

The survival function based on Bootstrap data is

$$\hat{S}_m(t) = \prod_{j=1}^{\delta_i} \left(1 - \frac{m_j}{M_j} \right)$$

Aim

“Bootstrap is a way to pull oneself up (from an unfavourable situation) by one’s Bootstrap, to provide trustworthy answers despite of unfavourable Circumstances” (Efron, 1979).

The aim of the study is to judge the performance (in terms of bias) of the quote by comparing the curves as well as the survival probabilities of two nonparametric survival functions for four data sets. For small sample Kaplan-Meier gives downward bias results (Whittemore & Keller, 1986). The performance of the two methods will be evaluated through the probabilities and also through graphical presentation on four different data sets containing different percentage of censoring. For the analysis R-package (2004) is used.

Brief Introduction of Data Sets

Leukaemia data set:

The famous leukaemia data set conducted by Freireich (Freireich et. al. 1963), which was reviewed by Gehan (1965) and also used by Borkowf (2005). The data consisted of 21 patients, including 9 events and 12(57%) censored observations. The data set consists of weeks in maintenance of remission. We ignore the placebo controls (containing no censored observation) here.

The weeks in remission are: 6, 6, 6, 6+, 7, 9+, 10, 10+, 11+, 13, 16, 17+, 19+, 20+, 22, 23, 25+, 32+, 32+, 34+ and 35+.

Where + denotes a censored observation.

Stanford Heart Transplant Data Analysis

The second data set is the Stanford heart transplant data (Kalbflesch and Printice, 1980) .The data set contains 103 patients 75of were events and 28(27.2%) censored. The data set is given below

1, 2, 2, 2, 3, 3, 3, 5, 5, 6, 6,
8, 9, 11+, 12, 16, 16, 16, 17, 18, 21,
21, 28, 30, 31+, 32, 35, 36, 37, 39,
39+, 40, 40, 43, 45, 50, 51, 53, 58,
61, 66, 68, 68, 69, 72, 72, 77, 78, 80,
81, 85, 90, 96, 100, 102, 109+, 110, 131+,
149, 153, 165, 180+, 186, 188, 207, 219,
263, 265+, 285, 285, 308, 334, 340, 340+,
342, 370+, 397+, 427+, 445+, 482+, 515+,
545+, 583, 596+, 630+, 670+, 675, 733, 841+,
852, 915+, 941+, 979, 995, 1032, 1141+, 1321+,
1386, 1400+, 1407+, 1571+, 1586+, 1799+

Where + denotes a censored observation.

An Application to a Lung Cancer Data

A data set from the Veterans Administration lung cancer trial in which chemotherapy was given to males with advanced inoperable lung cancer (presented by Prentice (1973)). This data set was also used by Gupta (1999). The data set consists of 97 patients out of which 91 were events and 6(6.2%) were censored. The survival times are given in days:

72	228	10	110	314	100*	42
	144	30	384	4	13	
123*	97*	59	117	151	22	18
	139	20	31	52	18	51
	122	27	54	7	63	392
92	35	117	132	162	3	95
	162	216	553	278	260	156
	182*	143	105	103		
112	87*	242	111	587	389	33
25	357	467	1	30		

283	25	21	13	87	7	24
99	8	99	61	25	95	80
29	24	83*	31	51	52	73
8	36	48	7	140	186	19
	45	80	52	53	15	133
	111	378	49			

Where + denotes a censored observation.

An Application to a Breast Cancer Data

A large data set of 1207 breast cancer patient is obtained from SPSS, version 11.5 (SPSS, 2004) containing 1135 censored (94.3 %) and 72 events only.

Method of Bootstrap Sample

Software for the program is prepared in R-package. A Bootstrap of sample “10* size of data set” is drawn with replacement. Sample is selected with the point to give each time enough chances to be included in the sample.

Results

The results of leukaemia data set are shown in table 1 and in Figure 1. For heart transplant data, we prepared table 2 and Figure 2. Table 3 and Figure 3 represent the facts of lung cancer data set. For a very large breast cancer data set, we show the results through Figure 4 and not prepared table of probabilities, due to the same conclusion obtained from the previous data sets.

Figure 1: Survival curves of Kaplan-Meier and Modified Kaplan-Meier for leukaemia data set

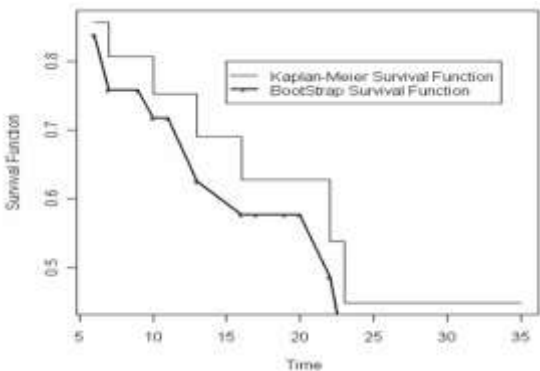
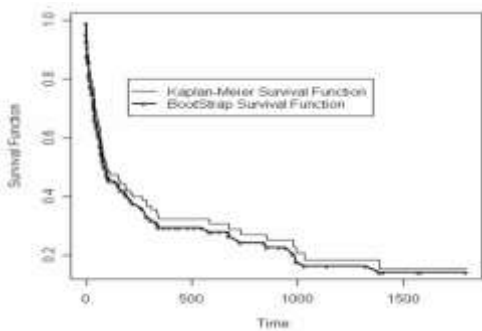


Figure 2: Survival curves of Kaplan-Meier and Modified Kaplan-Meier for heart transplant data Set.



Leukaemia data set containing 21 patients having 57% censored data. Stanford heart transplant data containing 27.2% censored data. Lung cancer data consists of 6.2% censored cases, while the breast cancer data 1207 patients having very high censoring (94.3%). The range of data sets is 21 to 1207 patients and the range of percentage censoring is 6.2 to 94.3. On the basis of these facts and also on the basis of 4 Figures and 3 Tables, we reach to the followings.

- Bootstrap function is a form of Kaplan-Meier survival function.

- Kaplan-Meier gives the underestimate (bias) results for small sample. By comparing the survival probabilities of given data sets and curves, we can say that Bootstrap function gives more bias results than Kaplan-Meier survival function.
- For large data set, Kaplan-Meier gives unbiased results (Maller and Zhou, 1996). If we compare the curves of breast cancer data, the Bootstrap still gives some bias.
- If the last observation is censored, one can not obtain the mean of survival function and the same result we obtained from the Bootstrap survival function.
- Both the survival functions have the same range i.e. 0 to 1.

Figure 3: Survival curves of Kaplan-Meier and Modified Kaplan-Meier for lung cancer data set.

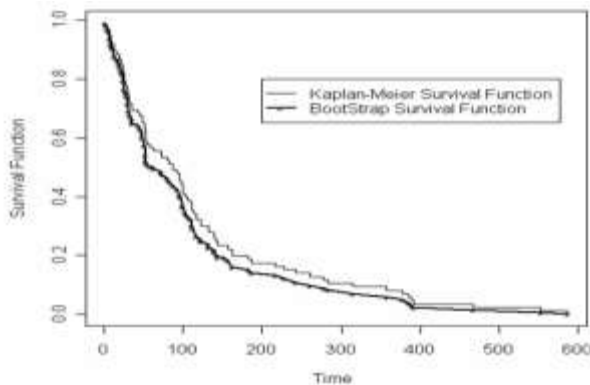
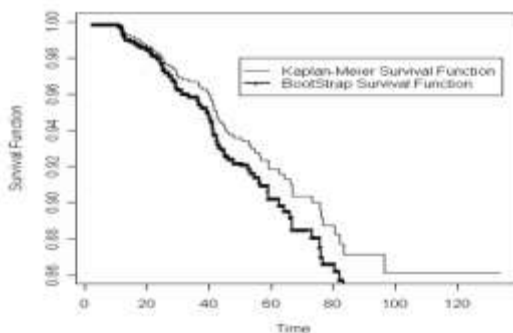


Figure 4: Survival curves of Kaplan-Meier and Modified Kaplan-Meier for breast cancer data set (SPSS data directory).



Discussion and Conclusion

Parametric approach produces better results, if we are able to find a specific parametric survival distribution (Exponential, Weibull etc.) that fits the data. As most of the survival data are skewed and some times, it is very difficult to find the appropriate distribution. The easiest way to avoid the possible error (which may occur due to the application of in appropriate distribution) is the non-parametric approach. The two non-parametric approaches for estimating the survival function are the Kaplan-Meier and Bootstrap approach.

In this article we compared the two approaches by applying them on different data sets and on the basis of analysis we reached to the conclusion that, Kaplan-Meier approach as

compared to the Bootstrap approach is easy to understand and to apply. The Kaplan-Meier approach can be easily apply to different data sets by the help of packages e.g. SPSS, R, S, SPLUS, while for applying the Bootstrap technique, there is no option available in commonly used packages. In some situations Kaplan-Meier gives bias results, which are smaller than the results obtained by applying the Bootstrap survival function.

Bootstrap survival function in all the four data sets gives the smaller probabilities of survival (for every censoring percentage).

If comparatively large data set, having moderate censoring is available, then for estimating the probabilities and for drawing the survival curve, Kaplan-Meier is considered as the conventional, easy and less time consuming method.

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Table 1. Estimated survival functions of Kaplan-Meier and Bootstrap for leukaemia data set

Time	Event	No. at risk	Kaplan-Meier Survival Function	Bootstrap Survival Function
6	3	21	0.8571	0.8382
7	1	17	0.8067	0.7578
9	0	16	0.8067	0.7578
10	1	15	0.7529	0.7179
11	0	13	0.7529	0.7179
13	1	12	0.6902	0.6257
16	1	11	0.6275	0.5772
17	0	10	0.6275	0.5772
19	0	9	0.6275	0.5772
20	0	8	0.6275	0.5772
22	1	7	0.5378	0.4861
23	1	6	0.4482	0.3797
25	0	5	0.4482	0.3797
32	0	4	0.4482	0.3797
34	0	2	0.4482	0.3797
35	0	1	0.4482	0.3797

Table 2. Estimated survival functions of Kaplan-Meier and Bootstrap for heart transplant data set

Time	Event	No. at risk	Kaplan-Meier Survival Function	Bootstrap Survival Function
1	1	103	0.9903	0.9874
2	3	102	0.9612	0.9500
3	3	99	0.9320	0.9247
5	2	96	0.9126	0.9062
6	2	94	0.8932	0.8788
8	1	92	0.8835	0.8697
9	1	91	0.8738	0.8616
11	0	90	0.8738	0.8616
12	1	89	0.8640	0.8515
16	3	88	0.8345	0.8164
17	1	85	0.8247	0.8066
18	1	84	0.8149	0.7968
21	2	83	0.7952	0.7738
28	1	81	0.7854	0.7633
30	1	80	0.7756	0.7510
31	0	79	0.7756	0.7510
32	1	78	0.7657	0.7448
35	1	77	0.7557	0.7297
36	1	76	0.7458	0.7155
37	1	75	0.7358	0.7084
39	1	74	0.7259	0.6933
40	2	72	0.7057	0.6756
43	1	70	0.6956	0.6616
45	1	69	0.6856	0.6519
50	1	68	0.6755	0.6449
51	1	67	0.6654	0.6361
53	1	66	0.6553	0.6256
58	1	65	0.6452	0.6159
61	1	64	0.6352	0.6045
66	1	63	0.6251	0.5940
68	2	62	0.6049	0.5766
69	1	60	0.5948	0.5662
72	2	59	0.5747	0.5474
77	1	57	0.5646	0.5423
78	1	56	0.5545	0.5346
80	1	55	0.5444	0.5219
81	1	54	0.5343	0.5159
85	1	53	0.5243	0.5040
90	1	52	0.5142	0.4955
96	1	51	0.5041	0.4819
100	1	50	0.4940	0.4700
102	1	49	0.4839	0.4615
109	0	48	0.4839	0.4615
110	1	47	0.4736	0.4513
131	0	46	0.4736	0.4513
149	1	45	0.4631	0.4417
153	1	44	0.4526	0.4312
165	1	43	0.4426	0.4181
180	0	42	0.4421	0.4181
186	1	41	0.4313	0.4073
188	1	40	0.4205	0.4010
207	1	39	0.4097	0.3866
219	1	38	0.3989	0.3748
263	1	37	0.3882	0.3577
265	0	36	0.3882	0.3577
285	2	35	0.3660	0.3303
308	1	33	0.3549	0.3187
334	1	32	0.3438	0.3088
340	1	31	0.3327	0.3008
342	1	29	0.3212	0.2909
370	0	28	0.3212	0.2909
397	0	27	0.3212	0.2909
427	0	26	0.3212	0.2909
445	0	25	0.3212	0.2909
482	0	24	0.3212	0.2909
515	0	23	0.3212	0.2909
545	0	22	0.3212	0.2909
583	1	21	0.3059	0.2780
596	0	20	0.3059	0.2780
630	0	19	0.3059	0.2780
670	0	18	0.3059	0.2780
675	1	17	0.2879	0.2652
733	1	16	0.2699	0.2410
841	0	15	0.2699	0.2410
852	1	14	0.2507	0.2247
915	0	13	0.2507	0.2247
941	0	12	0.2507	0.2247
979	1	11	0.2279	0.2010
995	1	10	0.2051	0.1757
1032	1	9	0.1823	0.1622
1141	0	8	0.1823	0.1622
1321	0	7	0.1823	0.1622
1386	1	6	0.1519	0.1403
1400	0	5	0.1519	0.1403
1407	0	4	0.1519	0.1403
1571	0	3	0.1519	0.1403
1586	0	2	0.1519	0.1403
1799	0	1	0.1519	0.1403

Table 2. Estimated survival functions of Kaplan-Meier and Bootstrap for lung cancer data set

Time	Event	No. at risk	Kaplan-Meier Survival Function	Bootstrap Survival Function
1	1	97	0.9897	0.9897
3	1	96	0.9794	0.9784
4	1	95	0.9691	0.9650
7	2	94	0.9485	0.9343
8	2	91	0.9276	0.9161
10	1	89	0.9172	0.9030
13	2	88	0.8963	0.8771
15	1	86	0.8859	0.8652
18	0	85	0.8859	0.8652
19	1	83	0.8753	0.8512
20	1	82	0.8646	0.8402
21	1	81	0.8539	0.8272
22	1	80	0.8432	0.8162
24	2	79	0.8219	0.7923
25	3	77	0.7899	0.7602
27	1	74	0.7792	0.7515
29	1	73	0.7685	0.7391
30	2	72	0.7472	0.7124
31	2	70	0.7258	0.6863
33	1	68	0.7151	0.6734
35	1	67	0.7045	0.6595
36	1	66	0.6938	0.6484
42	1	65	0.6831	0.6419
45	1	64	0.6724	0.6280
48	1	63	0.6618	0.6169
49	1	62	0.6511	0.5984
51	2	61	0.6298	0.5710
52	3	59	0.5977	0.5391
53	1	56	0.5871	0.5200
54	1	55	0.5764	0.5095
59	1	54	0.5657	0.5008
61	0	53	0.5657	0.5008
63	1	52	0.5548	0.4946
72	1	51	0.5440	0.4847
73	1	50	0.5331	0.4748
80	1	49	0.5222	0.4613
83	1	47	0.5111	0.4506
87	2	46	0.4889	0.4363
92	1	44	0.4778	0.4240
95	2	43	0.4555	0.4048
97	1	41	0.4444	0.3953
99	2	40	0.4222	0.3681
100	1	38	0.4111	0.3623
103	1	37	0.4000	0.3448
105	1	36	0.3889	0.3331
110	1	35	0.3778	0.3232
111	2	34	0.3555	0.2987
112	1	32	0.3444	0.2890
117	2	31	0.3222	0.2640
122	1	29	0.3111	0.2540
123	1	28	0.3000	0.2487
132	1	27	0.2889	0.2372
133	1	26	0.2778	0.2265
139	1	25	0.2667	0.2181
140	1	24	0.2555	0.2120
143	1	23	0.2444	0.2012
144	1	22	0.2333	0.1921
151	0	21	0.2333	0.1921
156	1	20	0.2217	0.1807
162	2	19	0.1983	0.1601
182	1	17	0.1867	0.1525
186	1	16	0.1750	0.1394
216	1	15	0.1633	0.1325
228	1	14	0.1517	0.1202
242	1	13	0.1400	0.1088
260	1	12	0.1283	0.0965
278	1	11	0.1167	0.0896
283	1	10	0.1050	0.0820
314	1	9	0.0933	0.0705
357	1	8	0.0817	0.0598
378	1	7	0.0700	0.0513
384	1	6	0.0583	0.0398
389	1	5	0.0467	0.0307
392	1	4	0.0350	0.0238
467	1	3	0.0233	0.0138
553	1	2	0.0117	0.0061
587	1	1	0.0000	0.0000