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E_k - cordial labeling of graphs

N. Sridharan[1] and R. Umarani[2]

[1]Department of Mathematics, Alagappa University, Karaikudi

[2]Department of Mathematics, Govt. Arts College for Women, Pudukkottai-1.

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ABSTRACT

Yilmag and Cahit defined E_k – cordial labelling of graphs by combining k-equitable labelling and edge-graceful labelling of graphs. In this paper, we investigate the E_k - Cordial labeling of the graphs P_n^+, F_n and C_n^+.

Introduction

Cordial graphs were first introduced by I.Cahit in 1987 as a weaker version of graceful and harmonious graphs and was based on {0, 1}- binary labeling of vertices [1 - 3]. On the other hand edge-graceful labeling of graphs was introduced by Lo in 1985. Edge – Cordial (E-cordial) graphs was introduced by Ng and Lee for graphs on 4n, 4n+1 and 4n+3 vertices in 1988. Combining k-equitable labeling and edge – graceful labeling of graphs, in[5] Yilmag and Cahit have defined a new graph labeling technique, called E_k – cordial labeling in 1997[89]. Let f be an edge labeling of a graph G = (V,E) such that f: $E(G) \to \{0,1,2,\ldots,k-1\}$ and the induced vertex labeling be given as $f(v) = \sum_u f(uv) \pmod k$, where u, v ∈ V and uv ∈E. The map f is called an E_k – cordial labeling of G, if the following conditions are satisfied for all i, j ∈ {0,1,….k-1}:

(1) $|e_f(i) - e_f(j)| \le 1$ and

(2) $|v_f(i) - v_f(j)| \le 1$

where $e_f(i)$, $e_f(j)$ denote the number of the edges labeled with i and j respectively and $v_f(i)$, $v_f(j)$ denote the number of vertices labeled with i and j respectively. The graph G is called E_k-cordial if it admits an E_k – cordial labeling. A graph is E – cordial if it is E_2 – cordial. For an extensive survey on graph labeling we refer to Gallian[4].

In this paper, we investigate the E_k - Cordial labeling of the graphs P_n^+, F_n and C_n^+.

Main Results

If G is a graph, then G^+ is the graph obtained from G by adding a (new) pendant vertex to each vertex of G. The paths, cycles on n vertices are denoted by P_n and C_n respectively.

Theorem 2.1. If n >1 is odd, then P_n^+ is E_k –cordial for all k ≥ 2.

Proof: Let n = mk + t, where 0 ≤ t ≤ k. Let

(i) $P_n = u_1 u_2 \ldots u_n$ be the path in P = n with $\deg(u_i) \neq 1$ for all i =1,2,…,n in P_n^+.

(ii) $v_1,\ldots,v_n$ be the pendant vertices of P_n^+ and $u_i v_i \in E(P_n^+)$.

(iii) $e_i = u_i u_{i+1}$ for all i = 1,2,…,n-1 and

(iv) e'_i === $= u_i v_i$ for all i =1,2,…,n

Case(1): If t = 0 or $t > \frac{k}{2}$, define f: $E(P_n^+) \to \{0,1,\ldots,k-1\}$as follows:

$f(e'_i)$ = □ and $f(e_i)$ = k-1- □

Where i-1 =□□ (mod k), 0 ≤ 1 ≤k. We claim that f induces an E_k – cordial labeling on P_n^+. Clearly the sequence {f(e_i)} is k-1, k-2,…,2, 1, 0, k-1, k-2,…,2,1, 0,… k-1,…1,0, k-1, k-2,…, k-t+1 and the sequence {$f(e'_i)$ } is 0,1,2,…,k-1,…0,1,…, k-1,0,1,2,…, t-1.

We note that

(1) If t = 0,

$$e_f(i) = \begin{cases} 2m & \text{if } i \neq 0 \\ 2m-1 & \text{if } i = 0. \end{cases}$$

(2)If t ≠ 0,

$$e_f(i) = \begin{cases} 2m+2 & \text{for } k-t < i < t \\ 2m+1 & \text{for } i \le k-t \text{ or } i \ge t \end{cases}$$

(3) $|e_f(i)-e_f(j)| \le 1$ for all i,j ∈ {0,1,..,k-1} .

(4) $f(u_i)+f(v_i)$ = k-1 for all i ≠ n $f(u_n) = 0$

(5) If t = 0, $v_k(i)$ = 2m for all i

(6) If t ≠ 0,

$$v_f(i) = \begin{cases} 2m+1 & \text{if } 1 \le i \le k-t \text{ or } t \le i \\ 2m+2 & \text{if } t = 0 \text{ or } k-t < i < t \end{cases}$$

(7) $|v_f(i)-v_f(j)| \le 1$, for all i, j ∈ {0,1….,k-1}

Thus in this case the map f induces an E_k – cordial labeling for P_n^+

Tele:

E-mail addresses: mrgopi1985@gmail.com

Case (2): If $2 \le t < \frac{k}{2}$, define f: $E(P_n^+) \to \{0,1,\ldots,k-1\}$ as

$f(e'_i)$=□□ and $f(e_i)$ = k-1-□. Where i ≤ mk and i = □(mod k)

$f(e'_{mk+i})$ = i and $f(e_{mk+i})$ = k-i-1, for 1 ≤ i ≤ t.

Clearly,

$v_f(i)$ = { 2m+1 if i = k-1,0 ≤ i ≤ t, k-t ≤ i ≤ k-3; 2m if i = k-2 and t < i < k-t.

$e_f(i)$ = { 2m+1 if i =1,2,…t, k-t, k-t+1,…,k-2; 2m if i = 0 or t < i <k-t, i ≠ k-1.

In this case f induces an E_k – cordial labeling.

Case (3): If t =1, define f: E (P_n^+).→{0,1,…,k-1}as

$f(e'_n)$ = k-1; $f(e'_{n-1})$ = 0

$f(e'_i)$=□□ for i ≤ n-2;

$f(e_i)$= k-□□ -1 for all i =1,2,…,n

Where i-1 =□□ (mod k), 0 ≤□□ ≤ k .

Clearly,

$e_f(i)$ = { 2m for all i ≠ 0; 2m+1 for i = 0

$v_f(i)$ = { 2m for all i ≠ 1,k-1; 2m+1 for i =1,k-1

This map f induces an E_k – cordial labeling.

Theorem 2.2 If n is even and n ≠ k/2(mod k), P_n^+ is E_k – cordial.

Proof: Let

(i) $u_1,u_2,\ldots u_n$ be the path in P_n^+ with deg(u_i) ≠1 in P_n^+.

(ii) $v,\ldots v_n$ be the pendant vertices of P_n^+ and $u_iv_i \in E(P_n^+)$.

(iii) $e_i = u_iu_{i+1}$ for all i=1,2,…,n-1 and $e'_i = u_iv_i$ for all i=1,2,…n.

(iv) n = mk+t, 0 ≤ t < k.

Case (i): If t=0 or t >k/2, define f on $E(P_n^+)$ as follows:

$f(e'_i)$ = □ and $f(e_i)$ = k-□□ -1 for all i where i =□□ (mod k),0 ≤□□ < k.

Clearly, if t = 0,

$e_f(i)$ = { 2m if i ≠ 0; 2m-1 if i = 0.

$v_f(i)$ = 2m for all i.

if t ≠ 0.

$e_f(i)$ = { 2m+2 for k-t < i < t; 2m+1 for i ≤ k-t or i ≥ t.

$v_f(i)$ = { 2m+2 for i = 0,k-t, < i < t; 2m+1 for 1 ≤ i ≤ k-t or i ≥ t.

Case(2): If t =1, define

$f(e'_n)$) = k-1; f((e'_{n-1}) = 0

$f(e'_i)$ = for all i ≤ n-2,

$f(e'_i)$ = k-1-□□ for all i =1,2,…,n-1

where i-1=□□ (mod k), 0≤□□ <k.

Clearly,

$e_f(i)$ = { 2m+1 if i = 0; 2m if i ≠ 0.

$v_f(i)$ = { 2m+1 if i =1 or k-1; 2m if i ≠ 1, k-1.

Case(3): If $2 \le t < \frac{k}{2}$, define f on $E(P_n^+)$.as follows:

$f(e'_n)$ = 2(t-1)

$f(e'_i)$ =□□ and $f(e_i)$ = k-1-□□ for all i ≤ n-1,

Where i-1 =□□ (mod k), 0 ≤□□ <k.

Then clearly,

$e_f(i)$ = { 2m+1 0≤ i ≤ t-1 or i = 2(t-1) or k-t ≤ i≤ k-1; 2m for all other i.

$v_f(i)$ = { 2m+1 0 ≤ i ≤ t-1 or i = 2(t-1) or k-t ≤ i ≤k-1; 2m for all other i.

Thus if n ≠ k/2 (mod k), P_n^+ is E_k – cordial.

Theorem 2.3. Let n ≥ 2 and k ≥ 2 be integers such that n ≠ k/2(mod k). Then C_n^+ is E_k – cordial.

Proof: Let $u_1,u_2,\ldots u_n\ u_1$ be the cycle in C_n^+ and $v_1,\ldots v_n$ be the pendant Vertices in C_n^+, let $u_iv_i \in E(C_n^+)$.Let e_i and e'_i denote the edges $u_i u_{i+1}$ and u_iv_i respectively for all i =1,2,…,n-1 and $e_n = u_nu_1$, $e'_n = u_nv_n$.

Let n = mk + t, 0 ≤ t < k

Let us define f: $E(C_n^+) \to \{0,1,\ldots,k-1\}$ as follows:

$f(e'_i)$ =□□ for all i=1,2,…,n ; $f(e_i)$ = k-□□ -1 for all i=1,2,…,n-1

where i-1 = □ (mod k),0 ≤□□ < k.

$f(e_n)$= { 0 if t = 0 or t > k/2; k-1 if t =1; $[(k+1)/2]$ if 1 ≤ t < k/2.

We note that

(1) If t = 0, $v_f(i) = e_f(i)$ = 2m

(2) If t =1,

$v_f(i)$ = { 2m+1 for i = 0, k-2; 2m for all other i.

$e_f(i)$ = { 2m+1 for i = 0, k-1; 2m for all other i.

(3) If $1 < t \le \frac{k}{2}$.

$v_f(i)$ = { 2m+1 if 1 ≤ i ≤ t-1 or i =, $[k/2]$ or $[(k+1)/2]$ k-t < i ≤ k-2; 2m for all other i.

$e_f(i)$ = { 2m+1 for 0 ≤ i≤ t-1, i = $[(k+1)/2]$, k-t < i ≤ k+1; 2m for all other i.

(4) if k/2 < t,

$v_f(i)$ = { 2m+2 for i = 0, k-t < i<t; 2m+1 for 1≤ i ≤ k-t or i ≥ t.

$e_f(i)$ = { 2m+2 for k-t < i < t and i=0; 2m+1 for i ≤ k-t or i ≥ t.

Thus C^+_n is E_k – cordial.

The graph obtained from P_n^+ by identifying all the pendant vertices of P_n^+ to new vertex w is denoted by F_n and is called the fan on (n+1) vertices. The wheel W_n is the graph obtained from C_n^+ by identifying all the pendant vertices of C_n^+ to a new vertex w. Infact $F_n= K_1+P_n$ and $W_n= K_1+C_n$.

Theorem 2.4 Let $k \geq 3$ and $n = mk+ t, 0 t < k ,m \geq 1$.
If $k = 2 \pmod 4$ and $m = 0 \pmod 2$ assume that $t \neq k-1$ Then F_n is E_k – cordial.

Proof:

Let $u_1u_2\ldots u_n$ be the path P_n and w be vertex which is adjacent to each u_i ($1\leq i\leq n$). The edges u_iu_{i+1} and wu_i are denoted by e_i and e'_i respectively. First we define $f(e_i)$ and $f(e'_i)$ for all i < mk-1. Then we consider various cases and extend f to $E(F_n)$ in each case.

Define $f(e'_i)$= □ and $f(e'_i)$= k-□□ -1 for $1\leq i \leq mk-1$,
Where i-1 = □□(mod k),0 ≤□□ < k.

Case(1): If t = 0 define $f(e'_i)$= k-1, $f(e'_{n-1})$ = k-2 and $f(e_{n-1})$=1

Case(2): If t =1, k = 4 and m is odd, define
$f(e'_{n-1})$ =3, $f(e'_{n-1})$= 0 and $f(e_{n-1})$= 0.

If t =1 and either $k \neq 4$ or m is even, define
$f(e'_n)$ = k-1, $f(e'_{n-1})$ = 1; $f(e_{n-2})$= $f(e_{n-1})$ = 0.

Case(3): Let 1< t < k.

We observe that $m\sum_{j=0}^{t-1} j$ = k/2 if k is even m is odd
= 0 if either k is odd or m is even

Let $(t(t-1))/2 = s \pmod k$, $0 \leq s < k$ and $s + k/2$ = a (mod k).

Assume that either k is odd or m is even.

Sub case(1):
Let s < k - t +1. Then define
$f(e_{mk-1})$ = 1, $f(e_{mk})$= 0; $f(e'_{mk-1})$= k-2; $f(e'_{mk})$= k-1
$f(e_{mk+i})$ = k-i $f(e'_{mk+i})$ = i - 1 for all $1 \leq i < t-1$.
and if s = 0, $f(e'_n)$ = t; $f(e'_{n-1})$= t-1; $f(e_{n-1})$) = k-t.
if $s \neq 0$, $f(e'_n)$ = t-1; $f(e'_{n-1})$ = t-2; $f(e_{n-1})$= k-t+1.

Sub case(2):
Let $s \geq k-t+1$ and t < k-t.
Define f as follows:
$f(e_{mk-1})$ =1, $f(e_{mk})$= 0; $f(e'_{mk-1})$ = k-2; $f(e'_{mk})$ = k-1

$f(e_{mk+i})$ = { k-i for all $1 \leq i \leq k-s-2$; k-i-1 for all $k - s - 1 \leq i \leq t-1$
$f(e'_{mk+i})$ = i -1 for all $1 \leq i < t-1$ $f(e'_n)$ = t.

Sub case(3):
Let $s \geq k - t + 1$; $2t \geq k + 2$ and $t \leq s$.
Then t -1< s and hence k –s < k- (t-1) and
(t-1) < (t-1) + (k-s) = k-(s-t)-1< k. Define f as follows:
$f(e_{mk-1})$ = 1, $f(e_{mk})$ = 0; $f(e'_{mk-1})$ = k-2;
$f(e'_{mk})$ = k-1
$f(e_{mk+i})$ = k-i for all $1\leq i \leq t-1$
$f(e'_{mk+i})$ = i-1 for all $1\leq i < t-1$
and $f(e'_n)$ = t -1 + k -s.

Sub case(4):
Let $s \geq k - t + 1$: 2t = k +1 (ork). Then $s \geq t$.
Then t -1 + k - s + k – t = k + k - (s +1) ≡ k- (s + 1)(mod k);
k - (s + 1) < k-t as $k > s \geq t$.
Define f as follows:
$f(e_{mk-1})$ =1, $f(e_{mk})$ = 0; $f(e'_{mk-1})$ = k-2;
$f(e'_{mk})$= k-1

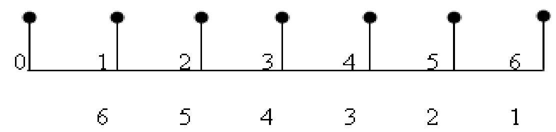
$f(e_{mk+i})$ = { k - i for all $1 \leq i \leq s - t$; k-i-1 for all $s-t+1 \leq i \leq t-1$
$f(e'_{mk+i})$ = { i -1 for all $1 \leq i < t-1$; (t-1) + (k-s) for i= t

Case(4):
Let m be odd and k be even. All the subcases proved in case 3 are valid, if we replace s by a, where $s+[k/2]$= a (mod k) $0 \leq a < k$.
One can easily verify that the map f induces an E_k – cordial labeling.

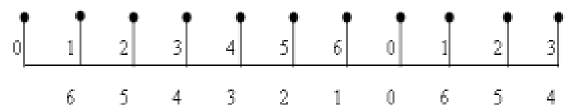
Illustration for E_k - cordial labeling of graphs

E_k – Cordial Labeling of P_n^+

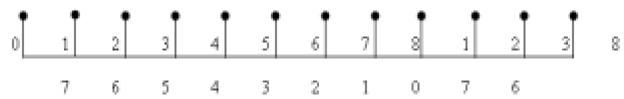
Case(i) E_7 – Cordial Labeling of P_7^+



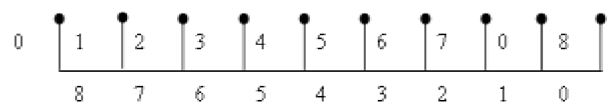
E_7 – Cordial Labeling of P_{11}^+



case(ii) E_9 – Cordial Labeling of P_{12}^+

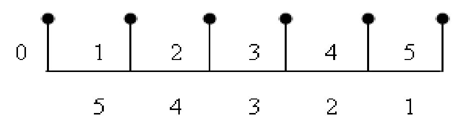


Case(iii) E_9 – Cordial Labeling of P_{10}^+

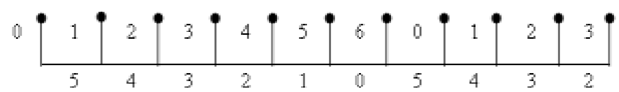


Case (ii) E_6 – Cordial Labeling of P_6^+

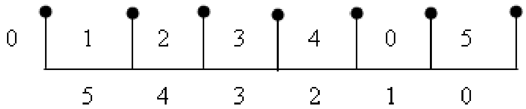
Subcase (i)



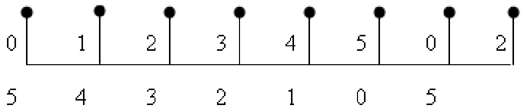
Subcase (ii) E_6 – Cordial Labeling of P_{11}^+



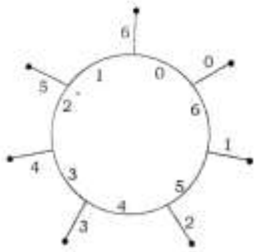
Subcase (iii) E_6 – Cordial Labeling of P_7^+



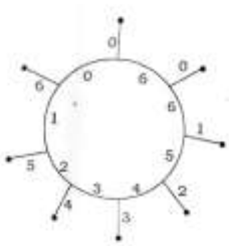
E_6 – Cordial Labeling of P_8^+



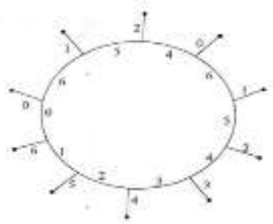
Case(i) E_7 – Cordial Labeling of C_7^+



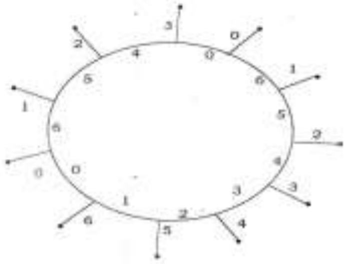
Case(ii) E_7 –Cordial labeling of C_8^+



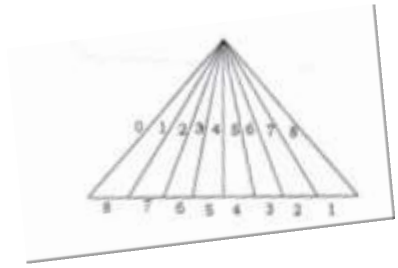
Case(iii) E_7 – Cordial labeling of C_{10}^+



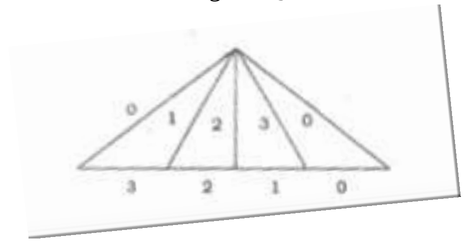
Case(iv)E_7 – Cordial labeling of C_{11}^+



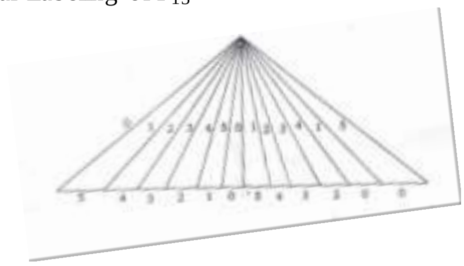
Theorem: 4 F_n is E_k – Cordial
Case (i) t = 0 m-odd k-odd
E_9 – Cordial Labeling of F_9



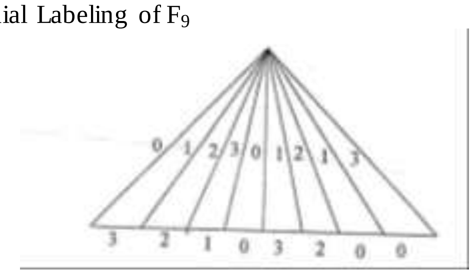
Case(ii) E_4 – Cordial Labeling of F_5



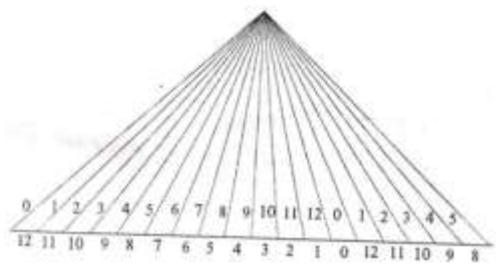
E_6 – Cordial Labeling of F_{13}



E_4 – Cordial Labeling of F_9



E_{13} – Cordial labeling of F_{19}



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