



Channel equalisation with an efficient variable step-size transform domain Adaptive algorithm

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ABSTRACT

Equalization techniques compensate for the time dispersion introduced by communication channels and combat the resulting inter-symbol interference (ISI) effect. The purpose of an adaptive equalizer is to operate on the channel output such that the cascade connection of the channel and the equalizer provides an approximation to an ideal transmission medium. This paper compares existing adaptive algorithm for channel equalisation for both DFT and DCT based OFDM system. Simulation results show that performance of Transform Domain Variable stepsize Griffith LMS (TVGLMS) algorithm in channel equalization performs with better convergence speed and better error misadjustment than the LMS algorithm. Further simulation results shows that DCT based OFDM system outperforms DFT -OFDM in terms of BER and PAPR.

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Introduction

Communication channels can distort data in two ways, that is by the channel noise and due to the time dispersion effect of the channel frequency response. The channel transfer function produces intersymbol interference (ISI), which puts limits to the maximum transmission speed of the data. To eliminate this effect of ISI, an adaptive channel equalization that uses a filter whose coefficients are adapted, is used at either transmitter or receiver. The implementation of equalizer is based on filter with finite impulse response (FIR) employing the well known LMS algorithm for adjusting its coefficients is addressed in literature.[1][5]

Adaptive filters adjust their transfer functions according to optimizing algorithm and adaptive filters will track the changes in the statistics of the signals or parameters of time-varying systems, differ from the non-adaptive Wiener filter in that their impulse are adjusted iteratively as data flow through the filters.[1][2]

An adaptive signal processing algorithm like the least mean squares (LMS) algorithm and the recursive least square (RLS) algorithm, are widely used algorithms with adaptation of filters. These adaptive algorithms are expected to be computationally simple, numerically robust and fast convergent.

The stability and convergence are important factors to be considered while choosing adaptive algorithms. The convergence speed of an adaptive algorithm measures the number of iterations to be taken for the algorithm to reach a desired state the stability of an adaptive algorithm measures its working reliability.[4].

Further selection of algorithm has to be cost effective also. With this goal in mind, we may identify three important issues like computational cost, performance, and robustness in designing adaptive equaliser.

In multipath communication systems, frequently the channel parameters are not known in advance and moreover they may vary with time. Hence, it is necessary to use the adaptive channel equalizers, which provide the means of tracking the channel characteristics. Equalization can be done in time domain or in frequency domain. In frequency domain equalization the equalizer tries to restore the spectrum the ideal signal and adaptation is achieved by monitoring some predefined frequencies. In frequency domain equalization, as the frequency content of the spectrum cannot be mapped onto the instantaneous time domain signal, wavelet can be used effectively.[12]

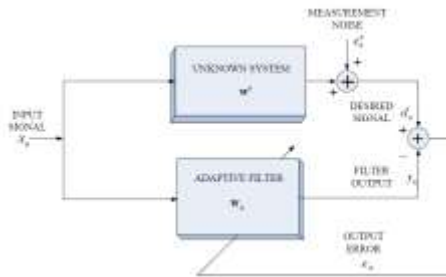
We present an efficient frequency domain channel equalisation algorithm, TVGLMS, for DCT based OFDM system and compare its performance with TLMS algorithm. The Paper is organised as follows: First, we present the comparative study of DCT and DFT as transforms used in LMS algorithm. In section II we discuss about Model for Adaptive filtering and in section III we compare DCT and DFT LMS and in Section IV Variable step size algorithm. In Section V and VI, we then discuss about TVGLMS algorithm, its mathematical model for channel equalisation for Rayleigh fading channel in OFDM transmission. Later in section VII, performance comparison with simulation results discussed.

Comparison of Discrete Cosine (DCT) and Discrete Fourier Transforms (DFT)

Though DFT is frequently used transform for most of the applications, In the DFT, a segment of the signal considered is truncated one and has periodic discontinuity due to truncation. This leads to Gibbs phenomenon. To overcome this effect, if smoother window is applied to more number of smaller segments, it will lead to reduced frequency resolution. Further for good approximation of signal large number of DFT coefficients are required. [20]

DCT is the DFT of a symmetrically extended signal. The symmetrical extension reduces the abruptness of truncation significantly and results in smooth truncation from one period to other. Hence it requires less number of coefficients for signal approximation. Further, DCT has good orthonormal, separable, and energy compaction property. Most of the signal information tends to be concentrated in a few low frequency components of the DCT. Although the DCT does not separate frequencies, it is a powerful signal decorrelator. It is a real valued function and thus can be effectively used in real-time operation. Therefore DCT based systems are preferred in communication signal processing.

Principle of adaptive filtering



A simple structure for building the model using an adaptive filter is shown in Figure 1. The input to the system is passed through the adaptive filter of N variable coefficients or weights. The filter output y_n is compared with the noisy observations of the system output d_n . The coefficients of the adaptive filter are adjustable with the modelling error e_n to produce the best possible match between the system output and filter output. The adaptive algorithms provide the filter with an algorithmic solution which allows for the tracking of time variations in the input statistics without having to solve the Wiener-Hopf equation. Typically, adaptive filters work as follows. The filter weights are initially set to zero or to arbitrary values. Then, at each iteration, the weights are adjusted by using an optimization method so as to improve the solution. Hence under the appropriate conditions, the filter weights converge to the solution of the Wiener-Hopf equation. Algorithms used can be time domain based or transform domain algorithms and DFT or DCT based algorithms.[2]

DFT-LMS verses DCT –LMS

One major advantage of the LMS algorithms is its simplicity and ease of implementation. The computational complexity of a regular LMS algorithm is $O(N)$, which is much more efficient than the steepest descent algorithm. Further the LMS algorithm has no memory. It therefore can more easily track a time-varying solution than the accumulative algorithms like the RLS-type algorithms. The conventional LMS adaptive algorithm [1] has the advantage of being simple to implement. However, the LMS algorithm with fixed step-size [1] have the disadvantage that for a small value of the step-size, its convergence speed is small. If the step-size is increased in order to increase the convergence speed, then the adaptation error will also increase.

Large Eigen value spreads of the channel restricts the maximum learning rate to be selected without causing stability issues. Best convergence can be obtained when all the eigenvalues are equal, which mean that autocorrelation matrix is proportional to identity matrix. This condition indicates inputs are having equal power. As the eigenvalue spread of the input autocorrelation matrix increases, convergence speed decreases in LMS algorithm. This is to ensure that for convergence, the

adaptation stepsize μ is limited by the maximum eigenvalue of the autocorrelation matrix of the input. Thus Eigen value spread of the signal decides rate of convergence for stationary signals and the tracking of statistics for the non-stationary signals.[9][11][13]

Convergence speed can be increased by pre-processing the input data with a well chosen fixed transformation which do not depend on the inputs. Further power normalisation can be done that cause the input eigen values to cluster around one, to speed up convergence of tap coefficients [3][4]. The MSE of LMS algorithm is a quadratic function of weights, which represents hyperellipsoid in n dimensional weight space. DCT and DFT are unitary matrices (rows are orthogonal to one another with norm one). Unitary transformation perform only rotations and symmetries, they do not modify shape of the object on which transformation is applied. When unitary transforms are applied to the input, rotates the hyperellipsoid, which may have slight imperfections due to leakage in the DCT/DFT transforms. With the power normalization, the new ellipse will be more round and has lower eigenvalue spread.[3][5]. Better option is to make step size variable.

Variable Step size LMS

The LMS algorithms with the variable step-size can achieve good trade-off between convergence speed and steady-state misadjustment. The algorithms can fast track the statistic changes of the inputs or the system variations. Initialization of parameters involved in the variable step-size adjustment schemes is easy and affected as less as possible by measurement noise. In addition, the increase of computation cost due to the adaptation of the step-size should be moderate compared to the implementation of the algorithms with fixed step-size.[2][6]

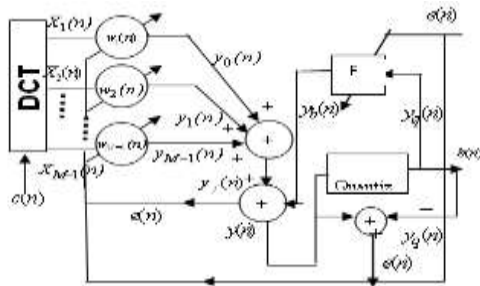
Transform domain LMS algorithms like DFT-LMS, DCT-LMS are already proposed in[3]. The delayed input samples are transformed before applying to LMS algorithm block. The transformed signals are normalized by the square root of their power and then the resulting equal power signals are input to an adaptive linear equaliser whose weights are adjusted using the DFT- LMS, DCT-LMS algorithm. In addition to fast convergence and robustness, DFT-LMS and DCT-LMS has low computational cost. The advantages of DCT-LMS as compared to normal LMS algorithm has better convergence [7]. Various variable step-size adjustment schemes have been proposed [7] to meet the conflicting requirements of fast convergence speed and low steady-state MSE or misadjustment. All these schemes are based on a specific criterion[8].[10][12]

Number of schemes based on the squared instantaneous error, on the time-averaged estimate of the error autocorrelation at adjacent time, on the sign of the instantaneous error, and on the estimate of gradient power have low computational costs, but their steady-state performance are highly dependent on the power of measurement noise[11][14][15]. As a result, these algorithms suffer from the applications where low signal-to-noise ratios (SNR) present. The variable step-size (VSS) LMS algorithm developed by Kwong [9] and Johnston is probably the best low-complexity variable step-size LMS algorithm available in the literature if well designed, except for its limited robustness to the measurement noise power. We next discuss very effective transform domain variable stepsize Griffiths algorithm.[16]

Transform Domain Variable Step Size Griffiths LMS Algorithm (TVGLMS) for Channel Equalisation

The TVGLMS, the robust variable step size has been achieved by using the Griffiths' gradient which uses cross-

correlation between the desired signal contaminated with observation noise and the input and the discrete cosine transform (DCT). A variable step size Griffiths LMS algorithm(VGLMS) which not only uses the step size but also gradient for weight vector which are robust to observation noise has been proposed [16]. The VGLMS achieves this by using the cross-correlation between the desired signal and the input. However, the VGLMS algorithm for a given maximum step size, has a slower convergence rate, due to replacement of instantaneous correlation between input and the error by the Griffiths averaged gradient. This motivates to apply TVGLMS algorithm with faster convergence rate for inputs with large eigenvalue spread and extend this algorithm to transform domain adaptation to provide additional convergence speed over the variable step size and a smaller misadjustment when applied to channel equalisation.[16-22]



Further DCT-OFDM is implemented instead of DFT as transformation at the transmitter, to take the advantage of energy compaction property of DCT. This helps to reduce Peak to average power Ratio (PAPR) in OFDM systems.

The frequency-domain implementation of the LMS adaptive algorithm is particularly efficient when long adaptive filters are used, due to the reduced computational complexity associated with the Fast Fourier Transform (FFT).[16]

The basic transform domain LMS algorithm using discrete cosine transform for channel equalization is shown in Fig. 3. For the input $x(n)$, the $2M$ point DCT produces orthogonal components $X_k(n), k = 0, K, M - 1$ (the remaining M points are symmetrical) and these form inputs to single coefficient adaptive filters $W_k(n), k = 0, K, M - 1$. The output of these adaptive filters $y_k(n), k = 0, K, M - 1$ are summed (with a gain factor of 2) to take care of symmetrical components) on sample to sample basis to get the estimate $y(n)$ of the desired signal $d(n)$.

The TLMS algorithm uses normalized LMS algorithm whose decomposed weight vector $w_k(n)$, is defined by the recursion

$$\hat{W}_k(n+1) = \hat{W}_k(n) + \frac{2\mu}{\sigma_k^2(n)} e(n) X_k^*(n) \quad k = 0, 1, 2, \dots, N - 1 \quad (1)$$

$$\sigma_k^2(n) = \beta \sigma_k^2(n-1) + (1-\beta) X_k^*(n) X_k(n) \quad 0 < \beta < 1 \quad (2)$$

Where β is close to one is a forgetting factor, where the input transformed vector (superscript * denotes complex conjugate) is $X(n) = [x_1(n), x_2(n), \dots, x_{N-1}(n)]^T$.

$\mu(n)$ is the step size used for the adaptation. $\sigma_k^2(n)$ is the input power to adaptive filter $w_k(n), k = 0, K, M - 1$ and is estimated recursively. The transform domain provides faster convergence compared to transversal filter since

$$\sigma_k^2(n) \ll \sum_{k=0}^{M-1} \sigma_k^2(n)$$

and the gradient step size will be relatively large for the former. However due to large step size, the convergence error of the transform domain LMS can be larger than that for transversal filter. Since $e(n) = d(n) - y(n)$

Equation. (1) Can be written as

$$w_k(n+1) = w_k(n) + \frac{2\mu(n)}{\sigma_k^2(n)} [d(n) - y(n)] X_k(n)$$

The estimation error is given by $e(n) = d(n) - y(n)$, where the filtered output $y(n) = W^T(n) X_T(n)$ and $d(n)$ is the desired signal. Griffiths' algorithm (TGLMS) modifies the Transform domain NLMS algorithm to remove the effect of noise. $G_k(n) = E[d(n)x(n-k)] = E\{[d'(n) + o(n)]x(n-k)\}$, (3)

then $G_k(n), k = 0, 1, 2, \dots, M - 1$, represents the cross correlation between the desired signal $d(n)$ and the input $x(n)$.

Further as $x(n)$ and $o(n)$ are independent,

$$E[o(n)x(n-k)] = 0$$

Therefore, $G_k(n) = E[d'(n)x(n-k)], k = 0, 1, 2, \dots, M - 1$

$$\text{and } E[e(n)x(n-k)] = G_k(n) - E[y(n)x(n-k)] \quad (3)$$

$$\text{If } Q_k(n) = [G_k(n) - y(n)x(n-k)], k = 0, 1, 2, \dots, M - 1 \quad (4)$$

$Q_k(n)$ is Griffiths cross-correlation in the context of transform domain LMS algorithm. Further $G_k(n)$ is estimated recursively as

$$\hat{G}_k(n+1) = \beta_g \hat{G}_k(n) + (1-\beta_g) d(n) X_k(n), 0 < \beta_g < 1 \quad (5)$$

Similar to VGLMS algorithm, the step size $\mu(n)$ can be adapted as

$$\mu(n) = \alpha \mu(n-1) + \gamma \sum_{k=0}^{M-1} Q_k^2(n) \quad 0 < (\alpha, \gamma) < 1 \quad (6)$$

Adaptive channel equalisation for DFT and DCT OFDM

For channels with severe delay spread, frequency domain equalization is computationally simpler than corresponding time domain equalization for the same reason OFDM is simpler: because equalization is performed on a block of data at a time, and the operations on this block involve an efficient FFT/DCT operation and a simple channel inversion operation. As the transmitted OFDM signal is a sum of a large number of slowly modulated subcarriers, OFDM has a high peak-to-average power ratio, therefore TVGLMS algorithm used provides better performance in OFDM.

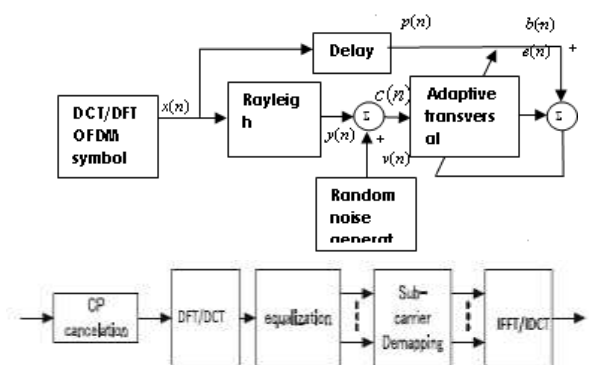


Figure 2. Model of Adaptive Equalisation in OFDM system

Figure 2, shows model for Adaptive equalisation for OFDM system. Random generator provides the input bit stream which is mapped into BPSK modulated data and converted to parallel

stream. Then IDCT/IFFT is applied which maps symbol onto subcarriers. Cyclic prefix (CP) is added to each symbol. The length of the CP added is taken to be 25% of frame length. [21]. This OFDM symbols are serially transmitted through Rayleigh channel and noise $v(n)$ of high SNR that corrupts the channel output.

The impulse response of the Rayleigh channel is given by $h(t) = 2a \exp(-a^2 (1:W))$

where, the parameter W controls the amount of amplitude distortion produced by the channel, with the distortion increasing with W . Equivalently, the parameter W controls the eigenvalue spread of the correlation matrix of the tap inputs of the equalizer, with the eigenvalue spread increasing with W .

At the receiver, Cyclic prefix of the corrupted signal is first removed and equalisation is done. An inverse FFT/DCT returns the equalized signal to the time domain prior to the detection of data symbols or adaptation of the frequency domain equalizer's transfer function is done with Transform Domain variable stepsize Griffith LMS, algorithm (TVGLMS). The adaptive equalizer has the task of correcting the distortion produced by the channel in the presence of additive random white noise. After equalisation, demapping and quantisation is done to get the estimate of transmitted signal.

Simulation Results

The performance of the proposed TVGLMS algorithm is illustrated for the channel equalization in both DCT and DFT based OFDM systems. Fig 4 and 5 shows simulated results MSE for TVGLMS algorithm used for equalisation in DCT based OFDM that performs better than TLMS algorithm. Simulation is carried for 8000 bits with binary PSK modulation 512 subcarriers were used for transmission. MSE curves shown for DFT and DCT based OFDM system with TVGLMS algorithm has faster convergence compared to TLMS. Further DCT based OFDM performs better and has faster convergence than DFT based OFDM system.

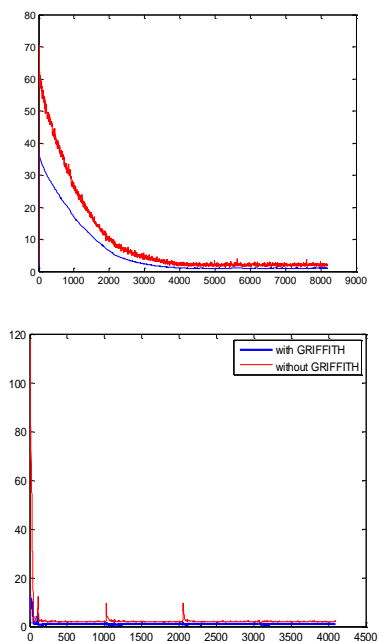
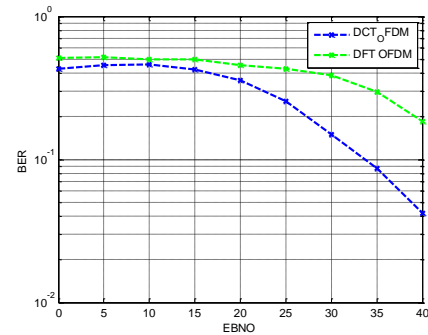


Figure 2. MSE curve for DFT and DCT OFDM



PAPR_dft_OFDM = 29.2610 PAPR_dct_OFDM = 24.1721

Conclusion

In this paper study of channel equalization with efficient Transform domain variable stepsize Griffiths' LMS (TVGLMS) algorithm is carried out and compared with TLMS algorithm, for DCT-OFDM and DFT-OFDM systems. The different step size used are significantly large as TLMS uses the total power of the input for stepsize normalization. The use of larger stepsize for different components though results in faster convergence rate, it results in higher convergence error/misadjustment.

The TVGLMS uses the cross-correlation between the desired signal and the input. TVGLMS algorithm has faster convergence rate for inputs with large eigenvalue spread and not only provides additional convergence speed over the variable step size but also a smaller misadjustment. The MSE performance curve shows faster convergence rate for DCT based OFDM than DFT -OFDM. Further BER plot and PAPR obtained indicate DCT OFDM outperforms DFT OFDM.

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