



Experimental and numerical realization of simple 4d – hyperchaotic circuit with “Cubic-Like” two ideal diodes

G. Kandiban¹, V. Balachandran² and S. Manimaran³¹Department of Physics, Thanthai Hans Roever College, Perambalur - 621 212, India.²Department of Physics, A.A Government Arts College, Musiri, Tiruchirappalli - 621 201, India.³Department of Physics, Roever Engineering College, Perambalur - 621 212, India.

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ABSTRACT

The simple 4D- hyperchaotic dynamics of an autonomous oscillator circuit was studied by measuring its responsible in the form of phase-portrait, power spectrum and hyperchaotic time series. The circuit consists of just four linear elements (two capacitors and two inductors), one linear negative conductance and two ideal diodes. The power spectrums are presented to confirm the strong hyperchaotic nature of the oscillations of the circuit. The performance of the circuit is investigated by means of experimental and numerical confirmation of the appropriate differential equations. The features of the obtained results are respected for various engineering system such as chaos based secure communication systems with robustness against various interferences.

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Introduction

In original Chua's circuit, a nonlinear resistor is called Chua's diode is the unique nonlinear electric element. It plays an important role in the circuit. Due to the existence of this nonlinear element Chua's circuit exhibits a variety of nonlinear phenomena, such as chaos, bifurcation and so on [1-5]. The characteristic of Chua's diode is described by a continuous piecewise - linear function with three segments and two non-differential break points [6-9]. However, the characteristics of nonlinear devices in practical circuits are always smooth and the implementation of piecewise-linear function requires a large amount of circuitry compared with smooth cubic function. Therefore, it is significant to investigate Chua's circuit with a smooth cubic nonlinearity from practical view point [14]. Hartley (1989) proposed to replace the piecewise-linear nonlinearity in Chua's circuit with a smooth cubic nonlinearity.

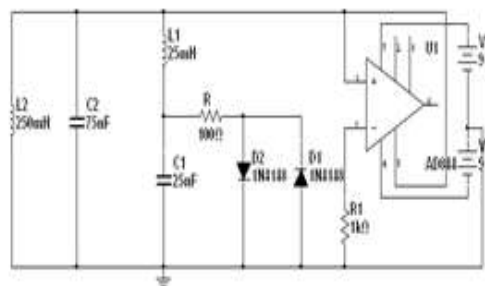
In the present report the behavior of a fourth-order autonomous hyperchaotic oscillator circuit has been studied. This circuit consists of two active elements, one linear negative conductance and smooth cubic nonlinearity exhibiting a symmetrical piecewise-linear $v-i$ characteristic. Two inductances (L_1, L_2) two capacitances (C_1, C_2) and one locally active resistor (R) is also included in the circuit, serve as the control parameters.

Hyperchaos is defined as a chaotic attractor with more than one positive Lyapunov exponents, i.e., its dynamics expand in more than one direction [5]. In other words, the dynamics expand not only small line segments, but also small area elements, there giving rise to a 'thick' chaotic attractor. Most hyperchaotic and bifurcation effects cited in the literature have been observed in electric circuits. They include the period-doubling route to chaos, the intermittency route to chaos, and the quasiperiodicity route to chaos and of course the crisis [7, 10-12]. This popularity is attributed to the advantages which electric circuits offer to

experimental hyperchaos studies, such complicated hyperchaotic wave forms are expected to be utilized for realization of several hyperchaotic applications such a chaos communication system with robustness against various interferences including multi-user access [9-14]. The plan of the paper is as follows. In section II, we present the details of realization of the proposed autonomous circuit. The results of the observations from the laboratory experimental simulation and the conformation through analytical calculation and numerical simulation on the dynamics of the circuit are presented in section III. Finally, in section IV, we summarize and conclude the results and indicate further direction.

Circuit description and simulation results

The fourth-order hyperchaotic oscillator circuit we have studied is presented in Fig. 1.



It consists of two active elements, one linear negative conductance (G_I) using current feedback op-amp and one smooth cubic nonlinearity with an odd symmetric piecewise-linear $v-i$ characteristic [14]. This fourth-order circuit is based on a third-order autonomous piecewise-linear circuit introduced by Chua and Lin, capable to realize every member of the Chua's circuit family [9]. Applying Kirchoff's laws, the set of four first-orders coupled autonomous differential equations as given below:

$$\begin{aligned}
 C_1 \frac{dV_1}{dt} &= i_{L_1} - \frac{V_1}{R} - i_N \\
 C_2 \frac{dV_2}{dt} &= G_1 V_2 - i_{L_1} - i_{L_2} \\
 L_1 \frac{di_{L_1}}{dt} &= V_2 - V_1 \\
 L_2 \frac{di_{L_2}}{dt} &= V_2
 \end{aligned}
 \tag{1}$$

While V_1 and V_2 are the voltages across the Capacitors C_1 and C_2 , i_{L_1} and i_{L_2} denotes the currents through the inductances L_1 and L_2 respectively, the term $f(V_1)$ representing the characteristic of the smooth cubic nonlinearity can be expressed mathematically:

$$i_N = f(V_1) = aV_1 + bV_1^3 \tag{2}$$

For our present experimental study we have chosen the following typical values of the circuit in Fig. 1. Were $L_1 = 25mH$, $L_2 = 250mH$, $C_1 = 25nF$, $C_2 = 75nF$ and $G_1 = 1.0mS$. Here the variable resistor 'R' is assumed to be the control parameter. By decreasing the value of 'R' from $2,100\Omega \geq R \geq 33\Omega$, the circuit behavior of Fig. 1 is found to transit from a period-doubling route to chaos and then to hyperchaotic attractor through border collision bifurcation behavior followed by period-doubling windows and boundary crisis [13-14], etc. The hyperchaotic attractors of fourth-order autonomous circuit with the smooth cubic nonlinearity projected onto different planes are shown in Fig. 2. Experimental time series were registered using a simulation storage oscilloscope for discrete values of C1 and C2 are shown if Fig. 3.

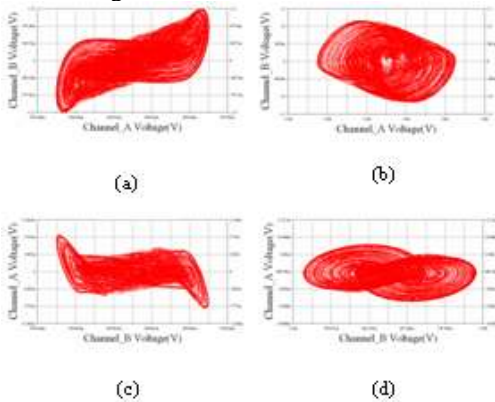


Fig. 2 Experimental results of the projections of hyperchaotic attractor onto different planes.

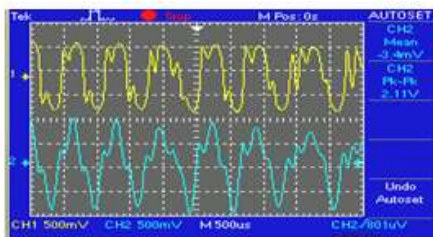


Fig. 3 Experimental results of the hyperchaotic time series.

The distribution of power in a signal $x(t)$ is the most commonly quantified by means of the power density spectrum or simply power spectrum. It is the magnitude-square of the Fourier transforms of the signal $x(t)$. It can detect the presence of hyperchaos when the spectrum is broad-banded. The power spectrum corresponding to the voltages $V_1(t)$ and $V_2(t)$

waveforms across the capacitors C_1 and C_2 for the hyperchaotic regimes are shown in Fig. 4 which resembles broad-band spectrum noise.

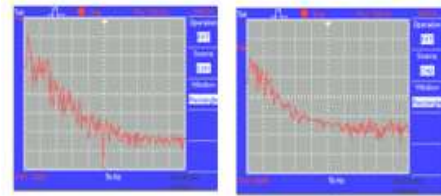


Fig. 4 Experimental results of the projections of hyperchaotic Power spectrum.

Numerical confirmation

The hyperchaotic dynamics of circuit as shown in Fig. 1 is studied by numerical integration of the normalized differential equations [14]. For a convenient numerical analysis of the experimental system given by Eq. (1), we rescale the parameters as

$$\begin{aligned}
 V_1 &= \frac{x_1}{\sqrt{bR}}, V_2 = \frac{x_2}{\sqrt{bR}}, i_{L_1} = \frac{x_3}{\sqrt{bR^3}}, i_{L_2} = \frac{x_4}{\sqrt{bR^3}} \\
 t &= \tau RC_2, v_1 = \frac{C_2}{C_1}, v_2 = \frac{C_2}{C_2}, \alpha = (1 + aR) \\
 \gamma &= G_1R, \beta_1 = \frac{C_2R^2}{L_1} \text{ and } \beta_2 = \frac{C_2R^2}{L_2}
 \end{aligned}$$

Eqs. (1) and (2) reduce to the following set of normalized equations of the fourth-order oscillator circuit as given below:

$$\begin{aligned}
 \bullet \quad x_1 &= v_1(x_3 - \alpha x_1 - x_1^3) \\
 \bullet \quad x_2 &= v_2(\gamma x_2 - x_3 - x_4) \\
 \bullet \quad x_3 &= \beta_1(x_2 - x_1) \\
 \bullet \quad x_4 &= \beta_2(x_2)
 \end{aligned}
 \tag{3}$$

The dynamics of Eq. (3) now depends upon the parameters $v_1, v_2, \gamma, \alpha, \beta_1$, and β_2 . The experimental results have been verified by numerical simulation of the normalized Eq. (3) using the standard fourth-order Runge-Kutta method for a specific choice of system parameters employed in the experimental simulation results. That is, in the actual experimental set up the resistor 'R' is varied from $R = 2,100\Omega \geq R \geq 33\Omega$. Therefore in the numerical simulation, we study the corresponding Eq. (3) for in the range $R = 2,100\Omega \geq R \geq 33\Omega$.

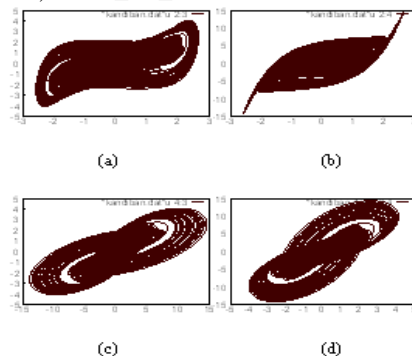


Fig. 5 Numerical results of the projections of hyperchaotic attractor onto different planes

When the value of 'R' is decreased to lower than 2,100Ω particularly in the range $R = (2,100\Omega \geq R \geq 33\Omega)$ the system

displays a period-doubling route to chaos and then to hyperchaos through boundary condition. These numerical results of the hyperchaotic attractor of fourth-order autonomous circuit with the smooth cubic nonlinearity projected onto different planes are shown in Fig. 5. It is gratifying to note that the numerical results agree qualitatively very well with that of the experimental simulation results.

Conclusions

We have presented a simple-4D hyperchaotic oscillator circuit which has symmetrical piecewise-linear elements. We can confirm hyperchaotic attractor on computer simulation or circuit experiments. The attractive feature of this circuit is the presence of hyperchaotic attractor over a range of parameter values, which might be useful for applications in controlling of hyperchaos, synchronization and in secure communication system.

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