Available online at www.elixirpublishers.com (Elixir International Journal)

Computer Science and Engineering

Elixir Comp. Sci. & Engg. 38 (2011) 4538-4540

ISSN: 2229-712X

Reoptimization of steiner tree a comparative analysis

Maitreyee Dutta and Rituraj Soni

Department of Computer Science Engineering, NITTTR, Panjab University Chandigarh ,India.

ARTICLE INFO

Article history:

Received: 22 August 2011;

Received in revised form: 26 August 2011;

Accepted: 31 August 2011;

Keywords

Reoptimization,

Steiner Minimal Tree,

Approximation Ratio,

Minimum Spanning Tree.

ABSTRACT

The problem of Reoptimization of Steiner Tree is a NP Hard problem. Given a graph and a optimal solution of the Steiner tree and then after a slight modification is done in the initial instance, then a new Steiner minimal tree is to be determined. This is known as reoptimization of Steiner Tree. Several cases of reoptimization of Steiner tree have been discussed. This paper presents a comparative analysis of some of the previous work done on reoptimization of Steiner Tree.

Introduction

The Steiner tree problem is one of the most famous combinatorial optimization problems in network design. In this problem a graph is given and it is required to calculate the minimum spanning tree over the given set of terminals. Since it is a optimization problem it is required to calculate the optimal solution to a Steiner Tree problem. There are certain cases of local modification to the initial instance and it is required to recompute the solution. This task can be achieved by calculating the solution with the help of the previous solution. This is called as the theory of reoptimization [30].

The reoptimization variants of different problems like Travelling Salesman [20,22], knapsack problem [21], Sum of subset problem, shortest common superstring problem [23] have been already proposed. The Steiner minimal Tree problem is a NP hard problem. Many approximation algorithms [24] have been proposed for the solution of the Steiner Tree. The solution is described in terms of the approximation ratio. The minimum spanning tree is used as the heuristics for the applying approximation algorithm [2].

The research of various scholars over the years on this problem of reoptimization of Steiner tree has reduced the approximation ratio to a great extent on different local modifications. In this paper we are presenting the comparative analysis of the reoptimization tree done over the years. Section I gives introduction to Steiner problem. Section II discusses the preliminaries and introduction of the Steiner minimal Tree problem. Section III discusses the theory of reoptimization. Section IV discusses the previous work done on the given problem of reoptimization. Section V of the paper discusses the conclusion and the future work.

Steiner Tree Minimal Problem

Basic Notation and facts

Let G be a Metric Graph, G = (V, E) where V is the set of Vertex and E is the set of edges. Let R be the set of terminals, R $\subset$ V. A Steiner Tree is to be designed for the given set R $\subset$ V. The Steiner Tree is another form of minimum spanning tree which spans the given subset R and the cost of the tree should be minimum. To further reduce the cost we need to include some non-terminal in the tree. These non terminals are known as Steiner Points [19]

The length of the tree is number of edges in the tree and the cost of the tree is the sum of the weights of the edges. The degree of the vertex is denoted by deg (v), v ε V. In shortest path the length of the path is minimized and in cheapest path the cost of the tree is minimized.

Approximation Algorithms

The Steiner Minimal tree problem is NP hard problem. Since it is a optimization problem in which it is needed to minimize the cost of the Steiner tree. The NP hard problems are solved with the help of the Approximation algorithm.

In computer science and operations research, approximation algorithms are algorithms used to find approximate solutions to optimization problems.

Approximation algorithms are often associated with NP-hard problems; since it is unlikely that there can ever be efficient polynomial time exact algorithms solving NP-hard problems, one settles for polynomial time sub-optimal solutions.

Ideally, the approximation is optimal up to a small constant factor (for instance within 5% of the optimal solution). Approximation algorithms are increasingly being used for problems where exact polynomial-time algorithms are known but are too expensive due to the input size.

These Approximation algorithms are used to solve many optimization problems like Travelling Salesman Problem, Knapsack problem, sum of subsets Problem etc.

Along with that if any variation occurs in the optimal solution then solution of these problems are also given by the approximation algorithms and by improved approximation ratio.

Approximation Ratio

The approximation algorithm as the name suggests are not the actual solution to the problem, but it gives a reasonably good solution to the problem.

The goodness of any solution is achieved by the approximation ratio [24]. The approximation ratio can be defined as the ratio of the solution obtained by the approximation algorithm and the optimal solution. It can be defined as

Tele:

E-mail addresses: d_maitreyee@yahoo.com

Approximation ratio $\rho \geq \frac{C(T)}{C(T_{opt})}$

Steiner Minimal Tree Problem

Let Graph G = (V, E, and c) be an undirected graph with nonnegative weights, Where V is the Vertex set, E is the Edge set, c is the cost function. Given a set $L \subset V$ of terminals, a Steiner minimal tree is the tree $T \subset G$ of minimal total edge weight such that T includes all vertices in L.

Problem: Steiner Minimal Tree.

Instance: A graph G = (V, E, c) and a set $L \subset V$ of terminals.

Goal: Find a tree T with $L \subset V(T)$ so as to minimize c (T).

There are various approaches have been proposed for the design of Steiner Minimal Tree Problem. These are discussed in the coming sections.

Theory of Reoptimization

The theory of reoptimization suggests the calculation of the optimal solution for the problem in which the original instance is modified. When some modification occurs, it is necessary to compute the optimal solution again. But the modification reflects some small change. Why does one require to fully computing the solution again for a little modification? There are methods so that we can learn from previous solution and by applying some approximation algorithm [24] a new optimized solution can be calculated. More precisely, we consider the model of reoptimization algorithms which handles problems where an instance together with one of its optimal solutions is given and the problem is to find a good solution for a locally modified instance. This concept of reoptimization was mentioned for the first time in [30] in the context of post optimality analysis for a scheduling problem. Since then, the concept of reoptimization has been investigated for several different problems like the travelling salesman problem [20, 22], knapsack problems [21], and the shortest common substring problem [23].

The reoptimization concept is applied in the case of the Reoptimization of Steiner minimal tree. In this case we assume that an optimal solution to Steiner Tree is already present with us. There are various modifications which can be done on the given Steiner Tree and with the help of the optimal solution and the approximation algorithms we can obtain a new optimized solution for the modified instances of the Steiner Tree Problem. The different modified instances of a Steiner tree can be

a) Addition of Terminal
b) Addition of Non Terminal
c) Increasing the weight of an edge
d) Decreasing the weight of an edge
e) Changing the terminal to non- terminal
f) Changing the non terminal to terminal

Previous work done

A lot of work has been done on the problem of reoptimization of Steiner Tree. There exist several heuristics for the Steiner Tree problem, but only certain heuristics has given a certain good results. The best approach for the calculation of Steiner Tree problem is to use Minimum Spanning Tree.

Dreyfus and Wagner [1] proposed an algorithm which computes the set of graph arcs of minimum total length needed to connect a specified set of k graph nodes. If the entire graph contains n node then the time taken by the algorithm is proportional to

$$n^3/2 + n^2(2^{k-1} - k - 1) + n(3^{k-1} - 2^k + 3)/2.$$

The fundamental technique in the above algorithm is called as Dynamic Programming and it calculates the optimal solution.

Bang Ye Wu and Kun Mao Chao [2] proposed the solution of the Steiner minimal with the help of the Approximation by minimum spanning tree. The given algorithm finds a 2-approximation of an SMT.

In this manner many theories and algorithms have been proposed for the reoptimization of tree. The target is to reduce the approximation ratio as minimum as possible. Initially Takahashi and Matsuyama were the first to prove that the well known minimum spanning tree algorithm achieves a performance ratio of 2. A large number of researchers and scholars have proposed the algorithm to reduce the approximation ratio which is shown in the given table.

Year	Performance Ratio (ρ)	Author
1980	2.000	Takahashi,Matasuyama [31]
1993	1.834	Zelikovsky [29]
1994	1.734	Berman, Ramayier [26]
1995	1.694	Zelikovsky [32]
1997	1.667	Promel, Steger [15]
1997	1.644	Karpinski,Zelikovsky [25]
1998	1.598	Hougard,Primel [4]
2000	1.55	Robins, G., Zelikovsky [3]
2002	1.6	Chung ling lu,Chuan tang [27]
2008	ρ < 1.4(for all cases)	Hans Joachim, David bilo [8]
2009	$2-\varepsilon$	Aaron Archan, Hossen [16]
2010	$1+\delta$, $0<\delta<1$	Panwar, Sunita Agarwal [11]
2010	½+ ß, ß < ½ +ln(3)/4	Joachim, Karin Friermuth [12]

The table above shows the various approximation ratios for the Steiner tree reoptimization. The approximation ratio is different for different modification cases like addition/removal of terminal/non-terminal, increasing/ decreasing the weight of edges.

Over the years the research from (table) shows that approximation ratio is decreasing. The decreasing ratio indicates that the solution obtained for different cases by approximation algorithm is near to optimal solution.

Conclusion and Future Work

Reoptimization of Steiner tree is NP hard Problem and also APX Hard problem for certain cases. In this paper, we have done a comparative analysis of previous work done on reoptimization problem for different local modification on the initial instance of the problem. Since a NP hard problem can be solved by approximation algorithms, approximation ratio is the main criteria for the comparative analysis.

The approximation ratio has been reduced from 2 to .775 over the years. The reoptimization of Steiner tree has been discussed for the modification cases like addition/removal of terminals/non- terminals, increasing / decreasing the weight of the edges.

There are certain instances of the approximation algorithms in which the input instances of the graph has been restricted like, the weight of the edges have been restricted to 1 & 2 or the weight of the edges are satisfying triangle inequality are also analysed. With the reduction of domain better approximation ratio has been obtained.

The future work for the reoptimization of Steiner Tree can be done with the help of reduction of domain by applying sharpened triangle inequality [9, 12] on the edges of the graph.

References

[1] s. E. Dreyfus and R. A. Wagner, “The Steiner problem in graphs” Networks, Vol.1 pp. 195– 207, 1972.

[2] Bang Ye Wu and Kun-Mao Chao, “Spanning tree and optimization problem”, in Chapman & Hall/CRC Press, USA, 2004.

[3] G.Robins, A. Zelikovsky, “Improved Steiner tree approximation in graphs” Proceedings of the Eleventh Annual ACM-SIAM Symposium on Discrete Algorithms ,SODA ,pp. 770–779. ACM Press, New York, 2000.
[4] Stefan Hougardy, Hans Jurgen Promel, “1.598 Approximation Algorithm for the Steiner problem in Grpahs”, Proceedings of Tenth annual ACM-SIAM Symposium on Discrete Algorithm, pp 448 -453, 1999.
[5] Gabriel Robins, Alexander Zelikovsky, “Improved Steiner tree approximation in graph”, In Proceedings of the 11th Annual ACM-SIAM Symposium on Discrete Algorithms (SODA 2000), pp. 770–779. ACM/SIAM, New York 2000.
[6] Bruno Escoffier, Martin Milani, Vangelis Th Paschos,”Simple and fast reoptimization for the Steiner tree problem”, Algorithmic Oper. Res. 4(2), pp 86–94 2009.
[7] B¨ockenhauer, H.J Hromkoviˇc, J. Kr´aloviˇc, R. M¨omke, T, Rossmanith, “Reoptimization of Steiner trees: Changing the terminal set”, Theoretical Computer Science Volume 410, Issue 36, Pages 3428-3435, 31 August 2009.
[8] David Bilo , Hans Joachim B¨ockenhauer, Juraj,Hromkovic, Richard Kralovic, Tobis M¨omke, Peter Widmayer, Anna Zych, “Reoptimization of Steiner trees” , Gudmundsson, J. (ed.) SWAT 2008, LNCS Springer, Heidelberg , vol. 5124, pp. 258–269, 2008.
[9] Hans-Joachim Böckenhauer, Juraj Hromkovic, Ralf Klasing, Sebastin Seibert walter Under, ”Approximation algorithms for the TSP with sharpened triangle inequality “ RWTH Aachen, 52056 Aachen, Germany, Information processing Letters 75,pp 133 -138 2000.
[10] Piotr Berman, Marek Karpinski, Alex Zelikovsky, “A Factor 3/2 Approximation for Generalized Steiner Tree Problem with Distances One and Two“, SpringerLink Lecture Notes in Computer Science, Volume 6506/2010, pp.15-24, 2010.
[11] Subhash Panwar, Suneeta Agarwal, “ Fast Reoptimization of Steiner Tree”, Advance Computing Conference (IACC), 2010 IEEE 2nd International,pp.1-3, 2010.
[12] Hans-Joachim Böckenhauer, Karin Freiermuth, Juraj Hromkovič, Tobias Mömke, Andreas Sprock and Björn Steffen, “The Steiner Tree Reoptimization Problem with Sharpened Triangle Inequality”, Lecture Notes in Computer Science, Volume 6078/2010, pp.180-191,2010.
[13] Hans-Joachim Bockenhauer, Juraj Hromkovic, Tobias M omke, Peter Widmaye, “On the Hardness of Reoptimization”,SOFSEM 2008. LNCS, vol. 4910, pp. 50–65. Springer, Heidelberg, 2008.
[14] Chin Lung Lua, Chuan Yi Tangb, Richard Chia-Tung Leec,” The Full Steiner Tree Problem”, Elsevier, Theoretical computer Science (306) pp.55-67, 2003.
[15] H.J.promel and A.Steger, RNC approximation algorithm for Steiner Tree Problem, Proceedings 14th Annual symposium on Theoretical aspects of Computer Science, 559-570, 1997
[16] Aaron Archer, MohammadHossein Bateni, MohammadTaghi Hajiaghayi,Howard Karloff, ”Improved Approximation Algorithms for prize collection Steiner tree and TSP”, Foundations of Computer Science, 2009, FOCS '09. 50th Annual IEEE Symposium on, pp. 427 – 436, 2009.
[17] YuanHsieh,Wen-Hao,”On the Partial Terminal Steiner Tree Problem “, International Symposium on Parallel Architectures, Algorithms, and Networks, 2008. I-SPAN 2008. , pp173 -177, 2008.
[18] PiotrBerman, MarekKarpinski, AlexanderZelikovsky,“1.25-Approximation Algorithm for Steiner Tree Problem with Distances 1 and 2”, Springer-Verlag Berlin Heidelberg ,LNCS 5664, pp. 86–97, 2009.
[19] F. Hwang, D.Richards, P.Winter, ”The Steiner tree problem, in Annals of Discrete Mathematics”, Vol.53 North-Holland,1992.
[20] C Archetti, L.Bertazzi, M.G.Speranza, “Reoptimizing the traveling salesman problem”, Networks 42(3),pp. 154–159, 2003.
[21] C. Archetti, L.Bertazzi, M.G.Speranza, ”Reoptimizing the 0-1 knapsack problem.” Tech. Rep. 267, University of Brescia ,2006.
[22] G.Ausiello, B.Escoffier, J.Monnot, V.T.Paschos, “Reoptimization of minimum and maximum traveling salesman’s tours”, Arge, L., Freivalds, R.V. (eds.) SWAT , LNCS, vol. 4059, pp. 196–207. Springer, Heidelberg, 2006.
[23] D.Bilo, H.J.Bockenhauer, D.Komm, R.Kralovi, T.Momke,S.Seibert, A.Zych, “Reoptimization of the shortest common superstring problem”, Kucherov, G., Ukkonen, E. (eds.) CPM, LNCS, vol. 5577, pp. 78–91. Springer, Heidelberg , 2009.
[24] V.V. Vazirani, “Approximation Algorithms” Springer, Heidelberg , 2004.
[25] M.Karpinski and A.Zelikovsky, New Approximation algorithms for Steiner Tree, Journal of Combinatorial Optimization, 47-65 ,1997
[26] P.Berman, V. Ramaiyer, improved approximation for the Steiner tree problem , journal of Algorithm, 33-41,1998.
[27] Chin Lung Lua, Chuan Yi Tangb, Richard Chia-Tung Leec,” The Full Steiner Tree Problem”, Elsevier, Theoretical computer Science (306) pp.55-67, 2003.
[28] Sun Yuan Hsieh, Huang-Ming Gao,” Hardness and Approximation of the Selected-Leaf-Terminal Steiner Tree Problem”, Parallel and Distributed Computing, Applications and Technologies, PDCAT '06, Seventh international conference pp. 565-568, 2006.
[29] A.Zelkivosky, An 11/6 Approximation algorithm for the network problem, Algorithmica 9,463-470 , 1993
[30] M.W. Schacter, “Scheduling with forbidden sets”, Discrete Appl. Math. 72(1-2),pp.,155–166 ,1997.
[31] H.Takahashi, A.Matsuyam,”An Approximate solution for Steiner Problem in Grpahs”, Math. Jap24, pp. 573-577, 1980.
[32] A.Zelikovsky, Better approximation bounds for the network, Technical report, CS 96-06, University of Virginia.