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On the tarig transform and ordinary differential equation with variable coefficients

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ABSTRACT

Tarig transform, whose fundamental properties are presented in this paper, is little known and not widely used .Here Tarig transform used to solve ordinary differential equation with variable coefficients without resorting to a new frequency domain.

Introduction

A new integral transform, called Tarig transform defined for functions of exponential order, is proclaimed. We Consider function in the set A, defined by:

$$A=\left\{ f(t): \exists M, k_1, k_2>0, \left|f(t)\right|<Me^{\frac{|t|}{k_j}}, \ if \ \ t\in(-1)^j \ X\left[0,\infty\right) \right\}$$

For a given function in the set *A*, the constant *M* must be finite number, and k_1 , k_2 may be finite or infinite.

Tarig transform defined by the integral equations,

$$T\left[f(t)\right]=F(u)=\frac{1}{u}\int_0^\infty f(t)e^{\frac{-t}{u^2}}dt \ , \quad t\geq 0 \ , u\neq 0$$

The variable u in this transform is used to factor of the variable t in the argument of the function f . this transform has deeper Connection with the Laplace transform. We also present many different of properties of this new transform and Sumudu transform.

The purpose of this study is to show the applicability of this interesting new transform and its efficiency to solving differential equations with the variable coefficients.

Theorem 1:

If $T\left[f(t)\right]=F(u)$ then:

$$(i) \ T\left[f'(t)\right]=\frac{F(u)}{u^2}-\frac{1}{u}f(0) \quad (ii) \ T\left[f''(t)\right]=\frac{F(u)}{u^4}-\frac{1}{u^3}f(0)-\frac{1}{u}f'(0)$$

$$(iii) \ T\left[f^{(n)}(t)\right]=\frac{F(u)}{u^{2n}}-\sum_{i=1}^{n}u^{2(i-n)-1}f^{(i-1)}(0)$$

Proof:

$(i)\ T\left[f'(t)\right]=\frac{1}{u}\int_0^\infty f'(t)e^{\frac{-t}{u^2}}dt$. Integrating by parts to find that:

$$T\left[f'(t)\right]=\frac{1}{u}\left\{-f(0)+\frac{1}{u}T\left[f(t)\right]\right\}$$

And $$T\left[f'(t)\right]=\frac{F(u)}{u^2}-\frac{1}{u}f(0)$$

$(ii)\ By(i)\ T\left[G'(t)\right]=\frac{T\left[G(t)\right]}{u^2}-\frac{1}{u}G(0)$. Let $G(t)=f'(t)$. then:

$$T\left[f''(t)\right]=\frac{T\left(f'(t)\right)}{u^2}-\frac{1}{u}f'(0)=\frac{1}{u^2}\left[\frac{F(u)}{u^2}-\frac{1}{u}f(0)\right]-\frac{1}{u}f'(0) \quad and$$

$$T\left[f''(t)\right]=\frac{F(u)}{u^4}-\frac{1}{u^3}f(0)-\frac{1}{u}f'(0)$$

The generalization to nth order derivatives in (iii) can be proved by using mathematical induction.

Theorem 2:

If $T\left[f(t)\right]=G(u)$, and $L\left[f(t)\right]=F(s)$ then:

$G(u)=\dfrac{F\left(\frac{1}{u^2}\right)}{u}$ where $F(s)$ is the Laplace transform of $f(t)$.

Proof:

$T\left[f(t)\right]=\int_0^\infty f(ut)e^{\frac{-t}{u}}dt=G(u)$ Let $w=ut$, then we have:

$$G(u)=\int_0^\infty f(w)e^{\frac{-w}{u^2}}\frac{dw}{u}=\frac{F\left(\frac{1}{u^2}\right)}{u}$$

Theorems 3:

If $T\left[f(t)\right]=\frac{1}{u}\int_0^\infty f(t)e^{\frac{-t}{u^2}}dt=F(u)$, then:

1- $T\left[tf(t)\right]=\frac{1}{2}\left[u^3\frac{d}{du}F(u)+u^2F(u)\right]$

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2- $T\left[tf'(t)\right]=\frac{1}{2}\left[u^3\frac{d}{du}\frac{F(u)}{u^2}+F(u)\right]$

3- $T\left[tf''(t)\right]=\frac{1}{2}\left[u^3\frac{d}{du}\frac{F(u)}{u^4}+\frac{F(u)}{u^2}+\frac{2f(0)}{u}\right]$

4- $T\left[tf^{(n)}(t)\right]=\frac{1}{2}\left[u^3\frac{d}{du}\frac{F(u)}{u^{2n}}+\frac{F(u)}{u^{2n-2}}+\sum_{i=0}^{n-1}\frac{\left[2(n-i-1)f^{(i)}(0)\right]}{u^{2n-(2i+3)}}\right]$

Proof:

1- $F(u)=\frac{1}{u}\int_0^\infty f(t)\,e^{\frac{-t}{u^2}}\,dt$

$$\frac{dF(u)}{du}=\frac{2}{u^4}\int_0^\infty t\,f(t)\,e^{\frac{-t}{u^2}}dt-\frac{1}{u^2}\int_0^\infty f(t)\,e^{\frac{-t}{u^2}}dt$$

$$\frac{1}{u}\int_0^\infty t\,f(t)e^{\frac{-t}{u^2}}dt=\frac{1}{2}\left[u^3\frac{dF(u)}{du}+u^2F(u)\right]$$

$$T\left[t\,f(t)\right]=\frac{1}{2}\left[u^3\frac{dF(u)}{du}+u^2F(u)\right]$$

2- Let $f(t)=f'(t)$ in to (1) we get:

$$T\left[t\,f'(t)\right]=\frac{1}{2}\left[u^3\frac{d}{du}\left(\frac{F(u)}{u^2}-\frac{f(0)}{u}\right)+F(u)-uf(0)\right]=\frac{1}{2}\left[u^3\frac{d}{du}\frac{F(u)}{u^2}+F(u)\right]$$

3- let $f(t)=f''(t)$ into (1) we get:

$$T\left[t\,f''(t)\right]=\frac{1}{2}\left[u^3\frac{d}{du}\frac{F(u)}{u^4}+\frac{F(u)}{u^2}+\frac{2f(0)}{u}\right]$$

4- Let $f(t)=f^{(n)}(t)$ into (1) we have:

$$T\left[tf^{(n)}(t)\right]=\frac{1}{2}\left[u^3\frac{d}{du}\left(\frac{F(u)}{u^{2n}}-\sum_{j=0}^{n-1}\frac{f^i(0)}{u^{2n-2i-1}}\right)+\frac{F(u)}{u^{2n-2}}-\sum_{j=0}^{n-1}\frac{f^i(0)}{u^{2n-2i-3}}\right]=$$

$$=\frac{1}{2}\left[u^3\frac{d}{du}\frac{F(u)}{u^{2n}}+\frac{F(u)}{u^{2n-2}}+\sum_{j=0}^{n-1}(2n-2i-1)\frac{f^{(i)}(0)}{u^{2n-2i-3}}-\sum_{i=0}^{n-1}\frac{f^{(i)}(0)}{u^{2n-2i-3}}\right]=$$

$$\frac{1}{2}\left[u^3\frac{d}{du}\frac{F(u)}{u^{2n}}+\frac{F(u)}{u^{2n-2}}+\sum_{i=0}^{n-1}\frac{2(n-i-1)f^{(i)}(0)}{u^{2n-(2i+3)}}\right]$$

Application of Tarig Transform to Ordinary Differential Equations with Variable Coefficients

As stated in the introduction of this paper, the Tarig transform can be used as an effective tool. For analyzing the basic characteristics of a linear system governed by the differential equation in response to initial data. The following examples illustrate the use of Tarig transform in solving certain initial value problems described by ordinary differential equations with variable coefficients.

Example 1:

Consider the following first order differential equation,

$$2ty'-y=3t^2$$

(1) With the initial condition:

$$y(0)=0$$

(2)

Solution:

Taking Tarig transform of eq (1) we have:

$$u^3\frac{d}{du}\left(\frac{y(u)}{u^2}\right)+y(u)-y(u)=6u^5$$

Where that $y(u)$ is Tarig transform of $y(t)$.

$$y'(u)-\frac{2}{u}y(u)=6u^4 \qquad (3)$$

The solution of eq (3) is $y(u)=2u^5+cu^2$

$$y(t)=F^{-1}\left[2u^5+cu^2\right]=t^2+c\sqrt{t} \qquad (4)$$

c is constant

Substituting eq (2) into eq (4) we get: $c=0\,then: y(t)=t^2$

Example 2:

Consider the following second differential equation,

$$ty''+y'=4t \qquad (5)$$

With the initial condition:

$$y(0)=1 \qquad (6)$$

Solution:

By using Tarig transform into eq (5) we have:

$$\frac{1}{2}\left[u^3\frac{d}{du}\frac{y(u)}{u^4}+\frac{y(u)}{u^2}+\frac{2y(0)}{u}\right]+\frac{y(u)}{u^2}-\frac{y(0)}{u}=4u^3 \qquad (7)$$

eq (7) can be written in the form $\frac{y'(u)}{u}-\frac{1}{u^2}y=8u^3$

Then: $\frac{d}{du}\left[\frac{1}{u}y\right]=8u^3$ or $y(u)=2u^5+cu$

And $y(t)=F^{-1}\left[2u^5+cu\right]=t^2+c$

By using $y(0)=1$ we get: $c=1$ then: $y(t)=1+t^2$

Example 3:

Consider the following differential equation with variable coefficients:

$$t\,y''+2y'+t\,y=t^3+6t \qquad (8)$$

With the initial condition:

$$y(0)=0 \qquad (9)$$

Solution:

Applying Tarig transform to eq (8) yields:

$$\frac{1}{2}\left[u^3\frac{d}{du}\frac{y(u)}{u^4}+\frac{y(u)}{u^2}+\frac{2y(0)}{u}\right]+$$

$$\frac{2y(u)}{u^2}-\frac{2y(0)}{u}+\frac{1}{2}\left[u^3\frac{d}{du}y(u)+u^2y(u)\right]=6u^7+6u^3$$

Or

$$\left(\frac{1}{u}+u^3\right)y'(u)+\left(\frac{1}{u^2}+u^2\right)y(u)=12u^7+12u^3$$

Or

$$y'(u)+\frac{1}{u}y=12u^4 \qquad (10)$$

The solution of eq(10) is,

$$y(u)=2u^5+\frac{c}{u}\ ,\quad \text{c is constant.}$$

Then: $$y(t)=F^{-1}\left[2u^5+\frac{c}{u}\right]=t^2+c\,\delta(t)$$

By using $y(0)=0$, we get: $c=0$, then: $y(t)=t^2$

Conclusion:

Application of Tarig transform to Solution of ordinary differential equation with variable Coefficients has been demonstrated.

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Appendix Tarig Transform of Some Functions

S.N0.	$f(t)$	$F(u)$
1	1	u
2	t	u^3
3	e^{at}	$\frac{u}{1-au^2}$
4	t^n	$n!\,u^{2n+1}$
5	t^a	$\Gamma(a+1)u^{2a+1}$
6	$\sin at$	$\frac{au^3}{1+a^2u^4}$
7	$\cos at$	$\frac{u}{1+a^2u^4}$
8	$\sinh at$	$\frac{au^3}{1-a^2u^4}$
9	$\cosh at$	$\frac{u}{1-a^2u^4}$
10	$H(t-a)$	$ue^{\frac{-a}{u^2}}$
11	$\delta(t-a)$	$\frac{1}{u}e^{\frac{-a}{u^2}}$