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A new decomposition method for elastic constant tensor to study the anisotropy of construction materials; tool steel and rock types

Cigdem Dinckal

Department of Civil Engineering, Cankaya University, Ankara, Turkey.

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ABSTRACT

An innovative method for the decomposition of the elastic constant tensor into its irreducible parts is presented. The norm concept of elastic constant tensor, norm ratios and irreducible decomposed parts of elastic constant tensor are used to study the anisotropy of tool steel, rock types and the relationship of their structural properties and other properties with their anisotropy are given. Finally the results are indicated.

Introduction

The constitutive relation for linear anisotropic elasticity, defined by using stress and strain tensors, is the generalized Hooke's law[1]

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl}, \qquad \varepsilon_{ij} = S_{ijkl}\sigma_{kl}. \qquad (1)$$

This formula demonstrates the well known general linear relation between the stress tensor whose components are and the strain tensor (symmetric second rank tensor) whose components are the components of elastic constant tensor (elasticity tensor) and is the elastic compliance tensor. satisfies three important symmetry restrictions. These are

$$C_{ijkl} = C_{jikl} \quad C_{ijkl} = C_{ijlk} \quad C_{ijkl} = C_{klij}, \qquad (2)$$

which follow from the symmetry of the stress tensor, the symmetry of the strain tensor and the elastic strain energy. These restrictions reduce the number of independent elastic constants from 81 to 21. Consequently, for anisotropic materials (with triclinic symmetry) the elastic constant tensor has 21 independent components. Elastic compliance tensor possesses the same symmetry properties as the elastic constant tensor and their connection is given by [2]:

$$C_{ijkl}S_{klmn} = \frac{1}{2}\left(\delta_{im}\delta_{jn} + \delta_{in}\delta_{jm}\right), \qquad (3)$$

Where is the Kronecker delta, the Einstein summation convention over repeated indices are used and indices run from 1 to 3 unless otherwise stated.

Schouten [3] has shown that can be decomposed into two scalars, two deviators, and one-nonor parts. The same decomposition in terms of the irreducible representations of the three dimensional rotation group has been given in as:

$$2D_0 + 2D_2 + D_4 \qquad (4)$$

Where the subscripts denote the weight of the representation.

By applying the symmetry conditions (2) to the decomposition results obtained for a general fourth-rank tensor, the following reduction spectrum for the elastic constant tensor is obtained. It contains two scalars, two deviators, and one nonor parts:

$$C_{ijkl} = C_{ijkl}^{(0;1)} + C_{ijkl}^{(0;2)} + C_{ijkl}^{(2;1)} + C_{ijkl}^{(2;2)} + C_{ijkl}^{(4;1)}, \qquad (5)$$

where

$$C_{ijkl}^{(0;1)} = \frac{1}{9}\delta_{ij}\delta_{kl}C_{ppqq}, \qquad (6)$$

$$C_{ijkl}^{(0;2)} = \frac{1}{90}(3\delta_{ik}\delta_{jl} + 3\delta_{il}\delta_{jk} - 2\delta_{ij}\delta_{kl})(3C_{pqpq} - C_{ppqq}), \qquad (7)$$

$$C_{ijkl}^{(2;1)} = \frac{1}{5}(\delta_{ik}C_{jplp} + \delta_{ik}C_{jplp} + \delta_{il}C_{jpkp} + \delta_{il}C_{ipkp}) - \frac{2}{15}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})C_{pqpq}, \qquad (8)$$

$$\begin{aligned} C_{ijkl}^{(2;2)} = {} & \frac{1}{7}\delta_{ij}(5C_{klpp} - 4C_{kplp}) + \frac{1}{7}\delta_{kl}(5C_{ijpp} - 4C_{ipjp}) - \frac{2}{35}\delta_{ik}(5C_{jlpp} - 4C_{jplp}) \\ & - \frac{2}{35}\delta_{jl}(5C_{ikpp} - 4C_{ipkp}) - \frac{2}{35}\delta_{il}(5C_{jkpp} - 4C_{jpkp}) - \frac{2}{35}\delta_{jk}(5C_{ilpp} - 4C_{iplp}) + \\ & \frac{2}{105}(2\delta_{jk}\delta_{il} + 2\delta_{ik}\delta_{jl} - 5\delta_{ij}\delta_{kl})(5C_{ppqq} - 4C_{pqpq}), \end{aligned} \qquad (9)$$

$$\begin{aligned} C_{ijkl}^{(4;1)} = {} & (C_{ijkl} + C_{iklj} + C_{iljk})/3 - [(C_{ijmm} + 2C_{imjm})\delta_{kl} + (C_{klmm} + 2C_{kmlm})\delta_{ij} + \\ & (C_{ikmm} + 2C_{imkm})\delta_{jl} + (C_{jlmm} + 2C_{jmlm})\delta_{ik} + (C_{ilmm} + 2C_{imlm})\delta_{jk} + (C_{jkmm} + C_{jmkm})\delta_{il}]/21 + \\ & (C_{ppmm} + 2C_{pmpm})(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})/105, \end{aligned} \qquad (10)$$

where $C_{ijkl}^{(0;1)}$, $C_{ijkl}^{(0;2)}$ are scalar parts, $C_{ijkl}^{(2;1)}$, $C_{ijkl}^{(2;2)}$ are deviators and $C_{ijkl}^{(4;1)}$ is the nonor part. These parts are orthonormal to each other. The indices are abbreviated according to the replacement rule given in the following table [1]:

The Norm Concept

Generalizing the concept of the modulus of a vector, norm of a Cartesian tensor (or the modulus of a tensor) is defined as the square root of the contracted product over all indices with itself:

Euclidean norm of a Cartesian tensor is defined as the square root of the contracted product over all the indices with itself, which is given as follows

Tele:

E-mail addresses: cigdemdinckal2004@yahoo.com

$$N = \|C\| = \{C_{ijkl...}C_{ijkl...}\}^{\frac{1}{2}} \quad (11)$$

Denoting elastic constant tensor C_{ijkl} by C_n, the square of the norm is expressed as [4]:

$$N^2 = \|C\|^2 = \sum_{j,q} \|C^{(j;q)}\|^2 = \sum_{(n)} C_{(n)}C_{(n)} = \sum_{(n),j,q} C^{(j;q)}_{(n)}C^{(j;q)}_{(n)}. \quad (12)$$

This definition is consistent with the reduction of the tensor in Cartesian formulation when all the irreducible parts are embedded in the original rank-n tensor space. Since the norm of a Cartesian tensor is an invariant quantity, The followings are suggested:

Rule1. The norm of a Cartesian tensor may be used as a criterion for representing and comparing the overall effect of a certain property of the same or different symmetry. The larger the norm value, the more effective the property is. It is known that the anisotropy of the materials, i.e., the symmetry group of the material and the anisotropy of the measured property depicted in the same materials may be quite different. Obviously, the property, tensor must show, at least, the symmetry of the material. For example, a property, which is measured in a material, can almost be isotropic but the material symmetry group itself may have very few symmetry elements. It is known that, for isotropic materials, the elastic constant tensor has two irreducible parts, i.e., two scalar parts, so the norm of the elastic constant tensor for isotropic materials depends only on the norm of the scalar parts, i.e. $N = N_s$. Hence the ratio $\frac{N_s}{N} = 1,$ for isotropic materials. For anisotropic materials, the elastic constant tensor additionally contains two deviator parts and one nonor part, so it can be defined $\frac{N_d}{N}$ for the deviator irreducible parts and $\frac{N_n}{N}$ for nonor parts. Generalizing this to irreducible tensors up to rank four, we can define the following norm ratios: $\frac{N_s}{N}$ for scalar parts, $\frac{N_v}{N}$ for vector parts, $\frac{N_d}{N}$ for deviator parts, $\frac{N_{sc}}{N}$ for septor parts and $\frac{N_n}{N}$ for nonor parts.

Norm ratios of different irreducible parts represent the anisotropy of that particular irreducible part, they can also be used to asses the anisotropy degree of
a material property as a whole, the following two more rules are suggested:

Rule 2. When N_s is dominating among norms of irreducible parts: the closer the norm ratio $\frac{N_s}{N}$ is to one, the closer the material property is isotropic.

Rule3. When N_s is not dominating or not present, norms of the other irreducible parts can be used as a criterion. But in this case the situation is reverse; the larger the norm ratio value we have, the more anisotropic the material property is.

The square of the norm of the elastic constant tensor C_{mn} is:

$$\|N\|^2 = \sum_{mn}\left(C^{(0;1)}_{mn}\right)^2 + \sum_{mn}\left(C^{(0;2)}_{mn}\right)^2 + 2\sum_{m,n}\left(C^{(0;1)}_{mn}\cdot C^{(0;2)}_{mn}\right) + \sum_{mn}\left(C^{(2;1)}_{mn}\right)^2 + \sum_{mn}\left(C^{(2;2)}_{mn}\right)^2$$

$$+ 2\sum_{mn}\left(C^{(2;1)}_{mn}\cdot C^{(2;2)}_{mn}\right) + \sum_{mn}\left(C^{(4;1)}_{mn}\right)^2 \quad (13)$$

Let us consider the irreducible decompositions of the elastic constant tensor in the following materials.

Results and Conclusions

1) From Table (3), considering the ratio $\frac{N_s}{N}$ it can be said that Micaschist is the most anisotropic material with the lowest ratio $\frac{N_s}{N}$ among the rock types. By regarding the effect of value of norm which is higher in the case of Eclogite, therefore it is obvious that Eclogite is the elastically strongest among the other rocks.

2) Also from Table (3), it is observed that Hardened tool steel is more anisotropic than the normal one with higher value of $\frac{N_n}{N}$. Furthermore, by considering the effect of N, Normal tool steel is elastically stronger than Hardened one. Since its norm value is higher than Hardened tool steel.

In conclusion, both materials (tool steel and rocks) are listed with increasing anisotropy degrees, that is from larger $\frac{N_s}{N}$ to smaller values. Among these five materials, normal tool steel is the elastically strongest and Slate is the elastically most anisotropic material.

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Table 1 Abbreviation of indices for four index and double index notations

four index notation	11	22	33	23, 32	13, 31	12, 12
double index notation	1	2	3	4	5	6

Table 2 Elastic Constants of the Materials (GPa) [5]

Materials(From Transversely Isotropic System)	c_{11}	c_{12}	c_{13}	c_{33}	c_{44}
Tool Steels:					
Normal	289	116	117	284	84.5
Hardened	277	113	112	272	80.3
Rocks:					
Micaschist	165	31.1	50	61.8	39.6
Slate	87	-54.6	47.5	94.1	14.9
Eclogite	116	42	41	50.9	19.6

Table 3 Norm and Norm Ratios for Materials

Materials	N_s	N_d	N_n	N	$\frac{N_s}{N}$	$\frac{N_d}{N}$	$\frac{N_n}{N}$
Tool Steels:							
Normal	592.461	4.658	0.384	592.479	0.99996	0.0079	0.0006
Hardened	567.798	4.438	1.461	567.817	0.99996	0.0078	0.0026
Rocks:							
Micaschist	152.791	16.753	5.078	153.790	0.99350	0.1089	0.0330
Slate	260.413	27.829	10.385	362.102	0.99356	0.1062	0.0396
Eclogite	181.220	63.857	23.217	193.539	0.93653	0.3299	0.1199