



# New algorithm for graph with graphs vertices

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**ABSTRACT**

In this paper we will compute a new algorithm for new graph which its vertex is a graph.

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**Introduction**

**Definition and Background**

**Definition 1:**

**Abstract graphs:** An abstract graphs G is a diagram consisting of a finite non empty set of the elements, called "vertices" denoted by  $V(G)$  together with a set of unordered pairs of these elements, called "edges" denoted by  $E(G)$ . The set of vertices of the graph G is called "the vertex .set of G " and the list of edges is called "the edge .list of G " [2,3].

**Definition 2:**

Consider a geometric graph  $G(V,E)$  where  $G(V)=\{\{v_0,e_0\},\{v_1,e_1\},\{v_2,e_2\},\dots,\{v_n,e_n\}\}$  and  $E(G) = \{e^1\}$ , we are called this graph (graph with complex vertices).

To represent these graphs we must show that there are three types of these graphs:

- 1- Null graphs which their vertices are graphs.
- 2- Graphs which their vertices are similar. [1].

**Definition 3:**

Null graphs which their vertices are graphs:

We know that a null graph is a graph containing no edges and every vertex is isolated. By definition(1) we can define a new null graphs (which their vertex are graphs), consider the graph  $G_n(V^{\square})$  such that  $V^{\square}(G) = \{\{v_0, e_0\}, \{v_1, e_1\}, \{v_2, e_2\}, \dots, \{v_n, e_n\}\}$  and  $E(G) = \Phi$ . See Fig.(1) [1].

**Definition 4:**

Graphs which their vertices are similar: Consider the geometric graph  $G(V, E)$  where  $V(G) = \{V^0, V^1\}$  where  $V^{\square} = \{\{v_0, e_0\}, \{v_1, e_1\}, \{v_2, e_2\}, \dots, \{v_n, e_n\}\}$  and  $V^1 = \{\{v_0, e_0\}, \{v_1, e_1\}, \{v_2, e_2\}, \dots, \{v_n, e_n\}\}$  where  $V^0$  is the same as  $V^1$  to represent this graph we will take the smallest circle which contains vertices of  $V^0$  and the smallest circle which contains vertices of  $V^1$  and connected between them this connect is  $E(G) = \{e^1\}$ . [1].

**Definition 5:**

Spanning tree for a graph G is a subgraph of G that contains every vertex of G and is a tree [4].

**Main results:**

We will discuss two new algorithms for graph with graphs vertices.

I. algorithm for null graph which vertices is a graph:

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Create a subgraph that visit each outer vertices  $V^n$  then its internal vertices  $V^{nm}$

Proceeding from vertex to vertex but moving along internal spanning tree T of that graph.

1. initialized T to have all vertices of G "which have outer vertices".

2. select the smallest superscript k for ,  $1 \leq k \leq i, 0 \leq n \leq j$ .  $V^{n \cdot nk}$  has not already been visited.

If no superscript is found , then,

Go to step 3 , otherwise,

Perform the following:

2a. attach the internal edge  $\{V^{1m1}, V^{n2m2}\}$  to T, and visit  $V^{1nk}$ .

2b. assign  $V^{1nk}$  to  $V^{nmk}$  and ,

2c. return to step 2.

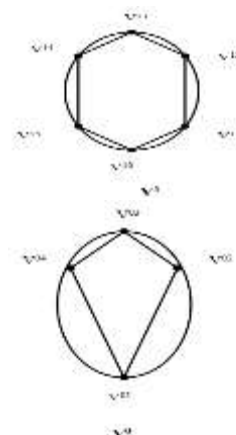
End while.

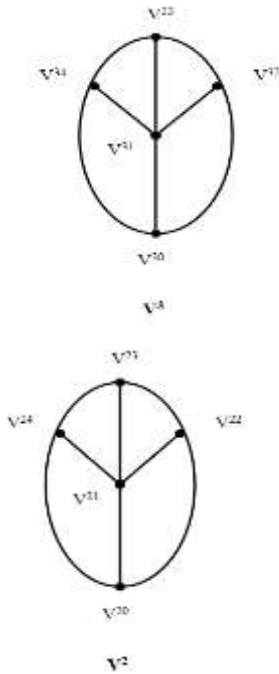
3. output T .

end algorithm.

**Example 1:**

For a null graph shown in fig(1) we can compute its algorithm as follows:





Fig(1)

**Input:**

Null graph with graphs vertices  $V^{nm}$ ,  $0 \leq n \leq 3$ ,  $0 \leq m \leq 5$ .

**Algorithm body:**

Create a subgraph that visit each outer vertices  $V^n$  then its internal vertices  $V^{nm}$

Proceeding from vertex to vertex but moving along internal spanning tree of that graph.

1. initialized  $T$  to have all vertices of  $G$  "which have outer vertices".

2. select  $V^0$  and visit all internal vertices  $V^{01}$  to  $V^{03}$ .

2a. attach the internal edges  $\{V^{01}, V^{02}, \dots, V^{04}, V^{01}\}$  to  $T$ .

2b. go to step 2 for the other vertices  $V^2$  to  $V^4$ .

End while.

3. output  $T$ .

end algorithm.

**Algorithm for graph which vertices is graph:**

**Input:**

Connected graph  $G(V, E)$ ,  $V(G) = \{V^0, V^1\}$ ,  $V^0 = \{V^0, e^0, \dots, V^n, e^n\}$ ,  $V^1 = \{V^1, e^1, \dots, V^n, e^n\}$ .

**Algorithm body:**

Create a subgraph that visit each outer vertices  $V^n$  then its internal vertices  $V^{nm}$

Proceeding from vertex to vertex but moving along internal spanning tree  $T$  of that graph, then along its outer spanning tree  $T'$ .

1. initialized  $T$  to have all the vertices of  $G$  and no edge.

2. let  $E$  the set of all edges of  $G$ ,  $m = 0$ .

3. while  $(m \leq n-1)$

3a. visit outer vertices  $V^n$ , then visit internal vertices  $V^{nmi}$ .

3b. attach the internal edge  $\{V^{1m1}, V^{n2m2}\}$  to  $T$ , and visit  $V^{1mk}$ .

3c. attach the outer edge  $\{V^1, V^2\}$  to  $T'$  and visit  $V^{nmk}$ ,

3d. return to step 3.

End while.

4. output  $T, T'$ .

end algorithm.

**Example 2:**

Consider a graph shown in fig(2).

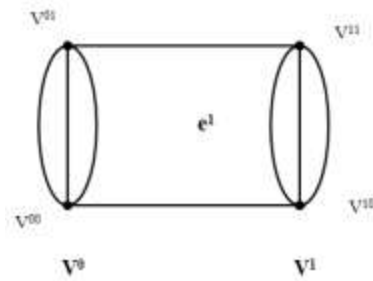


Fig (2)

**Input:**

Connected graph  $G(V, E)$ ,  $V(G) = \{V^0, V^1\}$ ,  $V^0 = \{V^{00}, V^{01}\}$ ,  $V^1 = \{V^{10}, V^{11}\}$ .

**Algorithm body:**

Create a subgraph that visit each outer vertices  $V^n$  then its internal vertices  $V^{nm}$

Proceeding from vertex to vertex but moving along internal spanning tree  $T$  of that graph, then along its outer spanning tree  $T'$ .

1. initialized  $T$  to have vertex  $v^0$ .

2. let  $E$  the set of all edges of  $G$ ,  $m = 0$ .

3. while  $(m \leq 1)$ .

3a. visit outer vertex  $V^0$

then visit  $V^{00}, V^{01}$ .

3b. attach the internal edge  $\{V^{00}, V^{01}\}$  to  $T$ .

3c. attach the outer edge  $e^1$  to  $T'$  and visit  $V^1$ .

3d. return to step 3.

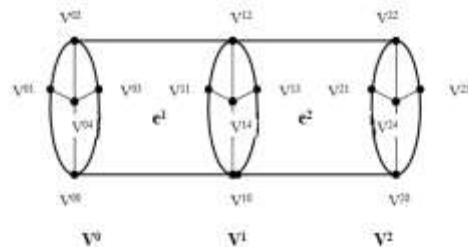
End while.

4. output  $T, T'$ .

end algorithm.

**Example 3:**

For a graph shown in fig(3).we have:



Fig(3)

**Input:**

Connected graph  $G(V, E)$ ,  $V(G) = \{V^0, V^1, V^2\}$ ,  $V^0 = \{V^{00}, V^{04}, \{V^{04}, V^{03}\}, \{V^{04}, V^{02}\}, \{V^{04}, V^{01}\}\}$ ,  $V^1 = \{V^{10}, V^{14}, \{V^{14}, V^{13}\}, \{V^{14}, V^{12}\}, \{V^{14}, V^{11}\}\}$ ,  $V^2 = \{V^{20}, V^{24}, \{V^{24}, V^{23}\}, \{V^{24}, V^{22}\}, \{V^{24}, V^{21}\}\}$ .

**Algorithm body:**

Create a subgraph that visit each outer vertices  $V^n$  then its internal vertices  $V^{nm}$

Proceeding from vertex to vertex but moving along internal spanning tree  $T$  of that graph, then along its outer spanning tree  $T'$ .

1. initialized  $T$  to have vertex  $v^0$ .

2. let  $E$  the set of all edges of  $G$ ,  $m = 0$ .

3. while  $(m \leq 2)$ .

3a. visit outer vertex  $V^0$

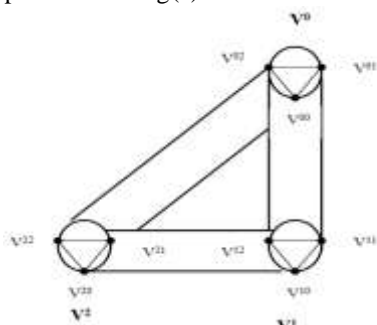
then visit  $V^{00}, V^{04}, V^{03}, V^{01}$ .

3b. attach internal vertices  $\{V^{00}, V^{04}\}, \{V^{04}, V^{03}\}, \{V^{04}, V^{02}\}, \{V^{04}, V^{01}\}$  to  $T$ .

- 3c. attach the outer edge  $e^l$  to  $T$  and visit  $V^l$ .
- 3d. return to step 3 for the other vertices.
- End while.
- 4. output  $T, T'$ .
- end algorithm.

**Example 4:**

For a graph shown in fig(4).



Fig(4)

**Input:**

Connected graph  $G(V, E)$ ,  $V(G) = \{V^0, V^1, V^2\}$ ,  $V^0 = \{\{V^{00}, V^{01}\}, \{V^{00}, V^{02}\}, \{V^{02}, V^{01}\}\}$ ,  
 $V^1 = \{\{V^{10}, V^{11}\}, \{V^{11}, V^{12}\}, \{V^{12}, V^{10}\}\}$ .  
 $V^2 = \{\{V^{20}, V^{21}\}, \{V^{21}, V^{22}\}, \{V^{20}, V^{22}\}\}$ .

**Algorithm body:**

Create a subgraph that visit each outer vertices  $V^n$  then its internal vertices  $V^{nm}_i$

Proceeding from vertex to vertex but moving along internal spanning tree  $T$  of that graph, then along its outer spanning tree  $T'$ .

- 1. initialized  $T$  to have vertex  $v^0$ .
- 2. let  $E$  the set of all edges of  $G$ ,  $m = 0$ .
- 3. while  $(m \leq 2)$ .
  - 3a. visit outer vertex  $V^0$ .  
then visit  $V^{00}, V^{01}, V^{02}$ .
  - 3b. attach internal vertices  $\{V^{00}, V^{01}\}, \{V^{01}, V^{02}\}, \{V^{00}, V^{02}\}$  to  $T$ .
  - 3c. attach the outer edge  $e^1$  to  $T'$  and visit  $V^1$ .
  - 3d. return to step 3 for the other vertices.
- End while.
- 4. output  $T, T'$ .
- end algorithm.

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