



## Combined effects of chemical reactions and heat generation/absorption on unsteady transient free convection MHD flow between two long vertical parallel plates through a porous medium with constant temperature and mass diffusion

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### ABSTRACT

The present work analyzes the influence of first order homogeneous chemical reactions on unsteady transient free convection flow of a viscous, incompressible, electrically conducting fluid between two long vertical parallel plates through a porous medium with heat generation/absorption in the presence of transverse magnetic field. The problem is solved analytically in closed form by Laplace transform technique and the expressions for velocity, temperature and concentration has been obtained. The velocity, temperature and concentration profiles are studied for different physical parameters like Schmidt number, Prandtl number, magnetic parameter, buoyancy ratio parameter, chemical reaction parameter, time, permeability parameter and heat generation/absorption parameter. The numerical values of skin-friction have been tabulated.

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### Introduction

The study of heat generation or absorption effects in moving fluid is important in view of several Physical problems, such as fluids undergoing endothermic or exothermic chemical reactions. There are two types of reactions - One is homogeneous chemical reactions and other is heterogeneous chemical reactions. Homogeneous reactions are chemical reactions in which the reactants are in the same phase, while heterogeneous reactions have reactants in two or more phases. Reactions that take place on the surface of a Catalyst of a different phase are also heterogeneous. A reaction is of order  $n$ , if the reaction rate is proportional to the  $n^{\text{th}}$  power of concentration. In particular, a reaction is of first order, if the rate of reaction is directly proportional to concentration itself. On the other hand, hydromagnetic incompressible viscous flow has many important engineering applications such as magnetohydrodynamic power generators and cooling of reactors. Also, its applications to problems in geophysics, astrophysics etc. Ostrach [9] has studied laminar free convection flow of a viscous incompressible fluid between two vertical walls with constant wall temperature. Ostrach [10] and Sparrow et al. [4] have studied the combined effect of a steady free and forced convection laminar flow and heat transfer between two vertical parallel walls. Singh et al. [1] have studied the transient free convection flow of a viscous incompressible fluid between two vertical parallel plates when the walls are heated asymmetrically. Lee [6] has studied a combined numerical and theoretical investigation of laminar natural convection heat and mass transfer in open vertical parallel plates with unheated entry

and unheated exit for various thermal and concentration boundary conditions.

Desrayaud and Lauriat [5] have studied the heat and mass transfer analogy for condensation of humid air in a vertical parallel plate channel. Narahari et al. [7] have studied the transient free convection flow between two vertical parallel plates with constant heat flux at one boundary and the other maintained at constant temperature. Jha et al. [3] have presented the transient free convection flow in a vertical channel as a result of symmetric heating of the channel walls. Sing and Paul [2] have presented the transient free convection flow of a viscous and incompressible fluid between two vertical parallel walls as a result of asymmetric heating or cooling of the walls. Narahari [8] has studied the transient free convection flow of a viscous incompressible fluid between two infinite vertical parallel plates in the presence of constant temperature and mass diffusion. We have already studied two MHD models, namely (i) the effect of a uniform transverse magnetic field on the unsteady transient free convection flow between two long vertical parallel plates with constant temperature and variable mass diffusion [11] and (ii) transient free convection MHD flow between two long vertical parallel plates through a porous medium with constant temperature and mass diffusion in the presence of chemical reactions [12].

The main object of the present investigation is to study the effect of first order homogeneous chemical reaction and heat generation/absorption on unsteady transient free convection MHD flow between two long vertical parallel plates with constant temperature and mass diffusion in a porous medium,

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when the fluid is viscous, incompressible and electrically conducting.

**Mathematical Analysis:**

Consider an unsteady transient free convection flow between two long vertical parallel plates through a porous medium with heat generation/absorption in the presence of first order homogeneous chemical reaction and transverse magnetic field. Assume that the magnetic Reynolds number is so small that the induced magnetic field can be neglected in comparison to the applied one. A magnetic field (fixed relative to the plates) of uniform strength  $B_0$  is assumed to be applied transversely to the plates. The  $x'$ -axis is considered along one of the vertical plates and the  $y'$ -axis is taken normal to the plates. Initially, at time  $t' \leq 0$ , the temperature of the fluid and the plates are same as  $T'_d$  and the concentration of the fluid is  $C'_d$ . At  $t' > 0$ , the temperature of the plate and concentration of the fluid at  $y' = 0$  are raised to  $T'_w$  and  $C'_w$  respectively, causing the flow of free convection currents. The governing equations under the usual Boussinesq's approximation are as follows:

$$\frac{\partial u'}{\partial t'} = g\beta(T' - T'_d) + g\beta^*(C' - C'_d) + \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho} u' - \frac{\nu}{K^*} u', \quad (1)$$

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} - Q^*(T' - T'_d), \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} - K'(C' - C'_d). \quad (3)$$

The initial and boundary conditions are as follows:

$$\left. \begin{aligned} t' \leq 0: & \quad u' = 0, T' = T'_d, C' = C'_d \quad \text{for } 0 \leq y' \leq d, \\ t' > 0: & \quad u' = 0, T' = T'_w, C' = C'_w \quad \text{at } y' = 0, \\ & \quad u' = 0, T' = T'_d, C' = C'_d \quad \text{at } y' = d, \end{aligned} \right\} \quad (4)$$

where  $u'$  is velocity of the fluid,  $g$  -acceleration due to gravity,  $\beta$  -volumetric coefficient of thermal expansion,  $t'$  -time,  $d$  -distance between two vertical plates,  $T'$  -temperature of the fluid,  $T'_d$  -temperature of the plate at  $y' = d$ ,  $\beta^*$  -volumetric coefficient of concentration expansion,  $C'$  -species concentration in the fluid,  $C'_d$  -species concentration at the plate  $y' = d$ ,  $\nu$  -the kinematic viscosity,  $y'$  -the coordinate axis normal to the plates,  $\rho$  -density,  $C_p$  -specific heat at constant pressure,  $k$  -thermal conductivity of the fluid,  $D$  -mass diffusion coefficient,  $T'_w$  -temperature of the plate at  $y' = 0$ ,  $C'_w$  -species concentration at the plate  $y' = 0$ ,  $B_0$  -uniform magnetic field,  $\sigma$  -electrical conductivity,  $K^*$  -permeability of the porous medium,  $Q^*$  -heat generation/absorption coefficient and  $K'$  -chemical reaction parameter.

Introducing the following non-dimensional quantities:

$$y = \frac{y'}{d}, \quad t = \frac{t' \nu}{d^2}, \quad u = \frac{u' \nu}{d^2 g \beta (T'_w - T'_d)} = \frac{u' d}{\nu Gr}, \quad Gr = \frac{g \beta (T'_w - T'_d) d^3}{\nu^2}, \quad \theta = \frac{T' - T'_d}{T'_w - T'_d}, \quad Pr = \frac{\mu C_p}{k},$$

$$C = \frac{C' - C'_d}{C'_w - C'_d}, \quad Gm = \frac{g \beta^* (C'_w - C'_d) d^3}{\nu^2}, \quad Sc = \frac{\nu}{D}, \quad N = \frac{Gm}{Gr}, \quad M = \frac{\sigma B_0^2 d^2}{\mu}, \quad \mu = \rho \nu,$$

$$K = \frac{K^*}{d^2}, \quad S = \frac{Q^* d^2}{\mu C_p}, \quad F = \frac{d^2 K'}{\nu}, \quad (5)$$

where  $u$  is the dimensionless velocity,  $y$  -dimensionless coordinate axis normal to the plates,  $t$  -dimensionless time,  $\theta$  -dimensionless temperature,  $C$  -dimensionless concentration,  $Gr$  -thermal Grashof number,  $Gm$  -mass Grashof number,  $\mu$  -coefficient of viscosity,  $Pr$  -the Prandtl number,  $Sc$  -the Schmidt number,  $N$  -buoyancy ratio parameter,  $M$  -magnetic parameter,  $F$  -dimensionless chemical reaction parameter,  $S$  -dimensionless heat generation/absorption coefficient and  $K$  -permeability parameter. then the model is transformed in to the following non-dimensional form of equations:

$$\frac{\partial u}{\partial t} = \theta + NC + \frac{\partial^2 u}{\partial y^2} - Mu - \frac{1}{K} u, \quad (6)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} - S\theta, \quad (7)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} - FC. \quad (8)$$

The initial and boundary conditions become:

$$\left. \begin{aligned} t \leq 0: & \quad u = 0, \theta = 0, C = 0 \quad \text{for } 0 \leq y \leq 1, \\ t > 0: & \quad u = 0, \theta = 1, C = 1 \quad \text{at } y = 0, \\ & \quad u = 0, \theta = 0, C = 0 \quad \text{at } y = 1. \end{aligned} \right\} \quad (9)$$

Applying the Laplace transform in equations (6), (7) and (8), we have:

$$\frac{d^2 \bar{u}}{dy^2} - (H + s)\bar{u} = -\bar{\theta} - N\bar{C}, \quad (10)$$

$$\frac{d^2 \bar{\theta}}{dy^2} - (S + s)Pr\bar{\theta} = 0, \quad (11)$$

$$\frac{d^2 \bar{C}}{dy^2} - (F + s)Sc\bar{C} = 0 \quad (12)$$

The boundary condition becomes:

$$\left. \begin{aligned} t > 0: & \quad \bar{u} = 0, \bar{\theta} = \frac{1}{s}, \bar{C} = \frac{1}{s} \quad \text{at } y = 0, \\ & \quad \bar{u} = 0, \bar{\theta} = 0, \bar{C} = 0 \quad \text{at } y = 1. \end{aligned} \right\} \quad (13)$$

The solution of equations (10), (11) and (12) with the boundary condition (13) are given by:

$$\bar{u}(y, s) = \frac{a_{11}}{sa_{12}} + \frac{Na_{11}}{sa_{13}} - \frac{\bar{\theta}}{a_{12}} - \frac{N\bar{C}}{a_{13}}, \quad (14)$$

$$\bar{\theta}(y, s) = \sum_{n=0}^{\infty} \frac{1}{s} \left[ \exp(-a\sqrt{(s+S)Pr}) - \exp(-b\sqrt{(s+S)Pr}) \right], \quad (15)$$

$$\bar{C}(y, s) = \sum_{n=0}^{\infty} \frac{1}{s} \left[ \exp(-a\sqrt{(s+F)Sc}) - \exp(-b\sqrt{(s+F)Sc}) \right], \quad (16)$$

$$\text{where } a_{11} = \sum_{n=0}^{\infty} \left[ \exp(-a\sqrt{H+s}) - \exp(-b\sqrt{H+s}) \right], \quad a_{12} = [s(Pr-1) - Z], \quad a_{13} = [s(Sc-1) - L]$$

$$a = (y + 2n) \text{ and } b = (2 + 2n - y).$$

Taking the inverse Laplace transform of equations (14), (15) and (16), the velocity, temperature and concentration fields are given by the expressions:

$$u(y,t) = \frac{e^{Rt}}{2Z} \sum_{n=0}^{\infty} [F_1(a,1,c_1,t) - F_1(b,1,c_1,t)] + \frac{Ne^{Qt}}{2L} \sum_{n=0}^{\infty} [F_1(a,1,c_2,t) - F_1(b,1,c_2,t)] - \left(\frac{1}{2Z} + \frac{N}{2L}\right) \sum_{n=0}^{\infty} [F_1(a,1,H,t) - F_1(b,1,H,t)] - \frac{e^{Rt}}{2Z} \sum_{n=0}^{\infty} [F_1(a,Pr,c_3,t) - F_1(b,Pr,c_3,t)] + \frac{1}{2Z} \sum_{n=0}^{\infty} [F_1(a,Pr,S,t) - F_1(b,Pr,S,t)] - \frac{Ne^{Qt}}{2L} \sum_{n=0}^{\infty} [F_1(a,Sc,c_4,t) - F_1(b,Sc,c_4,t)] + \frac{N}{2L} \sum_{n=0}^{\infty} [F_1(a,Sc,F,t) - F_1(b,Sc,F,t)] \quad (17)$$

$$\theta(y,t) = \sum_{n=0}^{\infty} \frac{1}{2} [F_1(a,Pr,S,t) - F_1(b,Pr,S,t)] \quad (18)$$

$$C(y,t) = \sum_{n=0}^{\infty} \frac{1}{2} [F_1(a,Sc,F,t) - F_1(b,Sc,F,t)] \quad (19)$$

**Skin-friction:**

The skin-friction  $\tau_0$  and  $\tau_1$  has been studied for  $Sc \neq 1$  and  $Pr \neq 1$ . Therefore using the expressions (17) the skin-friction in non-dimensional form are given by:

$$\tau_0 = \left(\frac{du}{dy}\right)_{y=0} = -\frac{e^{Rt}}{2Z} \sum_{n=0}^{\infty} [F_2(d_1,1,c_1,t) + F_2(d_2,1,c_1,t)] - \frac{Ne^{Qt}}{2L} \sum_{n=0}^{\infty} [F_2(d_1,1,c_2,t) + F_2(d_2,1,c_2,t)] + \left(\frac{1}{2Z} + \frac{N}{2L}\right) \sum_{n=0}^{\infty} [F_2(d_1,1,H,t) + F_2(d_2,1,H,t)] + \frac{e^{Rt}}{2Z} \sum_{n=0}^{\infty} [F_2(d_1,Pr,c_3,t) + F_2(d_2,Pr,c_3,t)] - \frac{1}{2Z} \sum_{n=0}^{\infty} [F_2(d_1,Pr,S,t) + F_2(d_2,Pr,S,t)] + \frac{Ne^{Qt}}{2L} \sum_{n=0}^{\infty} [F_2(d_1,Sc,c_4,t) + F_2(d_2,Sc,c_4,t)] - \frac{N}{2L} \sum_{n=0}^{\infty} [F_2(d_1,Sc,F,t) + F_2(d_2,Sc,F,t)] \quad (20)$$

$$\tau_1 = -\left(\frac{du}{dy}\right)_{y=1} = \sum_{n=0}^{\infty} \left[ \frac{e^{Rt}}{Z} F_2(d_3,1,c_1,t) + \frac{Ne^{Qt}}{L} F_2(d_3,1,c_2,t) - \left(\frac{1}{Z} + \frac{N}{L}\right) F_2(d_3,1,H,t) \right] - \sum_{n=0}^{\infty} \left[ \frac{e^{Rt}}{Z} F_2(d_3,Pr,c_3,t) - \frac{1}{Z} F_2(d_3,Pr,S,t) + \frac{Ne^{Qt}}{L} F_2(d_3,Sc,c_4,t) - \frac{N}{L} F_2(d_3,Sc,F,t) \right] \quad (21)$$

where

$$H = M + \frac{1}{K}, Z = (H - SPr), L = (H - FSc), R = \frac{Z}{(Pr-1)} \text{ with } Pr \neq 1, Q = \frac{L}{(Sc-1)} \text{ with } Sc \neq 1, c_1 = (H+R), c_2 = (H+Q), c_3 = (S+R), c_4 = (F+Q), d_1 = 2n, d_2 = 2+2n \text{ and } d_3 = 1+2n. \text{ Here } F_1(D_1, D_2, D_3, D_4) \text{ and } F_2(D_1, D_2, D_3, D_4) \text{ are defined in appendix.}$$

**Result and Discussions:**

The numerical values of the velocity, temperature, concentration and skin-friction are computed for different parameters like Prandtl number  $Pr$ , Schmidt number  $Sc$ , magnetic parameter  $M$ , time  $t$ , permeability parameter  $K$ , heat generation/absorption parameter  $S$ , chemical reaction parameter  $F$  and the buoyancy ratio parameter  $N$ . The values of main parameters considered are: Prandtl number  $Pr = 0.71$  (for air),  $7$  (for water) and  $3$  (for the saturated liquid Freon at  $273K$ ); Schmidt number  $Sc = 0.6$  (for Oxygen),  $0.78$  (for Ammonia) and  $2.01$  (for Ethyl Benzen); magnetic parameter  $M = 1, 2, 3$ ; time  $t = 0.1, 0.2, 0.4$ ; permeability parameter  $K = 0.3, 0.5, 1$ ; chemical reaction parameter  $F = 1, 3, 6$ ; heat generation/absorption parameter  $S = -0.5, 0, 0.5$  and the buoyancy ratio parameter  $N = 0.2, 0.4, -0.2, -0.4$ . When  $N = 0$ , there is no mass transfer and buoyancy force is due to

the thermal diffusion only.  $N > 0$  implies that mass buoyancy force acts in the same direction of thermal buoyancy force, while  $N < 0$  implies that mass buoyancy force acts in the opposite direction of thermal buoyancy force. Again, the term  $Q^*(T' - T'_d)$  is assumed to be the amount of heat generated or absorbed.  $Q^*$  is a constant, which may take either positive or negative values. The case of heat generated is when  $S < 0$  and the heat absorbed,  $S > 0$ . Graphs have been plotted for the velocity, temperature and concentration profiles to show the effects of different parameters.

Figure-1 shows the effects of heat generation/absorption parameter  $S$ , Prandtl number  $Pr$  and time  $t$  on the temperature profiles. The temperature increases when the heat is generated ( $S > 0$ ) and decreases when the heat is absorbed ( $S < 0$ ). Also, it is observed from this figure that the temperature decreases with the increasing of Prandtl number, but it increases with the value of time  $t$ . Figure-2 represents the effect of concentration profiles for different values of Schmidt number  $Sc$ , chemical reaction parameter  $F$  and time  $t$ . It is observed that near the wall, concentration increases with decreasing value of the Schmidt number and chemical reaction parameter, but it increases as the value of time  $t$  increases. The influence of heat generation/absorption parameter  $S$  and buoyancy ratio parameter  $N$  on the velocity profiles are presented in Figure-3. The velocity increases when the heat is generated ( $S < 0$ ) and decreases when the heat is absorbed ( $S > 0$ ). Also, It is observed that the velocity increases in the presence of aiding flows ( $N > 0$ ) and decreases in the presence of opposing flows ( $N < 0$ ). The influence of permeability parameter  $K$  and Schmidt number  $Sc$  on the velocity profiles are presented in Figure-4. It is observed that the velocity increases as the value of permeability parameter increases, however it decreases as the value of Schmidt number increases. Figure-5 shows the effects of Prandtl number  $Pr$  and magnetic parameter  $M$  on the velocity profiles. From this figure, we observe that the velocity decreases as the value of Prandtl number and magnetic parameter increase. Figure-6 shows the effects of time  $t$  and chemical reaction parameter  $F$  on the velocity profile. Which shows that the velocity increases as the value of time  $t$  increases, but it decreases as the value of chemical reaction parameter increases.

The numerical values of skin-friction  $\tau_0$  and  $\tau_1$  are presented in Table-1. From table, we observe that the values of skin-friction at the plate ( $y = 0$ ) is greater than the values of skin-friction at the plate ( $y = 1$ ). Skin-friction on both the plates increases in the presence of aiding flows ( $N > 0$ ), but it decreases in the presence of opposing flows ( $N < 0$ ). Further, skin-friction on both the plates increases when the heat is generated ( $S < 0$ ) and decrease when the heat is absorbed ( $S > 0$ ). It is also observed that the skin-friction increases as the value of time  $t$  and permeability parameter  $K$  increase. Also, from table it is clear that the skin-friction on both the plates decreases as the value of Schmidt number  $Sc$ , Prandtl

number  $Pr$ , magnetic parameter  $M$  and chemical reaction parameter  $F$  increase.

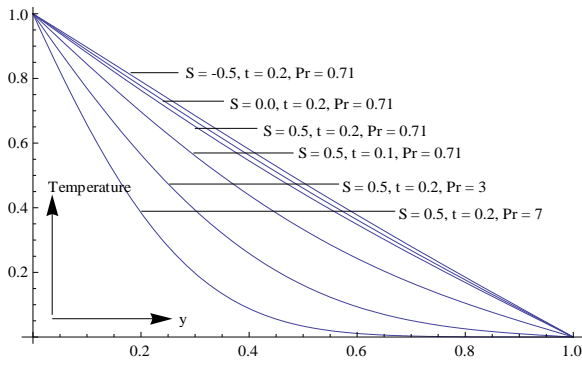


Figure-1: Temperature Profiles

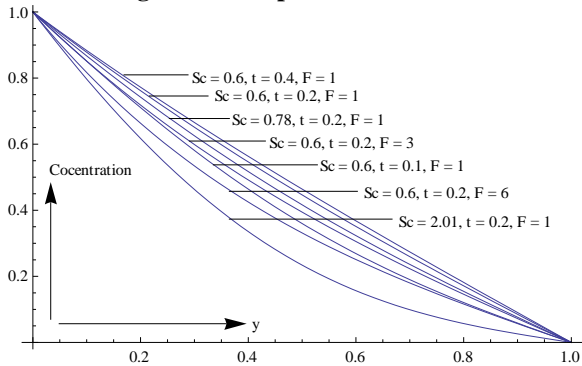


Figure-2: Concentration Profiles

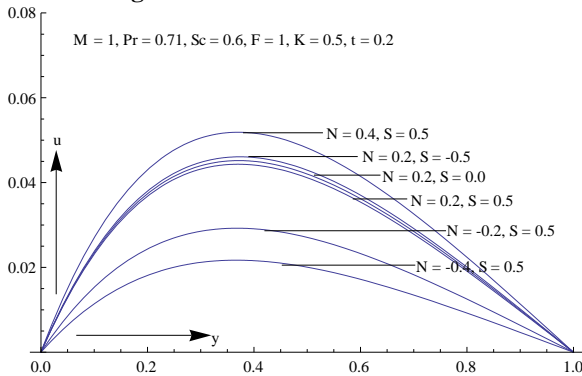


Figure-3: Velocity Profiles

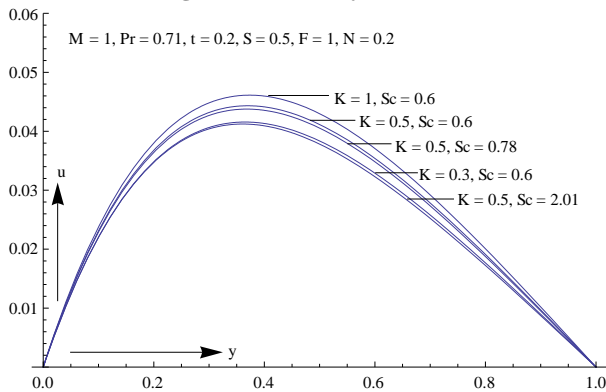


Figure-4: Velocity Profiles

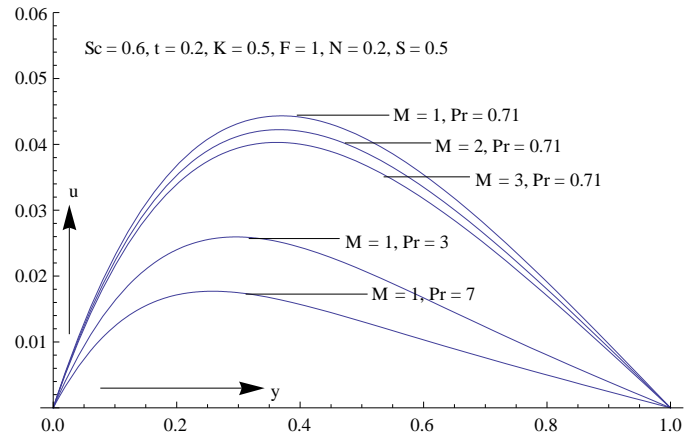


Figure-5: Velocity Profiles

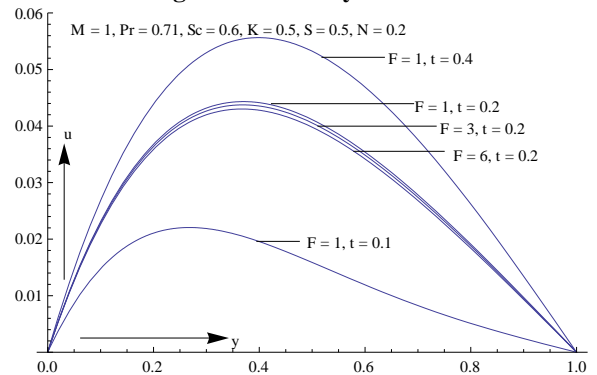


Figure-6: Velocity Profiles

**Conclusions:**

The governing equations for unsteady transient free convection flow between two long vertical parallel plates through a porous medium with heat generation/absorption in the presence of first order homogeneous chemical reaction and transverse magnetic field was formulated. The solutions for the model have been obtained by using Laplace transform method. The conclusions of the study are as follows:

- The temperature increases when heat is generated ( $S < 0$ ) and decreases when heat is absorbed ( $S > 0$ ). Further, It decreases when  $Pr$  is increased and increases when  $t$  is increased.
- The concentration decreases as the value of  $Sc$  and  $F$  increase, but it increases as the value of time  $t$  increases.
- The velocity and skin-friction increase when heat is generated ( $S < 0$ ) and decrease when heat is absorbed ( $S > 0$ ). Further, these decrease as the value of  $Pr$ ,  $Sc$ ,  $M$  and  $F$  increase.
- The velocity and skin-friction increase as the value of  $K$  and  $t$  increase.
- The velocity and skin-friction increase in case of aiding flows ( $N > 0$ ) and decreases in case of opposing flows ( $N < 0$ ).

**Appendix:**

$$F_1(D_1, D_2, D_3, D_4) = \exp(-a_3) \operatorname{erfc}(a_1) + \exp(a_3) \operatorname{erfc}(a_2),$$

$$F_2(D_1, D_2, D_3, D_4) = \frac{1}{\sqrt{\pi D_4}} e^{-a_3 - (a_1)^2} \sqrt{D_2} + \frac{1}{\sqrt{\pi D_4}} e^{-a_3 - (a_2)^2} \sqrt{D_2} + e^{-a_3} \sqrt{D_2 D_3} \operatorname{erfc}(a_1) - e^{-a_3} \sqrt{D_2 D_3} \operatorname{erfc}(a_2),$$

Where

$$a_1 = \left( \frac{D_1 \sqrt{D_2}}{2\sqrt{D_4}} - \sqrt{D_3 \cdot D_4} \right), a_2 = \left( \frac{D_1 \sqrt{D_2}}{2\sqrt{D_4}} + \sqrt{D_3 \cdot D_4} \right) \text{ and } a_3 = D_1 \sqrt{D_2 D_3} .$$

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**Table-1: Skin-friction for  $Sc \neq 1$  and  $Pr \neq 1$**

F	N	Sc	Pr	F	t	S	K	$\tau_0$	$\tau_1$
1.0	0.2	0.60	0.71	1.0	0.2	0.5	0.5	0.286836	0.097476
1.0	0.2	0.60	0.71	1.0	0.2	0.0	0.5	0.290179	0.099908
1.0	0.2	0.60	0.71	1.0	0.2	-0.5	0.5	0.293644	0.102449
1.0	0.4	0.60	0.71	1.0	0.2	0.5	0.5	0.335112	0.114308
1.0	-0.2	0.60	0.71	1.0	0.2	0.5	0.5	0.190252	0.063812
1.0	-0.4	0.60	0.71	1.0	0.2	0.5	0.5	0.141960	0.046981
1.0	0.2	0.60	0.71	1.0	0.4	0.5	0.5	0.324869	0.135424
1.0	0.2	0.60	0.71	1.0	0.1	0.5	0.5	0.201689	0.023719
1.0	0.2	0.60	0.71	3.0	0.2	0.5	0.5	0.284587	0.095815
1.0	0.2	0.60	0.71	6.0	0.2	0.5	0.5	0.281709	0.093761
1.0	0.2	0.60	0.71	1.0	0.2	0.5	0.3	0.275658	0.090011
1.0	0.2	0.60	0.71	1.0	0.2	0.5	1.0	0.293235	0.102519
1.0	0.2	0.78	0.71	1.0	0.2	0.5	0.5	0.284853	0.095591
1.0	0.2	2.01	0.71	1.0	0.2	0.5	0.5	0.275687	0.087562
1.0	0.2	0.60	3.0	1.0	0.2	0.5	0.5	0.217070	0.039175
1.0	0.2	0.60	7.0	1.0	0.2	0.5	0.5	0.176633	0.024562
2.0	0.2	0.60	0.71	1.0	0.2	0.5	0.5	0.278349	0.091793
3.0	0.2	0.60	0.71	1.0	0.2	0.5	0.5	0.270469	0.086601