



Development of certain Summation formulae associated to Hypergeometric Function

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ABSTRACT

The main objective of the present paper is to obtain certain new results in the aspects of Hypergeometric function. The results presented here are presumably new.

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Introduction

Generalized Gaussian Hypergeometric function of one variable is defined by

$${}_A F_B(a_1, a_2, \dots, a_A; b_1, b_2, \dots, b_B; z) = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \dots (a_A)_k z^k}{(b_1)_k (b_2)_k \dots (b_B)_k k!}$$

or

$${}_A F_B((a_A); (b_B); z) \equiv {}_A F_B((a_j)_{j=1}^A; (b_j)_{j=1}^B; z) = \sum_{k=0}^{\infty} \frac{((a_A))_k z^k}{((b_B))_k k!}$$

(2)

where the parameters b_1, b_2, \dots, b_B are neither zero nor negative integers and A, B are non negative integers.

Contiguous Relation is defined by

[Andrews p.363(9.16) , E.D. p.51(10), H.T.F.I. p.103(32)]

$$(a-b) {}_2F_1(a, b; c; z) = a {}_2F_1(a+1, b; c; z) - b {}_2F_1(a, b+1; c; z)$$

(3)

Recurrence relation is defined by

$$\Gamma(\xi+1) = \xi \Gamma(\xi) \quad (4)$$

Bailey summation theorem[Prud, p.491(7.3,7.8)]

$${}_2F_1(a, 1-a; c; \frac{1}{2}) = \frac{\Gamma(\frac{c}{2})\Gamma(\frac{c+1}{2})}{\Gamma(\frac{c+2}{2})\Gamma(\frac{c+1-2}{2})} = \frac{\sqrt{\pi}\Gamma(c)}{2^{c-1}\Gamma(\frac{c+2}{2})\Gamma(\frac{c+1-2}{2})}$$

(5)

Main Summation Formulae

$${}_2F_1(a, -a-29; c; \frac{1}{2}) = \frac{\sqrt{\pi}\Gamma(c)}{2^{c+29}} \left[\frac{-1(-3497196636753920000 + 5555901589554355200e)}{\Gamma(\frac{c+29}{2})\Gamma(\frac{c-1}{2})} + \frac{-2(-2361133612996790400a^2 + 296837573062012416a^3 + 8045509821284520a^4 - 2309495184101812a^5)}{\Gamma(\frac{c+29}{2})\Gamma(\frac{c-1}{2})} + \frac{-2(-52230988296494a^5 + 5767788455233a^7 + 229366815305a^8 - 1161910491a^9 - 180370407a^{10})}{\Gamma(\frac{c+29}{2})\Gamma(\frac{c-2}{2})} + \frac{-2(-2797949a^{11} - 3325a^{12} + 203a^{13} + a^{14} - 8048643072093388800c + 8669404405854904320ac)}{\Gamma(\frac{c+29}{2})\Gamma(\frac{c-2}{2})} + \frac{-2(-2559187026442174464a^2c + 179883929797479936a^3c + 15162881528017664a^4c)}{\Gamma(\frac{c+29}{2})\Gamma(\frac{c-2}{2})} \right]$$

$$\begin{aligned} & \frac{-2(-1073780850065280a^5c - 63174347268032a^6c + 1023904119168a^7c + 102035583552a^8c^2)}{\Gamma(\frac{c+29}{2})\Gamma(\frac{c-2}{2})} \\ & + \frac{-2(1301408640a^3c - 20035904a^{10}c - 545664a^{11}c - 3136a^{12}c - 7135437057322106880c^2)}{\Gamma(\frac{c+29}{2})\Gamma(\frac{c-1}{2})} \\ & + \frac{-2(5530409289193568256ac^2 - 1132617914353373184a^2c^2 + 31488534262945920a^3c^2)}{\Gamma(\frac{c+29}{2})\Gamma(\frac{c-2}{2})} \\ & + \frac{-2(6787990954128704a^4c^2 - 113605319650080a^5c^2 - 17919787048592a^6c^2 - 169540201824a^7c^2)}{\Gamma(\frac{c+29}{2})\Gamma(\frac{c-2}{2})} \\ & + \frac{-2(11836614384a^8c^2 + 275657760a^9c^2 + 864976a^{10}c^2 - 19488a^{11}c^2 - 112a^{12}c^2 - 3437604688802632704c^3)}{\Gamma(\frac{c+29}{2})\Gamma(\frac{c-2}{2})} \\ & + \frac{-2(1958183937501904896ac^3 - 271125878966476800a^2c^3 - 1819638403322880a^3c^3)}{\Gamma(\frac{c+29}{2})\Gamma(\frac{c-2}{2})} \\ & + \frac{-2(1358815980856320a^4c^3 + 14816758360320a^5c^3 - 2044645193472a^6c^3 - 48946060800a^7c^3)}{\Gamma(\frac{c+29}{2})\Gamma(\frac{c-2}{2})} \\ & + \frac{-2(290142720a^8c^3 + 16369920a^9c^3 + 112896a^{10}c^3 - 1037508672181223424c^4 + 43574987883907072ac^4)}{\Gamma(\frac{c+29}{2})\Gamma(\frac{c-2}{2})} \\ & + \frac{-2(-38579705221907456a^2c^4 - 1453332317393280a^3c^4 + 141397606888320a^4c^4 + 4326245868000a^5c^4)}{\Gamma(\frac{c+29}{2})\Gamma(\frac{c-2}{2})} \\ & + \frac{-2(-93210903072a^6c^4 - 3929755200a^7c^4 - 21161280a^8c^4 + 292320a^9c^4 + 2016a^{10}c^4)}{\Gamma(\frac{c+29}{2})\Gamma(\frac{c-2}{2})} \\ & + \frac{-2(-211454435938729984c^5 + 64931398533980160ac^5 - 3288417867356160a^2c^5)}{\Gamma(\frac{c+29}{2})\Gamma(\frac{c-2}{2})} \\ & + \frac{-2(-238107873075200a^3c^5 + 7421571171840a^4c^5 + 399949885440a^5c^5 + 166333440a^6c^5 - 130959360a^7c^5)}{\Gamma(\frac{c+29}{2})\Gamma(\frac{c-2}{2})} \\ & + \frac{-2(-1128960a^8c^5 - 3038953255526144c^6 + 6698752129804288ac^6 - 147310906293760a^2c^6)}{\Gamma(\frac{c+29}{2})\Gamma(\frac{c-2}{2})} \\ & + \frac{-2(-20618377898496a^5c^6 + 106680728448a^4c^6 + 18206469120a^5c^6 + 156522240a^6c^6 - 1559040a^7c^6)}{\Gamma(\frac{c+29}{2})\Gamma(\frac{c-2}{2})} \end{aligned}$$

$$\begin{aligned}
 & \frac{-2(-13440 a^8 c^6 - 3156646985203712 c^7 + 485153871003648 a c^7 - 323136135168 a^3 c^7)}{\Gamma\left(\frac{c+2+29}{2}\right)\Gamma\left(\frac{c-2}{2}\right)} \\
 & + \frac{-2(-1060658565120 a^3 c^7 - 8340541440 a^4 c^7 + 411586560 a^5 c^7 + 4730880 a^6 c^7 - 239741823107072 c^8)}{\Gamma\left(\frac{c+2+29}{2}\right)\Gamma\left(\frac{c-2}{2}\right)} \\
 & + \frac{-2(24603960539136 a c^8 + 361948967424 a^2 c^8 - 32519013120 a^3 c^8 - 471863040 a^4 c^8)}{\Gamma\left(\frac{c+2+29}{2}\right)\Gamma\left(\frac{c-2}{2}\right)} \\
 & + \frac{-2(3674880 a^5 c^8 + 42240 a^6 c^8 - 13317571018752 c^9 + 855551262720 a c^9 + 21544427520 a^2 c^9)}{\Gamma\left(\frac{c+2+29}{2}\right)\Gamma\left(\frac{c-2}{2}\right)} \\
 & + \frac{-2(-548782080 a^3 c^9 - 9461760 a^4 c^9 - 534865723392 c^{10} + 19428192256 a c^{10} + 613099520 a^2 c^{10})}{\Gamma\left(\frac{c+2+29}{2}\right)\Gamma\left(\frac{c-2}{2}\right)} \\
 & + \frac{-2(-3919872 a^3 c^{10} - 67584 a^4 c^{10} - 15112208384 c^{11} + 259424256 a c^{11} + 8945664 a^2 c^{11} - 284770304 c^{12})}{\Gamma\left(\frac{c+2+29}{2}\right)\Gamma\left(\frac{c-2}{2}\right)} \\
 & + \frac{-2(1544192 a c^{12} + 53248 a^2 c^{12} - 3211264 c^{13} - 16384 c^{14})}{\Gamma\left(\frac{c+2+29}{2}\right)\Gamma\left(\frac{c-2}{2}\right)} \\
 & + \frac{-23401744264394496000 a + 20545371093641345280 a^2 - 4904521386295587840 a^3}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{156338767211476080 a^4 + 45080795062004520 a^5 - 939925822046820 a^6 - 200972160567570 a^7}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{-2592856182210 a^8 + 264872851110 a^9 + 9518286750 a^{10} + 51454410 a^{11} - 2382870 a^{12}}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{-42630 a^{13} - 210 a^{14} + 23401744351572787200 c - 58591485964884096000 a c + 300206654190696487680 a^2 c}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{-428540022827758080 a^3 c - 87730053746096120 a^4 c + 34044520118735820 a^5 c}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{694521826011074 a^6 c - 87337054072815 a^7 c - 3381171326295 a^8 c + 18804899925 a^9 c + 2709866873 a^{10} c}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{41871795 a^{11} c + 49315 a^{12} c - 3045 a^{13} c - 15 a^{14} c + 38046115154708398080 c^2 - 51433340852463575040 a c^2}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{17217968620196090880 a^2 c^2 - 1358690011137043200 a^3 c^2 - 103661010083783040 a^4 c^2}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{8157514838155200 a^5 c^2 + 458470221367200 a^6 c^2 - 8044084560960 a^7 c^2 - 763071049440 a^8 c^2}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{-9637790400 a^3 c^3 + 151116000 a^{10} c^3 + 4092480 a^{11} c^3 + 23520 a^{12} c^3 + 26317197211846311936 c^3}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{-23523975044412641280 a c^3 + 5282522037460574720 a^2 c^3 - 171261064390903680 a^3 c^3}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{-32529809916870080 a^4 c^3 + 610652589986400 a^5 c^3 + 88661356663440 a^6 c^3 + 808458023520 a^7 c^3}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{-59394214320 a^8 c^3 - 1375365600 a^9 c^3 - 4304720 a^{10} c^3 + 97440 a^{11} c^3 + 560 a^{12} c^3}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{10435067990822092800 c^4 - 6560866486278021120 a c^4 + 975646586165314560 a^2 c^4}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{3910755752467200 a^3 c^4 - 5002283830118400 a^4 c^4 - 50588679960000 a^5 c^4 + 7669208877120 a^6 c^4}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{181910736000 a^7 c^4 - 1102147200 a^8 c^4 - 61387200 a^9 c^4 - 423360 a^{10} c^4 + 2679683878360186880 c^5}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{-1209155821033820160 a c^5 + 113494550585574912 a^2 c^5 + 4053904501507200 a^3 c^5}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{-422546430013440 a^4 c^5 - 12704000705440 a^5 c^5 + 281976403296 a^6 c^5 + 11765880000 a^7 c^5 + 63282240 a^8 c^5}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{-876960 a^9 c^5 + -6048 a^{10} c^5 + 475326771709870080 c^6 - 153963627309649920 a c^6}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{821223800736640 a^2 c^6 + 577038828764160 a^3 c^6 - 18690496526080 a^4 c^6 - 992671948800 a^5 c^6}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{-333043200 a^6 c^6 + 327398400 a^7 c^6 + 2822400 a^8 c^6 + 60437080794202112 c^7 - 13866465045719040 a^7}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{322776611857920 a^2 c^7 + 43532681264640 a^3 c^7 - 238024629120 a^4 c^7 - 38940364800 a^5 c^7}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{-334560000 a^6 c^7 + 3340800 a^7 c^7 + 28800 a^8 c^7 + 5623416147148800 c^8 - 890488513474560 a c^8}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{1088046812160 a^2 c^8 + 1976387212800 a^3 c^8 + 15425625600 a^4 c^8 - 771724800 a^5 c^8 - 8870400 a^6 c^8}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{386307577610240 c^9 - 40521490606080 a c^9 - 587941291520 a^2 c^9 + 54100858400 a^3 c^9 + 784748800 a^4 c^9}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{-6124800 a^5 c^9 - 70400 a^6 c^9 + 19562378035200 c^{10} - 1276192727040 a c^{10} - 32070635520 a^2 c^{10}}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{823173120 a^3 c^{10} + 14192640 a^4 c^{10} + 720829677568 c^{11} - 26446663680 a c^{11} - 834447360 a^2 c^{11}}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{5345280 a^3 c^{11} + 92160 a^4 c^{11} + 18785894400 c^{12} - 324280320 a c^{12} - 11182080 a^2 c^{12}}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{328007680 c^{13} - 1781760 a c^{13} - 61440 a^2 c^{13} + 3440640 c^{14} + 16384 c^{15}}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \quad (6)
 \end{aligned}$$

$$\begin{aligned}
 {}_2F_1(a, -a-30; \mathbf{C}; \frac{1}{2}) &= \frac{\sqrt{\pi} \Gamma(c)}{2c+30} \left[\frac{-46803488615967283200 a + 42767159087365735680 a^2}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \right. \\
 & + \frac{-1127601192070601856 a^3 + 66101591363194404 a^4 + 76584910384046512 a^5 - 441724660019672 a^6}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{-336434811648432 a^7 + 3976287855967 a^8 + 608379703391 a^9 + 9950104899 a^{10} - 147970213 a^{11}}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{-5745971 a^{12} - 49203 a^{13} - 7 a^{14} + a^{15} + 46803488703145574400 c - 118859389113316884480 a c}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{64102962284473595904 a^3 c - 10599521603190073344 a^4 c + 111032145402626688 a^5 c}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{70940427398809792 a^6 c - 407139854973664 a^7 c - 199425094414192 a^8 c - 3203075817296 a^9 c}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{144861951696 a^9 c + 4812296496 a^{10} c + 27275696 a^{11} c - 528752 a^{12} c - 6608 a^{13} c - 16 a^{14} c}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{76092230309416796160 c^2 - 105466755123938820096 a c^2 + 37807566181098061824 a^2 c^2}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{-3801519798063379968 a^3 c^2 - 122484531131280000 a^4 c^2 + 21183090923693504 a^5 c^2}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{525072890414880 a^6 c^2 - 33174563270480 a^7 c^2 - 1230849813984 a^8 c^2 + 1027790064 a^9 c^2}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{495596640 a^{10} c^2 + 5892880 a^{11} c^2 + 7392 a^{12} c^2 - 112 a^{13} c^2 + 52634394423692623872 c^3}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + \frac{-48739196162194857984 a c^3 + 11997982874998750208 a^2 c^3 - 6471203833003200 a^3 c^3}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)}
 \end{aligned}$$

$$\begin{aligned}
 & -54784666314553984a^4c^3 + 2650005405760320a^5c^3 + 149770214403680a^6c^3 - 1262779162176a^7c^3 \\
 & + \frac{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & - 137968653984a^8c^3 - 1507947840a^9c^3 + 12018720a^{10}c^3 + 237888a^{11}c^3 + 672a^{12}c^3 \\
 & + \frac{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + 20870135981644185600c^4 - 13743447729720811520ac^4 + 2316611729627405312a^2c^4 \\
 & + \frac{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & - 42837424766233856a^3c^4 - 995837120296320a^4c^4 + 9059085684640a^5c^4 + 1669549106040a^6c^4 \\
 & + \frac{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + 158053721328a^7c^4 - 5904024000a^8c^4 - 105934080a^9c^4 - 252000a^{10}c^4 + 2016a^{11}c^4 + 535936756720373760c^5 \\
 & + \frac{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & - 2564183483721285632ac^5 + 287511916583118848a^2c^5 + 2891094455400960a^3c^5 \\
 & + \frac{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & - 988409117368320a^4c^5 - 11460885179520a^5c^5 + 889877079168a^6c^5 + 17580998400a^7c^5 \\
 & + \frac{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & - 50023680a^8c^5 - 2378880a^9c^5 - 8064a^{10}c^5 + 950653543419740160c^6 - 331203488938483712ac^6 \\
 & + \frac{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + 23227972384509952a^2c^6 + 825626374939136a^3c^6 - 55557808244736a^4c^6 - 1476433967232a^5c^6 \\
 & + \frac{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + 19650247680a^6c^6 + 658533120a^7c^6 + 2472960a^8c^6 - 13440a^9c^6 + 120874161588404224c^7 \\
 & + \frac{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & - 3034968822075392ac^7 + 1175475851951104a^2c^7 + 75957743533056a^3c^7 - 1613337631488a^4c^7 \\
 & + \frac{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & - 72595015680a^5c^7 - 84395520a^6c^7 + 9968640a^7c^7 + 42240a^8c^7 + 11246832294297600c^8 \\
 & + \frac{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & - 1991999707934720ac^8 + 30885185565696a^2c^8 + 3902365108224a^3c^8 - 9744641280a^4c^8 \\
 & + \frac{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & - 1772770560a^5c^8 - 10264320a^6c^8 + 42240a^7c^8 + 772615155220480c^9 - 93305628573696ac^9 \\
 & + \frac{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & - 108440735744a^2c^9 + 119913953280a^3c^9 + 756828160a^4c^9 - 19937280a^5c^9 - 112640a^6c^9 \\
 & + \frac{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + 39124756070400c^{10} - 3061463646208ac^{10} - 37866008576a^2c^{10} + 2126170112a^3c^{10} + 20410968a^4c^{10} \\
 & + \frac{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & - 67584a^5c^{10} + 1441659355136c^{11} - 6761633824ac^{11} - 1271508992a^2c^{11} + 18849792a^3c^{11} + 159744a^4c^{11} \\
 & + \frac{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + 37571788800c^{12} - 930242560ac^{12} - 19222528a^2c^{12} + 53248a^3c^{12} + 656015360c^{13} - 6766592ac^{13} \\
 & + \frac{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & - 114688a^2c^{13} + 6881280c^{14} - 16384ac^{14} + 32768c^{15} \\
 & + \frac{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)}{\Gamma\left(\frac{c+2+30}{2}\right)\Gamma\left(\frac{c-2+1}{2}\right)} \\
 & + 202843204931727360000 - 322513180299113932800ac + 137081176976279704320a^2 \\
 & + \frac{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)}{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)} \\
 & - 16958441217372769664a^3 - 603183214997264496a^4 + 14675906054821328a^5 + 4133871935431448a^6 \\
 & + \frac{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)}{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)} \\
 & - 410385710727888a^7 - 19216954885103a^8 + 76930919809a^9 + 18767187789a^{10} + 359027053a^{11} \\
 & + \frac{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)}{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)} \\
 & + 584899a^{12} - 48237a^{13} - 457a^{14} - a^{15} + 473815891454924390400c - 514267672818039828480ac \\
 & + \frac{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)}{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)} \\
 & + 152733916790608472064a^2c - 10431151935471012096a^3c - 1042530101607866112a^4c \\
 & + \frac{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)}{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)}
 \end{aligned}$$

$$\begin{aligned}
 & + 70205646306447488a^5c + 4867596553619456a^6c - 72498049647248a^7c - 9301873967216a^8c \\
 & + \frac{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)}{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)} \\
 & - 143461234896a^9c + 2457464016a^{10}c + 89007184a^{11}c + 737968a^{12}c - 112a^{13}c - 16a^{14}c \\
 & + \frac{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)}{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)} \\
 & + 429952635468868976640c^2 - 338004961381278756864ac + 70059026697492178944a^2c^2 \\
 & + \frac{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)}{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)} \\
 & - 1759276041319756032a^3c^2 - 479277775728825600a^4c^2 + 7123495031408896a^5c^2 \\
 & + \frac{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)}{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)} \\
 & + 1469730527666880a^6c^2 + 17407942640720a^7c^2 - 1222948011264a^8c^2 - 35273078064a^9c^2 \\
 & + \frac{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)}{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)} \\
 & - 145104960a^{10}c^2 + 4630640a^{11}c^2 + 51072a^{12}c^2 + 112a^{13}c^2 + 213651946023436910592c^3 \\
 & + \frac{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)}{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)} \\
 & - 124357165551965761536ac^3 + 17532687851337924608a^2c^3 + 190507604230245120a^3c^3 \\
 & + \frac{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)}{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)} \\
 & - 101519794853401984a^4c^3 - 1414918873379520a^5c^3 + 184536822383840a^6c^3 + 5294045822016a^7c^3 \\
 & + \frac{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)}{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)} \\
 & - 3441721784a^8c^3 - 2670212160a^9c^3 - 26567520a^{10}c^3 + 4032a^{11}c^3 + 672a^{12}c^3 + 67050711362676224000c^4 \\
 & + \frac{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)}{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)} \\
 & - 29026509985070018560ac^4 + 2631784500033124352a^2c^4 + 117050823745715456a^3c^4 \\
 & + \frac{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)}{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)} \\
 & - 11408656870312320a^4c^4 - 412101963368640a^5c^4 + 9483271361760a^6c^4 + 501887904672a^7c^4 \\
 & + \frac{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)}{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)} \\
 & + 3510897600a^8c^4 - 69457920a^9c^4 - 917280a^{10}c^4 - 2016a^{11}c^4 + 14339374628808785920c^5 \\
 & + \frac{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)}{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)} \\
 & - 4586744602902335488ac^5 + 238537086089704448a^2c^5 + 20197806522800640a^3c^5 \\
 & + \frac{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)}{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)} \\
 & - 657184891672320a^4c^5 - 42870132090240a^5c^5 - 39115880832a^6c^5 + 21361670400a^7c^5 \\
 & + \frac{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)}{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)} \\
 & + 265681920a^8c^5 - 40320a^9c^5 - 8064a^{10}c^5 + 2185501760097976320c^6 - 50845256548447368ac^6 \\
 & + \frac{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)}{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)} \\
 & + 11307303767053312a^2c^6 + 1925277943980544a^3c^6 - 9760743879936a^4c^6 - 2321995498368a^5c^6 \\
 & + \frac{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)}{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)} \\
 & - 25841195520a^6c^6 + 370433280a^7c^6 + 6101760a^8c^6 + 13440a^9c^6 + 243864590252867584c^7 \\
 & + \frac{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)}{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)} \\
 & - 40222844632813568ac^7 - 22289722459136a^2c^7 + 113092549008384a^3c^7 + 1111218400512a^4c^7 \\
 & + \frac{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)}{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)} \\
 & - 67136593920a^5c^7 - 1113361920a^6c^7 + 168960a^7c^7 + 41240a^8c^7 + 2021819710617600c^8 \\
 & + \frac{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)}{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)} \\
 & - 2276064016568320ac^8 - 40567245886464a^2c^8 + 4142976178176a^3c^8 + 77685822720a^4c^8 \\
 & + \frac{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)}{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)} \\
 & - 873143040a^5c^8 - 19134720a^6c^8 - 42240a^7c^8 + 1251902765301760c^9 - 90907976146944ac^9 \\
 & + \frac{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)}{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)} \\
 & - 2799334866944a^2c^9 + 89515345920a^3c^9 + 2226780160a^4c^9 - 337920a^5c^9 - 112640a^6c^9 \\
 & + \frac{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)}{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)} \\
 & + 57657353994240c^{10} - 2473121226752ac^{10} - 100757651456a^2c^{10} + 931330048a^3c^{10} + 30547968a^4c^{10} \\
 & + \frac{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)}{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)} \\
 & + 67584a^5c^{10} + 1946239533056c^{11} - 42316292096ac^{11} - 2105372672a^2c^{11} + 319488a^3c^{11} \\
 & + \frac{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)}{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)}
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{159744a^4c^{12} + 46741094400c^{12} - 366878720ac^{12} - 24014848a^2c^{12} - 53248a^3c^{12} + 755793920c^{12}}{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)} \\
 & + \frac{-114688ac^{13} - 114688a^2c^{13} + 7372800c^{14} + 16384ac^{14} + 32768c^{15}}{\Gamma\left(\frac{c+2+31}{2}\right)\Gamma\left(\frac{c-2}{2}\right)} \quad (7)
 \end{aligned}$$

Derivations of formula (6) :

Putting $b = -a-29$, $z = \frac{1}{2}$ in known formula (3) , we get
 $(2a+29) {}_2F_1(a, -a-29 ; c ; \frac{1}{2}) = a {}_2F_1(a+1, -a-29 ; c ; \frac{1}{2}) + (a+29) {}_2F_1(a, -a-28 ; c ; \frac{1}{2})$

Now applying the same method of Ref [2] , we get both the results.

References

1. Abramowitz, Milton., A and Stegun, Irene ; *Handbook of Mathematical Functions with Formulas , Graphs , and Mathematical Tables.* National Bureau of Standards, 1970.
2. Salahuddin. ; Formation of certain summation formulae using hypergeometric function., *International Journal of mathematical Archive* 2(2011), 1122-1126
3. Salahuddin. ; A Summation Formula Related To Bailey Theorem, *Global Journal of Science Frontier Research.*, 11(2011), 53-67.
4. Salahuddin. ; Evaluation of a Summation Formula Involving Recurrence Relation , *Gen. Math. Notes.*, 2(2010), 42-59.
5. Salahuddin. ; Two Summation Formulae Based On Half Argument Associated To Hypergeometric function, *Global Journal of Science Frontier Research.*, 10(2010), 08-19.
6. Salahuddin. ; Evaluation of Certain Summation Formulae Involving Gauss Theorem, *Global Journal of Mathematical Sciences: Theory and Practical.*, 10(2010), 309-316