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Solution of linear and nonlinear system of partial differential equation by using projected differential transform method

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ABSTRACT

In this work, we introduced the novel computational algorithm for solving linear and nonlinear system of partial differential equation by using the projected differential transform method. Several illustrative examples are demonstrated to show the efficiency of the projected differential transform for solving initial value problems .All numerical results compared with those obtained by another analytic and numerical method; such as Adomain decomposition, variational iteration and spline method are found to be the same.

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Introduction

The projected differential transform method (PDTM) is one of the approximate methods which can be easily applied to many linear and nonlinear system and is capable of reducing the size of computational work. The concept of the projected differential transform method has been introduced to solve linear and nonlinear system in electric circuit analysis and mechanics [5,8,9].

Projected differential transform method is a semi-numerical analytic technique that formalizes the Taylor series in a totally different manner. With this method, the given system of partial differential equations and related initial conditions are transformed into a recurrence equation that finally transforms the system of partial differential equations to algebraic equations which can easily be solved. In this method no need for linearization or perturbations, much computational work and round- off errors are avoided. In resent years many researchers apply the PDTM for solving system of partial differential equations $[1,2]$.

This method constructs, for a system of partial differential equations an analytical solution in the form of a polynomial. Not like the traditional high order Taylor series method that requires symbolic computations. Another important advantage is that this method reduces the size of computational work while the Taylor series method is computationally taking long time for higher orders. This method is well addressed in $[4,6]$.

Projected differential transform:

The basic definitions and fundamental theorems of projected differential transform method are defined in $[3]$ and will be stated briefly in this paper.

Projected differential transform of function $y(x_1, x_2, \ldots, x_n)$ is defined as follows:

$$y(x_1, x_2, \ldots, x_{n-1}, k) = \frac{1}{k!}\left[\frac{d^k y(x_1, x_2, \ldots, x_n)}{dx_n^{\ k}}\right]_{x_n=0} \qquad (1)$$

Where $y(x_1, x_2, \ldots, x_n)$ the original is function and $y(x_1, x_2, \ldots, x_{n-1}, k)$ is the transformed function.

The inverse differential transform of $y(x_1, x_2, \ldots, x_{n-1}, k)$ is defined as.

$$y(x_1, x_2, \ldots, x_n) = \sum_{k=0}^{\infty} y(x_1, x_2, \ldots, x_{n-1}, k)\, x_n^{\ k} \qquad (2)$$

Combining eqs (1) and (2) we have

$$y(x_1, x_2, \ldots, x_n) = \sum_{k=0}^{\infty} \frac{1}{k!}\left[\frac{d^k y(x_1, x_2, \ldots, x_n)}{dx_n^{\ k}}\right]_{x_n=0} x_n^{\ k} \qquad (3)$$

It is worth noting that the projected differential transform method is close to the one dimensional differential transform method because the PDTM is considered as the standard of DTM $y(x_1, x_2, \ldots, x_n)$ with respect to variable x_n

The fundamental theorems of the projected differential transform are:

Theorems:

(1) $If\ z(x_1, x_2, \ldots, x_n) = u(x_1, x_2, \ldots, x_n) \pm v(x_1, x_2, \ldots, x_n)$

$Then\ z(x_1, x_2, \ldots, x_{n-1}, k) = u(x_1, x_2, \ldots, x_{n-1}, k) \pm v(x_1, x_2, \ldots, x_{n-1}, k)$

(2) $If z(x_1, x_2, \ldots, x_n) = cu(x_1, x_2, \ldots, x_n)$

$Then\ z(x_1, x_2, \ldots, x_{n-1}, k) = cu(x_1, x_2, \ldots, x_{n-1}, k)$

(3) $If z(x_1, x_2, \ldots, x_n) = \frac{du(x_1, x_2, \ldots, x_n)}{dx_n}$

$Then\ z(x_1, x_2, \ldots, x_{n-1}, k) = (k+1)u(x_1, x_2, \ldots, x_{n-1}, k+1)$

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$$(4)\ If\ z(x_1,x_2,......,x_n)=\frac{d^n u(x_1,x_2,......,x_n)}{dx_n^n}$$

$$Then\ z(x_1,x_2,......,x_{n-1},k)=\frac{(k+n)!}{k!}u(x_1,x_2,.....,x_{n-1},k+n)$$

$$(5)\ If\quad z(x_1,x_2,......,x_n)=u(x_1,x_2,......,x_n)\,v(x_1,x_2,......,x_n)$$

$$Then\quad z(x_1,x_2,......,x_{n-1},k)=\sum_{m=0}^{k}u(x_1,x_2,......,x_{n-1},m)\,v(x_1,x_2,......,x_{n-1},k-m)$$

$$(6)\ If\ z(x_1,x_2,......,x_n)=u_1(x_1,x_2,......,x_n)u_2(x_1,x_2,......,x_n).....u_n(x_1,x_2,......,x_n)\ Then$$

$$z(x_1,x_2,......,x_{n-1},k)=\sum_{k_{n-1}=0}^{k}\sum_{k_{n-2}=0}^{k_{n-1}}\ ......\ \sum_{k_2=0}^{k_3}\sum_{k_1=0}^{k_2}u_1(x_1,x_2,......,x_{n-1},k_1)u_2(x_1,x_2,......,x_{n-1},k_2-k_1)$$

$$\times.....u_{n-1}(x_1,x_2,......,x_{n-1},k_{n-1}-k_{n-2})u_n(x_1,x_2,......,x_{n-1},k-k_{n-1})$$

(7)

$$if\quad z(x_1,x_2,......,x_n)=f(x_1,x_2,.....,x_{n-1})x_n^m\quad then$$

$$z(x_1,......,x_{n-1},k)=f(x_1,.......,x_{n-1})\delta(m-k)=\begin{cases}f(x_1,.......,x_{n-1}) & k=m\\ 0 & k\neq m\end{cases}$$

Note that c is Constant and n is a nonnegative integer.

Example :(1)

We consider the following linear system

$$\begin{cases}u_t+u_x=2v\\ v_t+v_x=-2u\end{cases}\qquad(4)$$

With the initial conditions

$$u(x,0)=\sin x\quad ,v(x,0)=\cos x\qquad(5)$$

By taking the projected differential transform method of equation (4) we obtain

$$\begin{cases}(h+1)u(x,h+1)=2v(x,h)-\frac{\partial}{\partial x}u(x,h)\\ (h+1)v(x,h+1)=-2u(x,h)-\frac{\partial}{\partial x}v(x,h)\end{cases}\qquad(6)$$

Substituting eq (5) into eq (6) yields

$$u(x,1)=\cos x\quad ,u(x,2)=\frac{-\sin x}{2!},\ u(x,3)=-\frac{\cos x}{3!}$$

$$u(x,4)=\frac{\sin x}{4!}$$

$$v(x,1)=-\sin x\quad ,v(x,2)=\frac{-\cos x}{2!},\ u(x,3)=\frac{\sin x}{3!}$$

$$v(x,4)=\frac{\cos x}{4!}$$

And so on In general

$$u(x,h)=\begin{cases}(-1)^{\frac{h-1}{2}}\frac{\cos x}{h!} & h\ is\ \ odd\\ (-1)^{\frac{h}{2}}\frac{\sin x}{h!} & h\ is\ \ even\end{cases}$$

$$v(x,h)=\begin{cases}(-1)^{\frac{h+1}{2}}\frac{\sin x}{h!} & h\ is\ \ odd\\ (-1)^{\frac{h}{2}}\frac{\cos x}{h!} & h\ is\ \ even\end{cases}$$

Substituting $u(x,h),v(x,h)$ in to equation (2) yields

$$u(x,t)=\sum_{h=o}^{\infty}u(x,h)t^h=\sum_{h=0,2,4,...}^{\infty}(-1)^{\frac{h}{2}}\frac{\sin x}{h!}t^h+\sum_{h=1,3,5,...}^{\infty}\frac{(-1)^{\frac{h-1}{2}}\cos x}{h!}t^h$$

$$=\sum_{h=o}^{\infty}\frac{(-1)^h\,t^{2h}}{(2h)!}\sin\ x+\sum_{h=o}^{\infty}\frac{(-1)^{\frac{h}{2}}\,t^{2h+1}}{(2h+1)!}\cos\ x$$

$$=\cos t\sin x+\sin t\cos x=\sin(x+t)$$

$$v(x,t)=\sum_{h=0}^{\infty}v(x,h)\,t^h=\sum_{h=0,2,4,...}^{\infty}\frac{(-1)^{\frac{h}{2}}(\cos x)}{h!}t^h-\sum_{h=1,3,5,...}^{\infty}\frac{(-1)^{\frac{h+1}{2}}(\sin x)t^{2h+1}}{h!}=$$

$$\sum_{h=0}^{\infty}\frac{(-1)^h t^{2h}}{(2h)i}(\cos x)-\sum_{h=o}^{\infty}\frac{(-1)^h t^{2h+1}}{h!}(\sin\ x)=$$

$$\cos x\cos t-\sin x\sin t=\cos(x+t)$$

Example :-(2)

Consider the nonlinear system

$$\begin{cases}u_t+\ vu_x\ +u=1\\ v_t+\ uv_x\ +v=-1\end{cases}\qquad(7)$$

With the initial conditions

$$u(x,0)=e^{x}\ ,\ v(x,0)=e^{-x}\qquad(8)$$

Taking the projected differential transform method of eq (4.33) we get

$$\begin{cases}(h+1)u(x,h+1)=\delta(h)-\sum_{m=o}^{h}v(x,m)\frac{\partial}{\partial x}u(x,h-m)-u(x,h)\\ (h+1)v(x,h+1)=v(x,h)-\sum_{m=o}^{h}u(x,m)\frac{\partial}{\partial x}v(x,h-m)-s(h)\end{cases}\qquad(9)$$

Substituting eq (4.34) into eq (4.35) we have

$$u(x,1)=-e^{x}\ ,\ u(x,2)=\frac{e^{x}}{2!}\ ,\ u(x,3)=-\frac{e^{x}}{3!}$$

$$v(x,1)=-e^{-x}\ ,\ v(x,2)=\frac{e^{-x}}{2!}\ ,\ v(x,3)=\frac{e^{-x}}{3!}$$

And so on in general

$$u(x,h)=\frac{(-1)^{h}\ e^{x}}{h!}\ ,\ v(x,h)=\frac{e^{-x}}{h!}$$

Substituting $u(x,h),v(x,h)$ into eq (2) we have

$$u(x,t)=\sum_{h=0}^{\infty}u(x,h)t^{h}=\sum_{h=0}^{\infty}\frac{(-1)^{h}\ e^{x}t^{h}}{h!}=e^{x}\sum_{h=0}^{\infty}\frac{-t^{h}}{h!}=$$

$$e^{x}\cdot e^{-t}=e^{x-t}$$

$$v(x,t)=\sum_{h=0}^{\infty}v(x,h)t^{h}=\sum_{h=0}^{\infty}\frac{e^{-x}}{h!}t^{h}=e^{-x+t}$$

Example :-(3)

The following example deal with a system of three nonlinear partial differential equations in three unknown functions $u(x,y,t)$, $v(x,y,t)$ and $w(x,y,t)$ this problem can easily is solved by using projected differential transform which overcomes the difficulties of traditional methods [5].

$$\begin{cases} u_t+v_xw_y-v_yw_x=-u \\ v_t+w_xu_y+w_yu_x=v \\ w_t+u_xv_y+u_yv_x=w \end{cases} \qquad (10)$$

With the initial conditions

$$u(x,y,0)=e^{x+y}\ ,\ v(x,y,0)=e^{x-y}\ ,\ w(x,y,0)=e^{-x+y} \qquad (11)$$

By using the projected differential transform method of eq (10) we

$$\begin{cases} (h+1)u(x,y,h+1)=\sum_{m=0}^{h}\frac{\partial v(x,y,m)}{\partial y}\frac{\partial w(x,y,m-h)}{\partial x}-\sum_{m=0}^{h}\frac{\partial w(x,y,m)}{\partial y}\frac{\partial v(x,y,h-m)}{\partial x} \\ \quad -u(x,y,h) \\ (h+1)v(x,y,h+1)=v(x,y,h)-\sum_{m=0}^{h}\frac{\partial w(x,y,m)}{\partial x}\frac{\partial u(x,y,h-m)}{\partial y} \\ \quad -\sum_{m=0}^{h}\frac{\partial w(x,y,m)}{\partial y}\frac{\partial v(x,y,h-m)}{\partial x} \\ (h+1)w(x,y,h+1)=w(x,y,m)-\sum_{m=0}^{h}\frac{\partial u(x,y,m)}{\partial x}\frac{\partial v(x,y,h-m)}{\partial y} \\ \quad -\sum_{m=0}^{h}\frac{\partial u(x,y,m)}{\partial y}\frac{\partial v(x,y,h-m)}{\partial x} \end{cases}$$

(12)

Substituting eq (11) into eq (12) we get

$$u(x,y,1)=-e^{x+y}\ ,\ u(x,y,2)=\frac{e^{x+y}}{2!}\ ,\ u(x,y,3)=\frac{-e^{x+y}}{3!}$$

$$v(x,y,1)=e^{x-y}\ ,\ v(x,y,2)=\frac{e^{x-y}}{2!}\ ,\ v(x,y,3)=\frac{e^{x-y}}{3!}$$

$$w(x,y,1)=e^{-x+y}\ ,\ w(x,y,2)=\frac{e^{-x+y}}{2!}\ ,\ w(x,y,3)=\frac{e^{-x+y}}{3!}$$

And so on in general

$$u(x,y,h)=\frac{(-1)^{h}\ e^{x+y}}{h!}\ ,\ v(x,y,h)=\frac{e^{x-y}}{h!}\ ,\ w(x,y,h)=\frac{e^{-x+y}}{h!}$$

Substituting $u(x,y,h),v(x,y,h)$ and $w(x,y,h)$ into eq (2) yields

$$u(x,y,t)=\sum_{h=o}^{\infty}u(x,y,h)t^{h}=\sum_{h=o}^{\infty}\frac{e^{x+y}(-t)^{h}}{h!}=e^{x+y-t}$$

$$v(x,y,t)=\sum_{h=o}^{\infty}v(x,y,h)t^{h}=\sum_{h=o}^{\infty}\frac{e^{x-y}t^{h}}{h!}=e^{x-y+t}$$

$$w(x,y,t)=\sum_{h=0}^{\infty}w(x,y,h)t^{h}=\sum_{h=0}^{\infty}\frac{e^{-x+y}t^{h}}{h!}=e^{-x+y+t}$$

Conclusion:-

Projected differential transform have been applied to linear and nonlienear partial differential equations .The results for all examples can be obtained in Tayler's series form, all the calculations in the method are very easy. In summary, using projected differential transformation to solve PDE consists of three main steps. First, transforming PDE into algebraic equation, second, solving the equations, finally inverting the solution of algebraic equations to obtain a closed form series solution.

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