Available online at www.elixirpublishers.com (Elixir International Journal)

Applied Mathematics

Elixir Appl. Math. 40 (2011) 5168-5173

ISSN: 2229-712X

Effects of thermal radiation and chemical reaction on MHD convective flow of a polar fluid past a moving vertical plate with viscous dissipation

M. Sudheer Babu[1], P.V. Satya Narayana[2], T. Sankar reddy[3] and D.Umamaheswara reddy[4]

[1]Department of Mathematics, Sree Vidyanikethan Engineering College, Tirupati, A.P.
[2]Department of Mathematics, VIT University, Vellore – 632 014, T.N, India
[3]Department of Mathematics, RVP Engineering College for Women, Kadapa, A.P.,
[4]Department of Mathematics, S.V.University, Tirupati, A.P., India.

ARTICLE INFO

Article history:
Received: 19 August 2011;
Received in revised form:
17 October 2011;
Accepted: 27 October 2011;

Keywords
MHD,
Thermal radiation,
Chemical reaction,
Viscous dissipation.

ABSTRACT

The objective of the present investigation is to analyze the radiation and mass transfer effects on an unsteady two-dimensional laminar convective boundary layer flow of a viscous, incompressible, chemically reacting and dissipative fluid along a semi-infinite vertical plate with suction. The equations of continuity, linear momentum, energy and diffusion, which govern the flow field are solved by using a regular perturbation method. The behavior of the velocity, temperature, concentration has been discussed numerically and graphically for variations in the governing parameters.

Introduction

The study of heat and mass transfer with chemical reaction is of great practical importance to engineers and scientists because of its almost universal occurrence in many braches of science and engineering. Das et al. [1] have studied the effects of mass transfer on the flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. Diffusion of chemically reactive species from a stretching sheet is studied by Anderson et al. [2]. Anjalidevi and Kandaswamy [3, 4] have analysed the effects of chemical reaction, heat and mass transfer on laminar flow without or with along a semi-infinite horizontal plate. The effects of the chemical reaction and mass transfer on MHD unsteady free convection flow past a semi infinite vertical plate with constant/variable suction and heat sink was analyzed by [5-7]. Ghaly and Seddek [8] have discussed the Chebyshev finite difference method for the effects of chemical reaction, heat and mass transfer on the laminar flow along a semi-infinite horizontal plate with temperature dependent viscosity.

A new stage in the evaluation of fluid dynamic theory is in progress because of the increasing in the processing industries and elsewhere of materials whose flow shear behavior cannot be characterized by Newton relationships. The theory of micropolar fluids was first introduced and formulated by Eringen [9]. This theory displays the effects of local rotary inertia and couple stress. The theory is expected to a mathematical model for the non-Newtonian fluid behavior observed in certain fluid such as exotic lubricants, colloidal fluids, liquid crystals etc., which is more realistic and important from a technological point of view. The theory of thermomicropolar fluids was developed by Eringen [10] by extending his theory of micropolar fluid. The flow characteristics of the boundary layer of micropolar fluid over a semi-infinite plate in different situations have been studied by many authors in Refs. [11, 12]. We know that the radiation effect is important under many non-isothermal situations. If the entire system involving the polymer extrusion process is placed in a thermally controlled environment, then radiation could become important. The radiative flows of an electrically conducting fluid with high temperature in the presence of a magnetic field are encountered in electrical power generation, astrophysical flows, solar power technology, space vehicle re-entry, nuclear engineering applications and other industrial areas [13-16]. In the above authors studied the radiation on Newtonian and non-Newtonian fluids with and without magnetic field has been considered by many authors.

In all the studies mentioned above, viscous dissipation is neglected. But the viscous dissipation in the natural convective flow is important, when the flow field of extreme size or in high gravitational field. Gebhart [17] show the importance of viscous dissipative heat in free convection flow in the case of isothermal and constant heat flux at the plate. Gebhart and Mollendorf [18] considered the effects of viscous dissipation for the external natural convection flow over a surface. Soundalgekar [19] analyzed viscous dissipative heat on the two dimensional unsteady free convection flow past an infinite vertical porous plate when the temperature oscillates in time and there is constant suction at the plate.

However, the interaction of radiation with mass transfer in a chemically reacting and dissipative fluid has received little attention. So the objective of this paper is to study the effects of chemical reaction and thermal radiation on MHD convective flow of micro polar fluid past a semi infinite vertical plate with viscous dissipation.

Mathematical analysis:

Consider the steady, laminar, two-dimensional, free convection flow of a viscous incompressible, electrically conducting, and polar fluid occupying a semi-infinite region of the space bounded by an infinite vertical porous plate in the

Tele:
E-mail addresses: m_sudheerbabu@yahoo.co.in,
pvsatya8@yahoo.co.in

presence of viscous dissipation and subjected to thermal radiation. The x^*-axis is taken along the vertical plate in an upward and y^*-axis is taken normal to the plate. The transversely applied magnetic field and magnetic Reynolds number are assumed to be very small so that the induced magnetic field is negligible [20]. The governing equations for this physical situation are based on the usual balance laws of mass, laminar momentum, angular momentum, energy and mass diffusion modified to account for the physical effects mentioned above.

The equations are given by

continuity:

$$\frac{\partial v^*}{\partial y^*}=0, \tag{2.1}$$

linear momentum:

$$v^*\frac{\partial u^*}{\partial y^*}=(v+v_r)\frac{\partial^2 u^*}{\partial y^{*2}}+g\beta_f(T-T_\infty)+g\beta_c(C^*-C^*_\infty)-\left(\frac{\sigma B_0^2}{\rho}+\frac{v}{K^*}\right)u^*+2v_r\frac{\partial\omega^*}{\partial y^*}, \tag{2.2}$$

angular momentum:

$$v^*\frac{\partial w^*}{\partial y^*}=\frac{\gamma}{I}\frac{\partial^2 w^*}{\partial y^{*2}}, \tag{2.3}$$

energy:

$$v^*\frac{\partial T^*}{\partial y^*}=\alpha\left(\frac{\partial^2 T^*}{\partial y^{*2}}-\frac{1}{k}\frac{\partial q_r^*}{\partial y^*}\right)+\frac{\nu}{C_p}\left(\frac{\partial u^*}{\partial y^*}\right)^2, \tag{2.4}$$

diffusion:

$$v^*\frac{\partial C^*}{\partial y^*}=D^*\frac{\partial^2 u^*}{\partial y^{*2}}-K_lC^*, \tag{2.5}$$

where x^* and y^* are the dimensional distances along and perpendicular to the plate, respectively. u^*, v^* are the components of dimensional velocities along x^* and y^* directions, respectively, ρ is the density, ν is the kinematic viscosity, υ_r is the kinematic rotational viscosity, g is the acceleration of gravity, β_f and β_c is the coefficients of volumetric thermal and concentration expansion of the fluid, σ_c is the fluid electrical conductivity, B_0 is the magnetic induction, j^* is the micro inertia density, ω^* is the component of the angular velocity vector normal to the x^*y^*-plane, γ is the spin gradient viscosity, α is the effective thermal diffusivity of the fluid, k is the effective thermal conductivity, C_p is the specific heat at pressure, q_r is the radiative heat flux, T^* is the dimensional temperature, C^* is the dimensional concentration of the fluid, K_l is the chemical reaction parameter and D^* is the chemical molecular diffusivity. The second and third terms on the right hand side of the momentum equation (2.2) denotes thermal and concentration buoyancy effects and the forth is the MHD term. Also, the second term on the right hand side of the energy equation (2.4) represents the radiative heat flux.

Using the Rosseland approximation [21], the radiative heat flux in the y^* direction is given by

$$q^*_r=-\frac{4\sigma\partial T'}{3k_e\partial y'} \tag{2.6}$$

where σ and k_e are the Stefan-Boltzman constant and the mean absorption coefficient, respectively.

We assume that the temperature differences with within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 into the Taylor series about T and neglecting higher terms, then

$$T^4\cong 4T_\infty{}^4T-3T_\infty{}^4 \tag{2.7}$$

By substitution from equations (2.6) and (2.7) in equation (2.4), so

$$v^*\frac{\partial T^*}{\partial y^*}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y'^2}+\frac{16\sigma T_\infty^3}{3\rho c_p k_e}\frac{\partial^2 T}{\partial y'^2}+\frac{\nu}{C_p}\left(\frac{\partial u^*}{\partial y^*}\right)^2 \tag{2.8}$$

The appropriate boundary conditions for the velocity, microrotation, temperature and concentration fields are

$$u^*=u_p^*,\ \omega^*=-\frac{\partial u^*}{\partial y^*},\ T=T_w,\ C^*=C_w^* \text{ at } y^*=0$$

$$u^*\to 0,\ \omega^*\to 0,\ T\to T_\infty,\ C^*\to C_\infty^* \text{ as } y^*\to\infty$$

(2.9)

The integration of the continuity equation (1) yields

$$v^*=-V_0, \tag{2.10}$$

where V_0 is the scale of suction velocity which is a non-zero positive constant. The negative sign indicates that the suction is directed towards the plate.

It is convenient to employ the following non-dimensional variables:

$$y=\frac{V_0y^*}{\nu},\ v=\frac{v^*}{V_0},\ \theta=\frac{T-T_\infty}{T_w-T_\infty},\ C=\frac{C^*-C^*_\infty}{C^*_w-C^*_\infty},$$

$$\omega=\frac{\nu\omega^*}{V_0},$$

$$\Pr=\frac{\nu\rho C_p}{k}=\frac{\nu}{\alpha}=\frac{\mu C_p}{k},\qquad R=\frac{kk_e}{4\sigma_sT_\infty^3},\qquad \beta=\frac{\nu_r}{\nu},$$

$$Q=\frac{Q_0\nu}{\rho C_pV_0^2}$$

$$M=\frac{\sigma_cB_0^2\nu}{\rho V_0^2},\eta=\frac{I\nu}{\gamma},G_r=\frac{\nu\beta_fg(T_w-T_\infty)}{V_0^3},$$

$$Gc=\frac{\nu\beta_cg(C_w^*-C_\infty^*)}{V_0^3} \tag{2.11}$$

$$Sc=\frac{\nu}{D^*}\quad \Delta=\frac{K_l\nu}{V_0{}^2},\ E_c=\frac{V_0^2}{C_p(T_w-T_\infty)}.$$

In view of Equations (2.10) and (2.11) the governing Equations (2.2), (2.3), (2.5) and (2.8) reduce to the following non-dimensional form:

$$(1+\beta)\frac{\partial^2 u}{\partial y^2}+\frac{\partial u}{\partial y}-Nu=G_r\theta+G_cC+2\beta\frac{\partial\omega}{\partial y} \tag{2.12}$$

$$\frac{\partial^2 \omega}{\partial y^2} + \eta \frac{\partial \omega}{\partial y} = 0, \tag{2.13}$$

$$\frac{\partial^2 \theta}{\partial y^2} + \Gamma \frac{\partial \theta}{\partial y} = -\Gamma Ec \left(\frac{\partial u}{\partial y} \right)^2, \tag{2.14}$$

$$\frac{\partial^2 C}{\partial y^2} + Sc \frac{\partial C}{\partial y} - Sc \Delta C = 0. \tag{2.15}$$

where $N = M + \frac{1}{K}$, $\Gamma = \left(1 - \frac{4}{3R+4} \right) \Pr$

and Gr, Gc, M, K, Pr, R, Ec, Sc and Δ are the thermal Grashof number, solutal Grashof number, magnetic field parameter, permeability parameter, Prandtl number, radiation parameter, Eckert umber, Schmidt number and the chemical reaction parameter, respectively.

The boundary conditions (2.9) are then given by the following non-dimensional equations:

$$u = U_p, \theta = 1, \omega = -\frac{\partial u}{\partial y}, C = 1 \ at \ y = 0$$

$$u \to 0, \ \theta \to 0, \ \omega \to 0, \ C \to 0 \ as \ y \to \infty \tag{2.16}$$

The mathematical statement of the problem is now complete and embodies the solution of Eqs. (2.12)-(2.5) and subject to boundary conditions (2.16).

Solution of the problem

The solution of Equation (2.15) subject to the boundary conditions (2.16).

$$C(y) = e^{R_3 y} \tag{2.17}$$

where $R_3 = \frac{Sc}{2}\left[1 + \sqrt{1 + \frac{4\Delta}{Sc}} \right]$.

the problem posed in Eqs. (2.12)-(2.14) subjected to the boundary conditions presented in Equation (2.16) are highly non-linear, coupled equations and generally will involve a step by step numerical integration of the explicit finite difference scheme. However, analytical solutions are possible. Since viscous dissipation parameter is very small in most of the practical problems and therefore, we can advance an asymptotic expansion with as perturbation parameter for the velocity, microrotation and temperature as follows.

$$u = u_0(y) + Ec\, u_1(y) + O(\varepsilon^2) + ...$$
$$\omega = \omega_0(y) + Ec\, \omega_1(y) + O(\varepsilon^2) + ... \tag{2.18}$$
$$\theta = \theta_0(y) + Ec\, \theta_1(y) + O(\varepsilon^2) + \ldots$$

Substituting Eqs. (2.18) into Eqs. (2.12)-(2.14), equating the coefficients of the same power of E and neglecting terms in E^2 and higher order, we get

$$(1+\beta)u_0'' + u_0' - Mu_0 = -G_r\theta_0 - G_c C - 2\beta\omega_0' \tag{2.19}$$

$$(1+\beta)u_1'' + u_1' - Mu_1 = -G_r\theta_1 - 2\beta\omega_1' \tag{2.20}$$

$$\omega_0'' + \eta\omega_0' = 0 \tag{2.21}$$

$$\omega_1'' + \eta\omega_1' = 0 \tag{2.22}$$

$$\theta_0'' + \Gamma\theta_0' = 0 \tag{2.23}$$

$$\theta_1'' + \Gamma\theta_1' = -\Gamma(u_0'^2) \tag{2.24}$$

where a prime denotes differentiation with respect to y.

With the corresponding boundary conditions

$$u_0 = U_p, \ u_1 = 0, \ \omega_0 = -u_0', \ \omega_1 = -u_1', \ \theta_0 = 1, \ \theta_1 = 0, \ at \ y = 0$$

$$u_0 = 0, \ u_1 = 0, \omega_0 \to 0, \omega_1 \to 0, \ \theta_0 \to 0, \ \theta_1 \to 0, \ as \ y \to \infty \tag{2.25}$$

Without going into details, the solutions of Eqs. (2.19)-(2.24) subject to Equation (2.25) can be shown to be

$$u_0(y) = a_1 e^{-R_1 y} + a_2 e^{-R_2 y} + a_3 e^{-\Gamma y} + a_4 e^{-\eta y}$$

$$u_1(y) = b_1 e^{-R_1 y} + b_2 e^{-\Gamma y} + b_3 e^{-2R_1 y} + b_4 e^{-2R_2 y} + b_5 e^{-2\Gamma y} + b_6 e^{-2\eta y} + b_7 e^{-(R_1+R_2) y}$$
$$+ b_8 e^{-(R_1+\Gamma) y} + b_9 e^{-(R_1+\eta) y} + b_{10} e^{-(R_2+\Gamma) y} + b_{11} e^{-(R_2+\eta) y} + b_{12} e^{-(\Gamma+\eta) y} + b_{13} e^{-\eta y}$$

$$\omega_0(y) = C_1\, e^{-\eta y}$$

$$\omega_1(y) = C_2\, e^{-\eta y}$$

$$\theta_0(y) = e^{-\Gamma y}$$

$$\theta_1(y) = a_5 e^{-\Gamma y} + a_6 e^{-2R_1 y} + a_7 e^{-2R_2 y} + a_8 e^{-2\Gamma y} + a_9 e^{-2\eta y} + a_{10} e^{-(R_1+R_2) y} + a_{11} e^{-(R_1+\Gamma) y} + a_{12} e^{-(R_1+\eta) y}$$
$$+ a_{13} e^{-(R_2+\Gamma) y} + a_{14} e^{-(R_1+R_2) y} + a_{15} e^{-(\Gamma+\eta) y}$$

$$C(y) = e^{R_3 y}$$

where

$$R_1 = \frac{1}{2(1+\beta)}\left[1 + \sqrt{1 + \frac{4M}{(1+\beta)}} \right]$$

and the remaining mathematical expressions involved in the above equations are given in appendix.

In view of the above solutions, the streamwise velocity, microrotation, temperature and concentration in the boundary layer become

$$u(y) = a_1 e^{-R_1 y} + a_2 e^{-R_2 y} + a_3 e^{-\Gamma y} + a_4 e^{-\eta y}$$
$$+ Ec \left\{ \begin{array}{l} b_1 e^{-R_1 y} + b_2 e^{-\Gamma y} + b_3 e^{-2R_1 y} + b_4 e^{-2R_2 y} + b_5 e^{-2\Gamma y} + b_6 e^{-2\eta y} + b_7 e^{-(R_1+R_2) y} \\ + b_8 e^{-(R_1+\Gamma) y} + b_9 e^{-(R_1+\eta) y} + b_{10} e^{-(R_2+\Gamma) y} + b_{11} e^{-(R_2+\eta) y} + b_{12} e^{-(\Gamma+\eta) y} + b_{13} e^{-\eta y} \end{array} \right\} \tag{2.26}$$

$$\omega(y) = C_1\, e^{-\eta y} + Ec\{C_2\, e^{-\eta y}\} \tag{2.27}$$

$$\theta(y) = e^{-\Gamma y} + Ec \left\{ \begin{array}{l} a_5 e^{-\Gamma y} + a_6 e^{-2R_1 y} + a_7 e^{-2R_2 y} + a_8 e^{-2\Gamma y} + a_9 e^{-2\eta y} + a_{10} e^{-(R_1+R_2) y} \\ + a_{11} e^{-(R_1+\Gamma) y} + a_{12} e^{-(R_1+\eta) y} + a_{13} e^{-(R_2+\Gamma) y} + a_{14} e^{-(R_1+R_2) y} + a_{15} e^{-(\Gamma+\eta) y} \end{array} \right\} \tag{2.28}$$

The skin-friction, the couple stress coefficient, the Nusselt number and the Sherwood number are important physical quantities for this type of boundary-layer flow. These parameters can be defined and determined as follows:

$$C_f = \frac{\tau_w^*}{\rho U_0 V_0} = \left. \frac{\partial u}{\partial y} \right|_{y=0} = \left[\frac{\partial u_0}{\partial y} + Ec \frac{\partial u_1}{\partial y} \right]_{y=0}$$
$$= -a_1 R_1 - a_2 R_2 - a_3 \Gamma - a_4 \eta \tag{2.29}$$
$$- Ec \left\{ \begin{array}{l} b_1 R_1 + b_2 \Gamma + 2b_3 R_1 + 2b_4 R_2 + 2b_5 \Gamma + 2b_6 \eta + b_7 (R_1 + R_2) + b_8 (R_1 + \Gamma) \\ + b_9 (R_1 + \eta) + b_{10} (R_2 + \Gamma) + b_{11} (R_2 + \eta) + b_{12} (\Gamma + \eta) + b_{13} \eta. \end{array} \right\}$$

$$Nu_x = x \frac{\left. \partial T / \partial y^* \right|_{y^*=0}}{T_w - T_\infty} \Rightarrow Nu_x \operatorname{Re}_x^{-1} = -\left. \frac{\partial \theta}{\partial y} \right|_{y=0} = \left[\frac{\partial \theta_0}{\partial y} + \frac{\partial \theta_1}{\partial y} \right]_{y=0} \tag{2.30}$$
$$= \Gamma + Ec \left\{ \begin{array}{l} a_5 \Gamma + 2a_6 R_1 + 2a_7 R_2 + 2a_8 \Gamma + 2a_9 \eta + a_{10}(R_1 + R_2) + a_{11}(R_1 + \Gamma) \\ + a_{12}(R_1 + \eta) + a_{13}(R_2 + \Gamma) + a_{14}(R_2 + \eta) + a_{15}(\Gamma + \eta) \end{array} \right\}$$

Results and discussion

In order to study the behavior of velocity u, microrotation ω, temperature θ and concentration C fields, a comprehensive numerical computation is carried out for various values of the parameters that describe the flow characteristics viz., viscosity ratio β, plate velocity U_p, the thermal Grashof number *Gr*, the solutal Grashof number *Gc*, Prandtl number *Pr*, the radiation parameter *R*, Eckert number *Ec*, Schmidt number *Sc* and chemical reaction parameter Kr and results reported in in terms of graghs. The effect of dimensionless viscosity ratio β, on the velocity and microrotation past a porous plate is presented in Fig. 1.

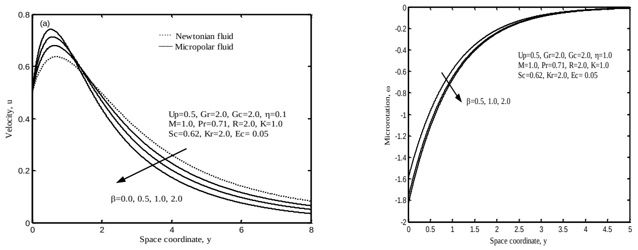

Fig 1 Velocity and Microrotation profiles for different values of β

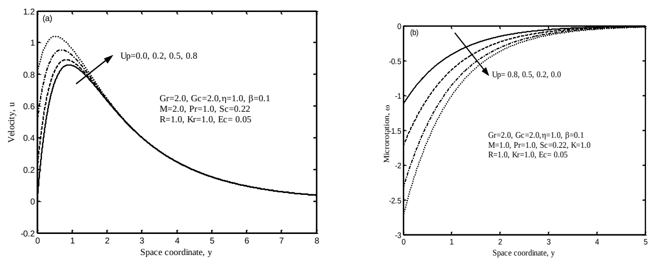

Fig. 2. Velocity and Microrotation profiles for different values of Up

The numerical results show that the peak value of the velocity distribution across the boundary layer is smaller for a Newtonian fluid $(\beta = 0)$ with the fixed flow and material parameters, as compared with a micropolar fluid. In addition, the value of the angular velocity on the porous plate increases as the viscosity ratio β increases.

Fig. 2 illustrates the variation of velocity and microrotation distribution across the boundary layer for the various values of the plate velocity U_p. It is observed that the values of translational velocity and microrotation on the porous plate increase, as the plate moving velocity U_p increases.

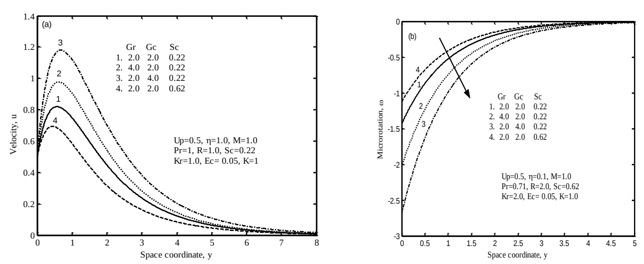

Fig 3 Velocity and Microrotation profiles for different values of Gr, Gc and Sc

The translational velocity and the microrotation profiles against spanwise coordinate y for different values of Grashof number Gr, Solutal Grashof number Gc and Schmidt number Sc are described in Fig. 3. It is observed that the velocity increases as Gr, Gc increase while it decreases as Sc increase, but opposite behavior is found to microrotation. Here the positive values of Gr corresponds to a cooling of the surface by natural convection.

Fig. 4 shows the translational velocity and the microrotation profiles across the boundary layer for different values of $\Pr$ and R. We observe that the effect of increasing values $\Pr$ or R in a decreasing the velocity and magnitude of microrotation.

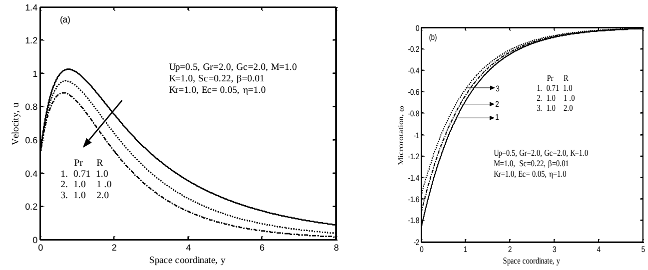

Fig. 4. Velocity and Microrotation profiles for different values of $\Pr$ and R

For different values of the Schmidt number Sc and chemical reaction parameter Kr translational velocity and the microrotation profiles are plotted in Fig. 5. It is obvious that the effect of increasing values of Sc or Kr results in a decreasing velocity distribution across the boundary layer. Furthermore, the results show that the magnitude of microrotation on the porous plate is decreased as Sc or Kr increases.

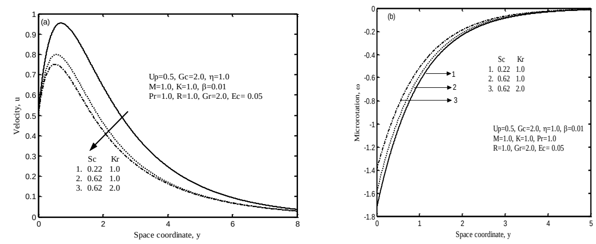

Fig. 5. Velocity and Microrotation profiles for different values of Sc and Kr

The effects of the viscous dissipation parameter i.e., Eckert number on the velocity and microrotation are shown in Fig. 6. It is clear that from these results an increase in the values of Eckert number Ec leads to rise in the velocity and magnitude of microrotation.

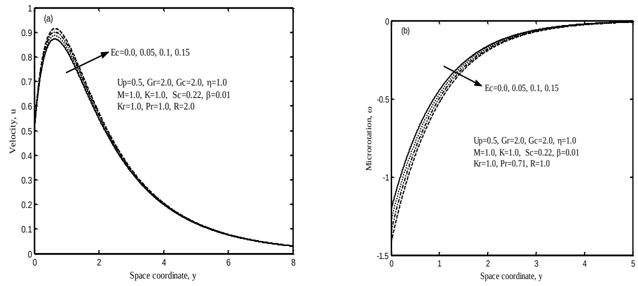

Fig. 6 Velocity and Microrotation profiles for different values of Ec

Fig. 7 illustrates the influence of Eckert number Ec on the dimensionless temperature θ. It is observed that an increase in Ec leads to a fall in the velocity.

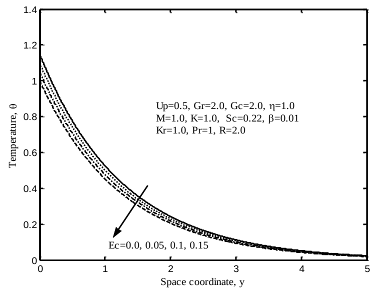

Fig 7 Temperature profiles for different values of Ec

For different values of Prandtl number $\Pr$ and radiation parameter R, the temperature profiles are plotted in Fig. 8. These results show that an increase in Pr or R results in a decrease thermal boundary layer thickness.

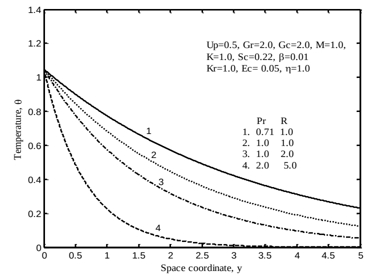

Fig. 8. Temperature profiles for different values of $\Pr$ **and** R

Fig .9 shows typical variations in the concentration profiles for different values of the Schmidt number Sc and the chemical reaction parameter Kr. It is clear from Fig. 9 that the concentration boundary layer thickness decreases as the Schmidt number Sc and the chemical reaction parameter Kr.

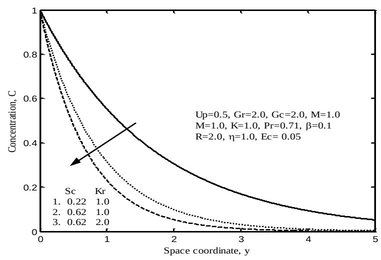

Fig. 9. Concentration profiles for different values of Sc **and** Kr

References

1. U.N. Das, R.K. Deka , V.M. Soundalgekar, 37(1994) 659.
2. H.I. Anderson, O.R. Hansen B. Holmedal, Int. J. Heat Mass Transfer 37 (1994) 465.
3. S.P Anjalidevi, R. Kandaswamy, Heat Mass Transfer 35(1999)465.
4. S.P Anjalidevi, R. Kandaswamy, Heat Mass Transfer 80(2000)697.
5. P.V Satya Narayana, D.Ch. Kesavaiah and S.Venkataramana" International Journal of Mathematical Archive,2(4) (2011) 476.
6. D.Ch. Kesavaiah, P. V. Satya Narayana and S. Venkataramana, Int. J. of Appl. Math and Mech. 7 (1) (2011) 52.
7. M. Sudheer Babu and P.V. Satya Narayana, J.P. Journal of Heat and mass transfer,3 (2009) 219..
8. A.Y. Ghaly, M.A. Seddek, Chaos Solutions Fractals 19(2004) 61.
9. Eringen A.C. Theory of micropolar fluids, J. Math. Mech. Vol 16, (1966), pp. 1-18.
10. Eringen A.C., J. Math. Anal. Appl. Vol 38(1972) pp 480-496.
11. Ahmadi G. Int. J. Eng. Sci., Vol. 14, (1976), pp. 639-646.
12. V. M. Soundalgekar, H. S Takhar, Int. J. Eng. Sci., Vol. 21, (1983)961-965.
13. Perdikis C and Raptis A. Heat Mass Transfer, Vol. 31, (1996), pp. 381-382.
14. Raptis A, Int. J. Heat Mass Transfer, Vol. 41, (1998), pp. 2865-2866.
15. El- Arabavy E.A.M. Int. J. Heat and Mass Transfer, Vol. 46, (2003), pp. 1471-1477.
16. Kim Y.J. and Fodorov A.G. Int. J. Heat and Mass Transfer, Vol. 46, (2003), pp.1761- 1758.
17. Gebhart. B., J. Fluid Mech., Vol.14, (1962), pp.225-232.
18. Gebhart. B. and Mollendorf. J., J. Fluid Mech., Vol.38, (1969), pp.97-107.
19. Soundalgekar V.M., Int. J. Heat Mass Transfer, Vol.15, (1972), pp.253-1261.
20. Cowling T.G., Magnetohydynamics, Interscince Publishers, New York (1957).
21. M.Q. Brewster (1992), Thermal Radiative Transfer and Properties. John Wilely & Sons, New York.

Appendix

$$a_1 = U_p - (a_2 + a_3 + a_4), \quad a_2 = \frac{-Gc}{(1+\beta)R_2^2 - R_2 - M}$$

$$a_3 = \frac{-Gr}{(1+\beta)\Gamma^2 - \Gamma - M},$$

$$a_4 = \frac{2\beta\eta C_1}{(1+\beta)\eta_2^2 - \eta - M} = \theta_1 C_1$$

$$a_5 = 1 - \sum_{n=6}^{15} a_n,$$

$$C_1 = \frac{U_p R_1 + a_2(R_2 - R_1) + a_3(\Gamma - R_1)}{1 + \theta(R_1 - \eta)}$$

$$a_6 = \frac{-\Gamma a_1^2 R_1}{4R_1 - 2\Gamma}, \quad a_7 = \frac{-\Gamma a_2^2 R_2}{4R_2 - 2\Gamma}, \quad a_8 = \frac{-\Gamma a_3^2}{2}$$

$$a_9 = \frac{-\Gamma a_4^2 \eta}{4\eta - 2\Gamma}, \quad a_{10} = \frac{-2\Gamma a_1 a_2 R_1 R_2}{(R_1 + R_2)^2 - \Gamma(R_1 + R_2)}$$

$$a_{11} = \frac{-2\Gamma^2 a_1 a_3}{(R_1 + \Gamma)}, \quad a_{12} = \frac{-2\Gamma a_1 a_4 R_1 \eta}{(R_1 + \eta)^2 - \Gamma(R_1 + \eta)}$$

$$a_{13} = \frac{-2\Gamma^2 a_2 a_3}{(R_2 + \Gamma)}, \quad a_{14} = \frac{-2\Gamma a_2 a_4 R_2 \eta}{(R_2 + \eta)^2 - \Gamma(R_2 + \eta)}$$

$$a_{15} = \frac{-2\Gamma^2 a_3 a_4}{(R_2 + \eta)}$$

$$b_1 = -(b_2 + b_3 + b_4 + b_5 + b_6 + b_7 + b_8 + b_9 + b_{10} + b_{11} + b_{12})$$

$$b_2 = \frac{-Gra_5}{(1+\beta)\Gamma_2^2 - \Gamma - M}, \quad b_3 = \frac{-Gra_6}{4(1+\beta)R_1^2 - 2R_1 - M}$$

$$b_4 = \frac{-Gra_7}{4(1+\beta)R_2^2 - 2R_2 - M}, \quad b_5 = \frac{-Gra_8}{4(1+\beta)\Gamma_1^2 - 2\Gamma - M}$$

$$b_6 = \frac{-Gra_9}{4(1+\beta)\eta_1^2 - 2\eta - M},$$

$$b_7 = \frac{-Gra_{10}}{(1+\beta)(R_1+R_2)^2 - (R_1+R_2) - M}$$

$$b_8 = \frac{-Gra_{11}}{(1+\beta)(R_1+\Gamma)^2 - (R_1+\Gamma) - M},$$

$$b_9 = \frac{-Gra_{12}}{(1+\beta)(R_1+\eta)^2 - (R_1+\eta) - M}$$

$$b_{10} = \frac{-Gra_{13}}{(1+\beta)(R_2+\Gamma)^2 - (R_2+\Gamma) - M},$$

$$b_{11} = \frac{-Gra_{14}}{(1+\beta)(R_2+\eta)^2 - (R_2+\eta) - M}$$

$$b_{12} = \frac{-Gra_{15}}{(1+\beta)(\Gamma+\eta)^2 - (\Gamma+\eta) - M},$$

$$b_{13} = \frac{2\beta C_2\eta}{(1+\beta)\eta^2 - \eta - M} = \theta_2 C_2$$

$$C_3 = b_2(\Gamma - R_1) + b_3R_1 + b_4(2R_2 - R_1) + b_5(2\Gamma - R_1) + b_6(2\eta - R_1) + b_7R_2 + b_8\Gamma + b_9\eta + b_{10}(R_2 - R_1 + \Gamma) + b_{11}(R_2 - R_1 + \eta) + b_{12}(\Gamma - R_1 + \eta)$$

$$C_2 = \frac{C_3}{(1+\theta_2(R_1-\eta))}$$