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Effect of chemical reaction on free convection mhd flow through a porous medium bounded by vertical surface

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ABSTRACT

Effect of chemical reaction on free convection MHD flow through a porous medium bounded by vertical surface is studied here. The governing equations involved in the present analysis are solved by the two-term perturbation method. The velocity, temperature, concentration, skin friction and Nusselt number are studied for different parameters like thermal Grash of number, mass Grash of number, Schmidt number, magnetic field parameter, Permeability parameter, Prandtl number, Eckert number and chemical reaction parameter.

Introduction

Study of MHD flow with heat and mass transfer plays an important role in chemical, mechanical and biological Sciences. Some important applications are cooling of nuclear reactors, liquid metals fluid, power generation system and aero dynamics. The response of laminar skin friction and heat transfer to fluctuations in the stream velocity was studied by Lighthill [5]. Free convection effects on the oscillating flow past an infinite vertical porous plate with constant suction - I, was studied by Soundalgekar [15] which was further improved by Vajravelu et al. [17].

Further researches in these areas were done by Gupta et al. [1], Jaiswal et al. [3] and Soundalgekar et al. [16] by taking different models. Some effects like radiation and mass transfer on MHD flow were studied by Muthucumaraswamy et al. [7, 8] and Prasad et al. [9].

Radiation effects on mixed convection along a vertical plate with uniform surface temperature were studied by Hossain and Takhar [2].

Mass transfer effects on the flow past an exponentially accelerated vertical plate with constant heat flux was studied by Jha, Prasad and Rai [4].

Earlier we have studied some MHD flow models [10, 11, 12] and [13] considering variable temperature alongwith mass diffusion [11] and rotation effects [12].

Radiation and free convection flow past a moving plate was considered by Raptis and Perdikis [14].

In this paper, we are considering the rotation and chemical reaction effects on MHD flow past an impulsively started vertical plate with variable mass diffusion.

We are considering effect of chemical reaction on free convection MHD flow through a porous medium bounded by vertical surface.

The results are shown with the help of graphs (Fig-1 to Fig-13) and table-1.

Mathematical Analysis:

We consider a steady flow of an incompressible viscous fluid through a porous medium occupying a semi-infinite region of the space bounded by a vertical infinite surface in the presence of an imposed uniform magnetic field B_0, normal to the plate. The y' axis is taken along the surface in an upward direction. The fluid properties are assumed to be constant except for the density in the body force term.

A chemically reactive species is emitted from the vertical surface into a hydrodynamic flow field. It diffuses into the fluid, where it undergoes a homogeneous chemical reaction.

The reaction is assumed to take place entirely in the stream. Under the above assumptions, the flow is governed by the following set of equations:

$$\frac{\partial v'}{\partial y'}=0, \qquad (1)$$

$$v'\frac{\partial u'}{\partial y'}=g\beta\left(T'-T'_\infty\right)+g\beta^*\left(C'-C'_\infty\right)+\upsilon\frac{\partial^2 u'}{\partial y'^2}-\frac{\sigma B_0^2 u'}{\rho}-\frac{\upsilon u'}{K_p}, \qquad (2)$$

$$v'\frac{\partial T'}{\partial y'}=\frac{\alpha}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2}+\frac{\upsilon}{C_p}\left(\frac{\partial u'}{\partial y'}\right)^2, \qquad (3)$$

$$v'\frac{\partial C'}{\partial y'}=D\frac{\partial^2 C'}{\partial y'^2}-K_c\left(C'-C'_\infty\right). \qquad (4)$$

Equation (1) gives:

$$v'=-v_0, \qquad (5)$$

where $v_0>0$ and v' is the steady normal velocity of suction on the surface.

The boundary conditions are as follows:

$$\left.\begin{array}{l} u'=0,\ T'=T'_w,\ C'=C'_w \text{ at } y'=0\ ; \\ u'\to 0,\ T'\to T'_\infty,\ C'\to C'_\infty \text{ as } y'\to\infty\ . \end{array}\right\} \qquad (6)$$

Using (7), equations (1)-(4) reduce to:

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$$\frac{\partial^2 u}{\partial y^2}+\frac{\partial u}{\partial y}-\left(M+\frac{1}{K}\right)u=-G_r\theta-G_mC, \quad (8)$$

$$\frac{\partial^2 \theta}{\partial y^2}+P_r\frac{\partial \theta}{\partial y}=-P_rE\left(\frac{\partial u}{\partial y}\right)^2, \quad (9)$$

$$\frac{\partial^2 C}{\partial y^2}+S_c\frac{\partial C}{\partial y}-S_ck_0C=0. \quad (10)$$

Also, the boundary condition (6) reduces to:

$$\left.\begin{array}{l} u=0,\ \theta=1,\ C=1 \text{ at } y=0\ ; \\ u\to 0,\ \theta\to 0,\ C\to 0 \text{ as } y\to\infty\ . \end{array}\right\} \quad (11)$$

The dimensionless governing equations (8), (9) and (10), subject to the boundary conditions (11), are solved by the perturbation method. Expanding u,θ and C in the power of the Eckert number E (assuming that E is very small). We can write:

$$u=u_0+Eu_1+O\left(E^2\right),$$
$$\theta=\theta_0+E\theta_1+O\left(E^2\right), \quad (12)$$
$$C=C_0+EC_1+O\left(E^2\right).$$

Substituting the equation (12) into equations (8)-(10), equating the coefficients at the terms with the same power of E, and neglecting terms of E^2 and higher orders, we get the following equations:

Zero order;

$$\frac{\partial^2 u_0}{\partial y^2}+\frac{\partial u_0}{\partial y}-mu_0=-G_r\theta_0-G_mC_0, \quad (13)$$

$$\frac{\partial^2 \theta_0}{\partial y^2}+P_r\frac{\partial \theta_0}{\partial y}=0, \quad (14)$$

$$\frac{\partial^2 C_0}{\partial y^2}+S_c\frac{\partial C_0}{\partial y}-S_ck_0C_0=0. \quad (15)$$

First order;

$$\frac{\partial^2 u_1}{\partial y^2}+\frac{\partial u_1}{\partial y}-mu_1=-G_r\theta_1-G_mC_1, \quad (16)$$

$$\frac{\partial^2 \theta_1}{\partial y^2}+P_r\frac{\partial \theta_1}{\partial y}=-P_r\left(\frac{\partial u_0}{\partial y}\right)^2, \quad (17)$$

$$\frac{\partial^2 C_1}{\partial y^2}+S_c\frac{\partial C_1}{\partial y}-S_ck_0C_1=0. \quad (18)$$

The corresponding boundary conditions are as follows:

$$\left.\begin{array}{l} u_0=0,u_1=0,\ \theta_0=1,\theta_1=0, \\ \quad C_0=1,C_1=0 \text{ at } y=0\ ; \\ u_0\to 0,u_1\to 0,\ \theta_0\to 0,\theta_1\to 0, \\ \quad C_0\to 0,C_1\to 0 \text{ as } y\to\infty\ , \end{array}\right\} \quad (19)$$

Solving equations (13)-(18) under the boundary conditions (19) and then using (12), we get the solution, which is as under:

$$\begin{aligned} u=&\left(G+EA_2\right)e^{-\lambda y}-G_1e^{-\mu y}-G_2e^{-yP_r} \\ &-EG_r\left(A_9e^{-yP_r}-A_3e^{-2\lambda y}+A_5e^{-y(\lambda+P_r)}\right) \\ &+EG_r\left(A_4e^{-2\mu y}+A_6e^{-y(\mu+P_r)}+A_7e^{-y(\mu+\lambda)}\right) \quad (20) \\ &+EG_rA_8e^{-2P_ry}, \end{aligned}$$

$$\begin{aligned} \theta=&\left(1+EA_1\right)e^{-yP_r}-\frac{P_rEG_2^2e^{-2yP_r}}{2} \\ &-\frac{P_rE}{2}\left(A_{10}e^{-2\lambda y}+A_{11}e^{-2\alpha y}\right) \quad (21) \\ &+2P_rEG_1\left(A_{12}e^{-y(\lambda+P_r)}-A_{13}e^{-y(\mu+P_r)}\right) \\ &+2P_rEG_1A_{14}e^{-y(\mu+\lambda)}, \end{aligned}$$

$$C=e^{-\mu y}, \quad (22)$$

where

$$\lambda=\frac{1+\sqrt{1+4m}}{2},\ G=G_1+G_2,\ \mu=\frac{S_c+\sqrt{S_c^2+4k_0S_c}}{2},$$

$$G_1=\frac{G_m}{\mu^2-\mu-m},\ G_2=\frac{G_r}{P_r^2-P_r-m},\ m=M+\frac{1}{K},$$

$$A_1=\frac{P_rE}{2}\left[\begin{array}{l} A_{10}-A_{11}+4A_{12}G_1-4A_{13}G \\ +\dfrac{4\mu\lambda GG_1}{(\mu+\lambda)^2-P_r(\mu+\lambda)}-G_2^2 \end{array}\right],$$

$$A_2=G_r(A_9-A_3-A_8-A_4+A_5-A_6-A_7),$$

$$A_3=\frac{\lambda G_1^2P_r}{2(2\lambda-P_r)(4\lambda^2-2\lambda-m)},$$

$$A_4=\frac{\mu G_1^2P_r}{2(2\mu-P_r)(4\mu^2-2\mu-m)},$$

$$A_5=\frac{2\lambda P_r^2GG_1}{(\lambda^2-P_r^2)\left[(\lambda+P_r)^2-(\lambda+P_r)-m\right]},$$

$$A_6=\frac{2\mu P_r^2G_2G_1}{(\mu^2-P_r^2)\left[(\mu+P_r)^2-(\mu+P_r)-m\right]},$$

$$A_7=\frac{2\mu A_{14}P_rG_1}{\alpha G_rE\left[(\mu+\lambda)^2-(\mu+\lambda)-m\right]},$$

$$A_8=\frac{G_2^2P_r}{2(4P_r^2-2P_r-m)},\ A_9=\frac{A_1}{P_r^2-P_r-m},$$

$$A_{10}=\frac{\lambda G_1^2}{(2\lambda-P_r)},\ A_{11}=\frac{\mu G_1^2}{(2\mu-P_r)},\ A_{12}=\frac{P_rG}{(\lambda+P_r)},$$

$$A_{13}=\frac{P_rG_2}{(\mu+P_r)} \quad \text{and}$$

$$A_{14}=\frac{\alpha\lambda GG_rE}{(\mu+\lambda)^2-P_r(\mu+\lambda)}.$$

Skin friction:

The non-dimensional skin friction is given by:

$$\tau=\left(\frac{\partial u}{\partial y}\right)_{y=0}=\mu G_1+P_rG_2-\lambda\left(G+EA_2\right)$$
$$+EG_r\left[A_9G_r-2A_8P_r-2A_3+\frac{A_5}{\lambda+P_r}\right] \qquad (23)$$
$$-EG_r\left[2\mu A_4+\frac{A_6}{\lambda+P_r}\right]$$
$$+\frac{2\mu\lambda P_rGG_1EG_r}{\left(\mu+\lambda-P_r\right)\left[\left(\mu+\lambda\right)^2-\left(\mu+\lambda\right)-m\right]}.$$

Nusselt number:
The non-dimensional Nusselt number is given by:

$$Nu=-\left(\frac{\partial\theta}{\partial y}\right)_{y=0}=P_r\left(1+EA_1\right)$$
$$-PE\left(\lambda A_{10}+\mu A_{11}+P_rG_2^2+2P_rG_1G_2\right) \qquad (24)$$
$$+2EP_rGG_2\left[P_r+\frac{2\lambda\mu}{\mu+\lambda-P_r}\right].$$

Results and Discussion:

The velocity profiles for different parameters $M, G_r, G_m, K, k_0, P_r, S_c$ and E are shown by figures-1 to 7.

From figure-1, it is clear that the velocity increases when Eckert number E is increased (keeping other parameters $M=2, G_r=5, G_m=5, K=1, k_0=1, P_r=0.71, S_c=2.01$ constant). Velocity profile for different values of Eckert number E is shown in figure-2. It shows that the velocity increases with increasing mass Grashof number G_m, thermal Grashof number G_r and Prandtl number P_r (keeping other parameters constant). Similar pattern is observed in figure-1 and figure-3, i.e. the velocity increases when chemical parameter k_0 is increased.

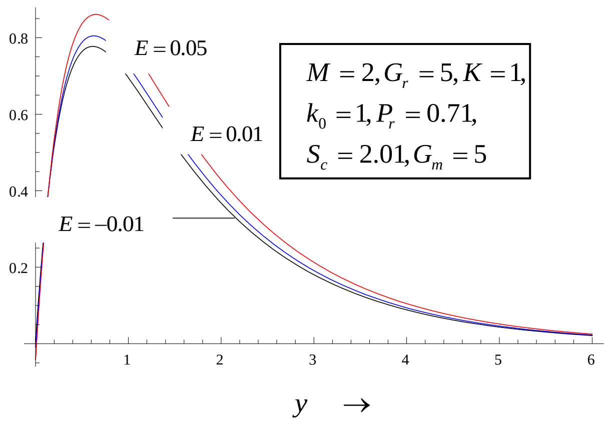

Figure -1: Velocity profile for different Eckert number E

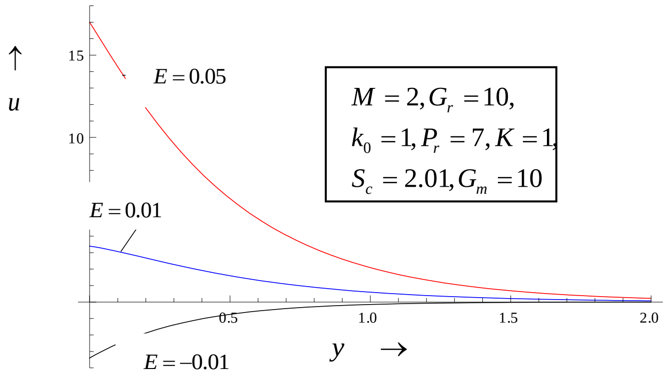

Figure- 2: Velocity profile for different Eckert number E

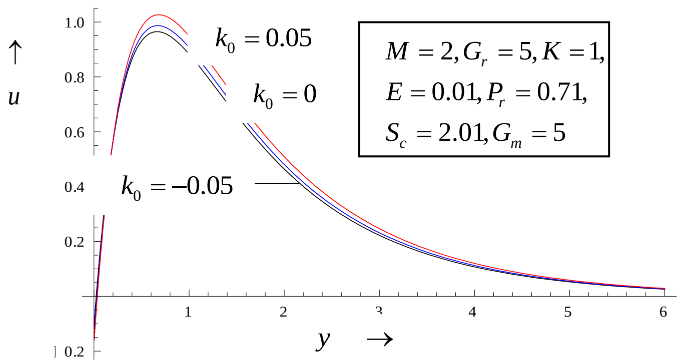

Figure-3: Velocity profile for different chemical parameter k_0

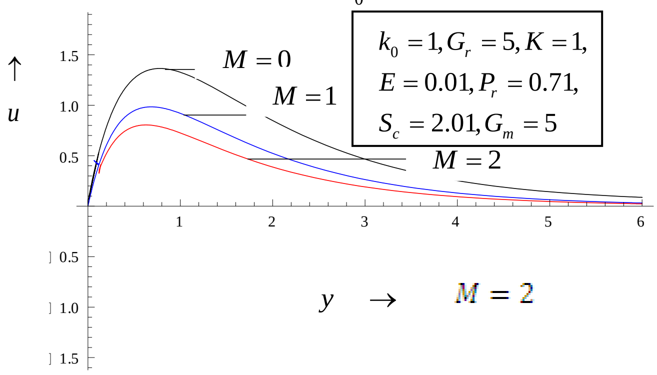

Figure-4: Velocity profile for different magnetic field parameter M

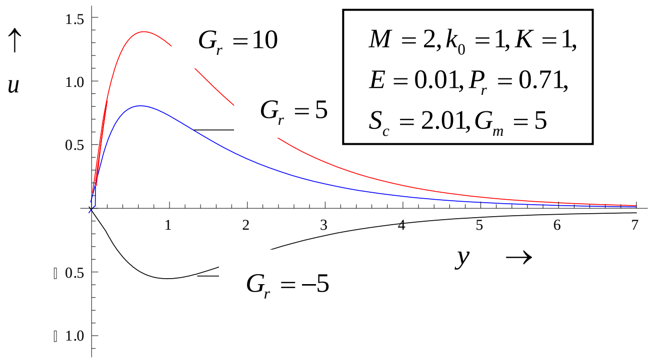

Figure-5: Velocity profile for different thermal Grashof number G_r

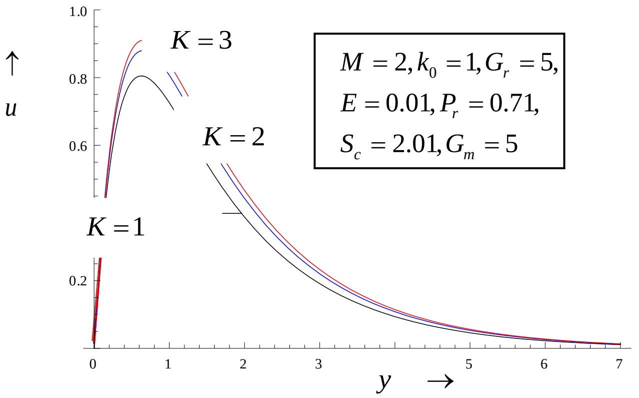

Figure-6: Velocity profile for different permeability parameter K

But in figure-4 the velocity decreases when magnetic field parameter M is increased.

In figure-5 it is observed that velocity increases when thermal Grash of number G_r is increased. Similarly in figure-6 and figure-7, the velocity increases when the value of permeability parameter K and mass Grash of number G_m is increased respectively.

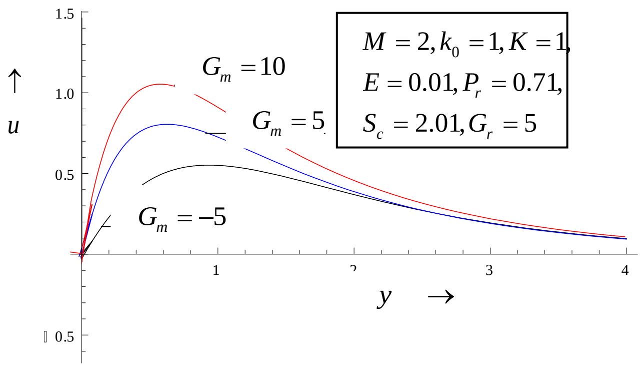

Figure-7: Velocity profile for different mass Grashof number G_m

Temperature profile is shown in figures-8 to 12. From figure-8, it is clear that temperature decreases when Eckert number E is increased (keeping other parameters $M=2, G_r=5, G_m=5, K=1, k_0=1, P_r=0.71,$ $S_c=2.01$ constant). In figure-9, it is shown that temperature increases when magnetic field parameter M is increase

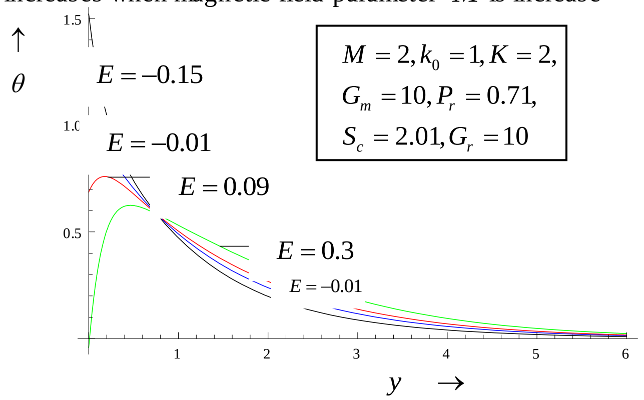

Figure-8: Temperature profile for different Eckert number

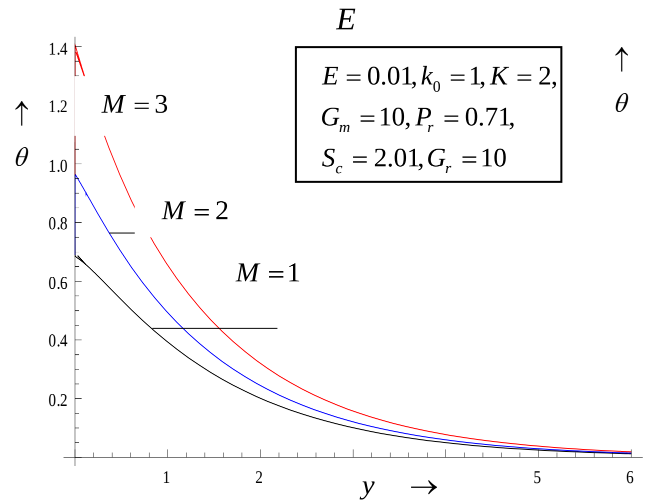

Figure-9: Temperature profile for different magnetic field M

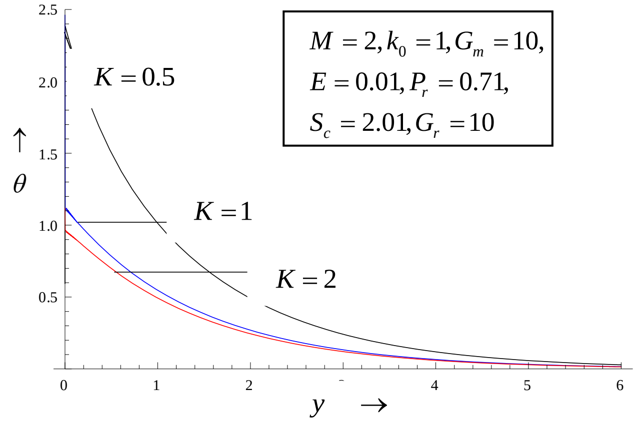

Figure-10: Temperature profile for different permeability parameter K

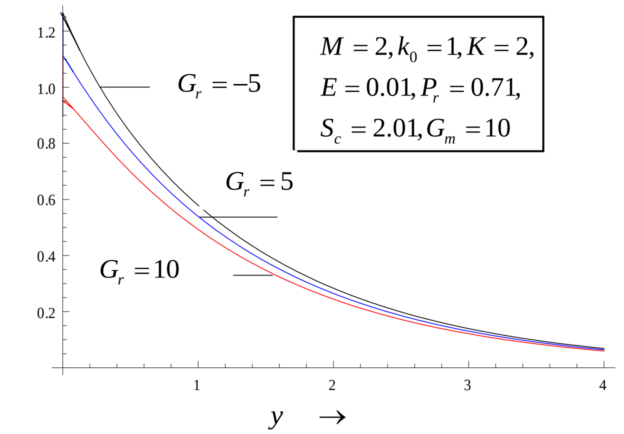

Figure-11: Temperature profile for different thermal Grashof number G_r

Figure-10 shows that temperature decreases with increasing the permeability parameter K. Similar pattern is observed in figure-11, the temperature decreases with increasing the thermal Grashof number G_r.

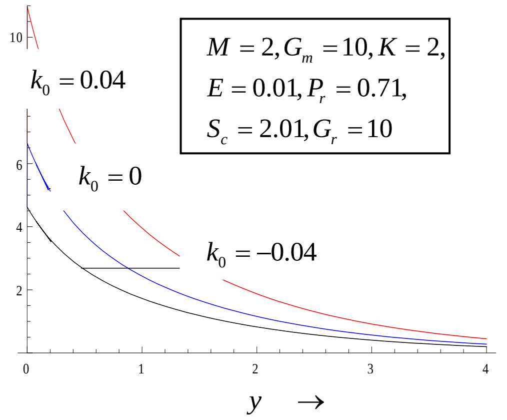

Figure-12: Temperature profile for different chemical parametric k_0

In figure-12, temperature increases with increasing the values of chemical parameter k_0.Figure-13 represents the concentration profile for different chemical parameter k_0. It is shown in figure-13, concentration decreases when increasing the value of chemical parameter k_0 (keeping $S_c = 2.01$ fixed).

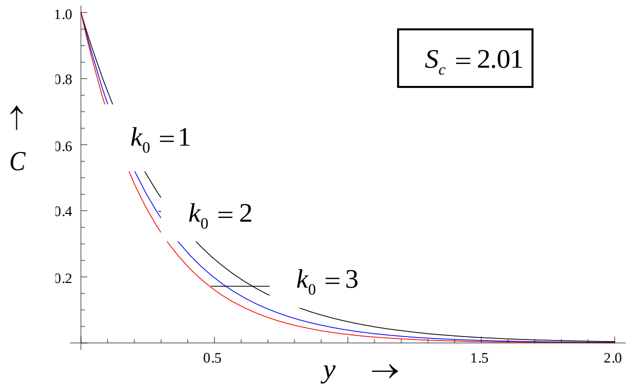

Figure -13: Concentration profile for different chemical parameter k_0

The values of skin friction and Nusselt number are tabulated in table-1 for different parameters. When the values of M, E, G_r and G_m are increased (keeping other parameters constant) the value of τ also gets increased. But if values of K, k_0, P_r and S_c are increased, the value of τ gets decreased (keeping other parameters constant).

When the values of M, G_m and P_r are increased (keeping other parameters constant) the value of Nu is also get increased. But if values of K, k_0, G_r, E and S_c are increased, the value of τ gets decreased (keeping other parameters constant).

Conclusions:

In this paper a theoretical analysis has been done to study the effect of chemical reaction on free convection MHD flow through a porous medium bounded by vertical surface. Solutions for the model have been derived by using two- term perturbation method. Some conclusions of the study are as below:

- Velocity increases with the increase in E, k_0, G_r, K and G_m and decreases with increase in M.
- Temperatures of the fluid increase when M and k_0 are increased.
- Concentration of the fluid decreases when k_0 is increased.
- Skin friction increases when magnetic field parameter, Eckert number, thermal Grashof number and mass Grashof number are increased but decreases when chemical parameter, Prandtl number, Schmidt number and permeability parameter are increased.
- Nusselt number increases when magnetic field parameter, mass Grashof number and Prandtl number are increased but decreases when thermal Grashof number, Eckert number, chemical parameter, permeability parameter and Schmidt number are increased.

Notation

C - non-dimensional fluid concentration; C' - concentration, mol/m^3; C_∞ - fluid concentration far away from the wall, mol/m^3; C_p - specific heat at a constant pressure, J/(kg.deg); D - mass diffusivity, m^2/sec; E - Eckert number; G_m - mass Grashof number; G_r - thermal Grashof number; g - gravitational acceleration, m/sec^2; K - non-dimensional permeability coefficient of a porous medium; k_0 - non-dimensional rate of a chemical reaction; K_c - rate of chemical reaction, sec^{-1}; K_p - permeability of a porous medium, m^2; M - magnetic field parameter; Nu - Nusselt number; P_r - Prandtl number; S_c - Schmidt number; T_∞ - fluid temperature far away from the wall, ^{0}C; T' - temperature, ^{0}C; u', v' velocity components, m/sec; u - non-dimensional velocity; v_0 - suction velocity, m/sec; x', y' - space coordinates, m; y - non-dimensional space coordinate; α - thermal conductivity, W/(m.deg); β - coefficient of volume expansion, 1/deg; β^* - coefficient of volume expansion with concentration, m^3/mol; θ - non-dimensional temperature; υ - kinematic viscosity, m^2/sec; ρ - fluid density, kg/m^3; τ - non-dimensional skin friction.

Subscripts and Superscripts:

w - wall; ∞ - far away from the wall; 0 and 1 - zero and first orders.

Acknowledgment

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References:

[1]Gupta AS, Pop I, Soundalgekar VM. Free convection effects on the flow past an accelerated vertical plate in an incompressible dissipative fluid. Rev. Roum. Sci. Techn.-Mec. Apl. 1979; 24: 561-568.

[2]Hossain MA, Takhar HS. Radiation effect on mixed convection along a vertical plate with uniform surface temperature. Heat and Mass Transfer. 1996; 31: 243-248.

[3]Jaiswal BS, Soundalgekar VM. Oscillating plate temperature effects on a flow past an infinite porous plate with constant suction and embedded in a porous medium, Heat and Mass Transfer, 2001; 37: 125-131.

[4]Jha BK, Prasad R, Rai S. Mass transfer effects on the flow past an exponentially accelerated vertical plate with constant heat flux, Astrophysics and Space Science. 1991; 181: 125-134.
[5]Lighthill MJ. The response of laminar skin friction and heat transfer to fluctuations in the stream velocity. Proc. R. Society. 1954; 224, A: 1-23.
[6]Manivannan K, Muthucumaraswamy R, Thangaraj V. Radiation and chemical reaction effects on isothermal vertical oscillating plate with variable mass diffusion. Thermal Science. 2009; no. 2, 13: 155-162.
[7]Muthucumaraswamy R, Janakiraman. MHD and Radiation effects on moving isothermal vertical plate with variable mass diffusion. Theoretical Applied Mechanics. 2006; no. 1, 33: 17-29.
[8]Muthucumaraswamy R, Sathappan KE, Natarajan R. Mass transfer effects on exponentially accelerated isothermal vertical plate. International Journal of Applied Mathematics and Mechanics. 2008; no. 6, 4: 19-25.
[9]Prasad VR, Reddy NB, Muthucumaraswamy R. Radiation and mass transfer effects on two- dimensional flow past an impulsively started infinite vertical plate. Int. Journal of Thermal Science. 2007; no. 12, 46: 1251-1258.
[10]Rajput US, Kumar S. Radiation effects on MHD flow through porous media past an exponentially accelerated vertical plate with variable temperature. ISST Journal of Mathematics & Computing System. 2010; no. 2, 5: 1-7.
[11]Rajput US, Kumar S. MHD flow past an impulsively started vertical plate with variable temperature and mass diffusion. Applied Mathematical Sciences. 2011; no. 3, 5: 149-157.
[12]Rajput US, Kumar S. Rotation and radiation effects on MHD flow past an impulsively started vertical plate with variable temperature. Int. Journal of Math. Analysis. 2011; no. 24, 5: 1155-1163.
[13]Rajput US, Sahu PK. Combined effects of chemical reactions and heat generation/absorption on unsteady transient free convection MHD flow between two long vertical parallel plates through a porous medium with constant temperature and mass diffusion. Elixir Appl. Math. 2011; 39: 4855- 4859.
[14]Raptis A, Perdikis C. Radiation and free convection flow past a moving plate. Int. Journal of Applied Mechanics and Engineering. 1999; 4: 817-821.
[15]Soundalgekar VM. Free convection effects on the oscillatory flow an infinite, vertical porous, plate with constant suction I. Proc. R. Society. 1973; A, 333: 25- 36.
[16]Soundalgekar VM, Takhar HS. Radiation effects on free convection flow past a semi-infinite vertical plate. Modeling, Measurement and Control. 1993; B 51: 31- 40.
[17]Vajravelu K, Sastri KS. Correction to `Free convection effects on the oscillatory flow an infinite, vertical porous, plate with constant suction I'. Proc. R. Society. 1977; A, 51: 31-40.

Table-1: Skin friction and Nusselt number for different parameters

M	K	G_r	G_m	E	k_0	P_r	S_c	τ	Nu
2	1	5	5	0.01	1	0.71	2.01	3.79	0.70
3	1	5	5	0.01	1	0.71	2.01	4.08	1.13
2	2	5	5	0.01	1	0.71	2.01	3.65	0.66
2	1	10	5	0.01	1	0.71	2.01	6.02	0.55
2	1	5	10	0.01	1	0.71	2.01	5.50	0.96
2	1	10	10	0.01	1	0.71	2.01	8.02	0.69
2	1	5	5	0.3	1	0.71	2.01	5.89	0.62
2	1	5	5	0.01	0	0.71	2.01	4.86	1.26
2	1	5	5	0.01	1	7	2.01	1.06	17.24
2	1	5	5	0.01	1	0.71	3	3.43	0.67