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Configurational modeling and analysis of multicomponent parallel system with imperfect failure detection, repair/replacement and common cause failure Lakhan Singh and Aqil Ahmad

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____ ABSTRACT

This paper investigates a two multi-component unit parallel system model with imperfect detection and common cause failure. A single repair facility is always available with the system but whenever a regular detector fails in detection of the failure cause, then the unit goes for replacement for which a single replacement facility is always available. Using regenerative point technique various measure of system effectiveness are obtained. The behaviour of MTSF and profit function have been studied in a particular case.

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Keywor ds

System states, Sojourn times, Reliability analysis.

Introduction

Repair maintenance, inspection and replacement are the some important means for increasing the availability of the system. Under these tools, a number of authors including (2-5) have carried out the stochastic analysis of various redundant systems. Most of these studies are concentrated at the stochastic analysis of two unit redundant systems. A very few authors including (1) have analysed the multi-component redundant systems by using supplementary variable technique.

In the literature of reliability commonly, perfect pre repair inspection is considered. But there are some situations where pre repair inspection may be imperfect. In case of imperfect inspection the failed unit should be replaced.

Under the above facts in view, the purpose of present study is to analyse a two multicomponent unit parallel system with imperfect failure detection, repair/replacement and common cause failure.

2. Model Description and Assumptions

In the present model we assume:

(i)The system consists of two identical units arranged in parallel configuration. Each unit consists of 'c', ($c \ge 1$) repairable independent components arranged in series network.

(ii)Each unit of the system has two modes normal (N) and total failure (F).

(iii)Upon failure of a unit it goes for the detection to determine which component failure is the cause of unit failure. For this purpose a detector is always available with the system. But the detector is not always successful in detection.

(iv)If detection is successful, then the failed component goes for repair otherwise the failed unit goes for replacement.

(v)A single repair facility as well as a single replacement facility is always available with the system.

(vi)During the repair the detector is busy in monitoring of the repair process.

(vii)Unit/component fails either due to its normal failure or due to common cause failure. Common cause failure is defined as any instance where multiple units or components fail due to a single cause. A common cause failure may occur due to voltage fluctuation, temperature, fire, operational and maintenance error, etc.

(viii)The repaired discipline is FCFS and the repaired unit is as good as new.

(ix)Detection time and failure time distributions are taken as negative exponential where as repair time distributions are taken as general.

Using regenerative point technique, the following economic measures of interest to system designers and managers have been obtained.

(1)Reliability of the system and mean time to system failure (MTSF)

(2)Expected up time of the system during (o, t) and in steady state.

(3)Expected busy period of the repairman during (o, t) and in steady state.

(4)The cost benefit analysis of the system is carried out by using the above characteristics.

3. Notations and symbols for the system states

- E : set of regenerative states, i.e., $E = \{S_0 S_9\}$
- α : failure rate of the operative unit

 γ : detection rate of the failed unit

 η : rate of replacement of the failed component

p/q : probability of success/failure of detection

pc: probability with which the cth component found failed

 β : common cause failure rat

 $G_c(\bullet)$: c.d.f. of repair time of the failed cth component

 $H(\bullet)$: c.d.f. of repair time due to common cause

*, ~ : symbols for Laplace and Laplace Stieltjes transform i.e.,

$$Q_{ij}(s) = {}^{\Delta}\int e^{-st}d Q_{ij}(t),$$

$$q_{ij}^*(s) = \int e^{-st} q_{ij}(t) dt$$

$$A(t) B(t) = \int_{0}^{t} B(t-u) A(u) du$$

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Symbols for the states of the system

No. : unit is operative

 F_D/F_{WD} : unit in failure mode and under detection/waiting for detection

 F_{cr} : unit in failure mode and under repair due to the failure of cth component.

 F_R/F_{WR} : unit in failure mode and under replacement/waiting for replacement

Using the above notations and symbols the possible states and the possible transitions among the states are shown in fig. 1. **4. Transition probabilities and sojourn times**

The non-zero elements p_{ij} of the transition probability matrix (tpm) for the considered system model are as follows:

$$p_{01} = \frac{2\alpha}{(\beta + 2\alpha)}, \qquad p_{09} = \frac{\beta}{(\beta + 2\alpha)},$$
$$p_{1,2c} = p_c \frac{p\gamma}{(\alpha + \beta + \gamma)},$$

^{Δ}The limit of integration when O to ∞ are not mentioned.

$$p_{13} = \frac{q\gamma}{(\alpha + \beta + \gamma)}, \qquad p_{16} = \frac{\alpha}{(\alpha + \beta + \gamma)},$$
$$p_{19} = \frac{\beta}{(\alpha + \beta + \gamma)},$$
$$p_{2c,0} = \mathfrak{G}_{c}^{0}(\alpha + \beta),$$

$$p_{2c,1}^{(4c)} = \frac{\alpha}{(\alpha + \beta)} [1 - \mathcal{O}(\alpha + \beta)],$$



$$p_{39} = \frac{\beta}{(\alpha + \beta + \eta)}, \quad p_{51} = \frac{\eta}{(\eta + \gamma)}, \quad p_{5,7c} = p_c \frac{p\gamma}{(\eta + \gamma)},$$

$$p_{58} = p_c \frac{q\gamma}{(\eta + \gamma)}, \quad p_{6,4c} = p_c \cdot p, \quad p_{65} = q,$$

$$p_{7c,2c} = 1 - \mathfrak{C}_c(\eta), \quad p_{7c,3} = \mathfrak{C}_c(\eta),$$
It can easily be verified that
$$p_{01} + p_{09} = 1, \quad p_{1,0} + {}^{1}\Sigma p_{1,2c} + p_{13} + p_{19} = 1,$$

$$p_{30} + p_{35} + p_{39} = 1, p_{51} + \Sigma p_{5,7c} + p_{58} = 1,$$

$$p_{7c,2c} + p_{7c,3} = 1$$

Using the formula $\psi_i = \int P(T_i > t)dt$ for the mean sojourn time in state $S_i \in E$, its values for various states are:

$$\begin{split} \psi_{0} &= \frac{1}{(2\alpha + \beta)}, \ \psi_{1} = \frac{1}{(\alpha + \beta + \gamma)}, \\ \psi_{2c} &= \frac{[1 - \mathring{G}_{c}^{b}(\alpha + \beta)]}{(\alpha + \beta)}, \\ \psi_{3} &= \frac{1}{(\alpha + \beta + \eta)}, \ \psi_{4c} = 1 = \psi_{9}, \ \psi_{5} = \frac{1}{(\eta + \gamma)}, \\ \psi_{6} &= \frac{1}{\gamma}, \ \psi_{7c} = \frac{[1 - \mathring{G}_{c}^{b}(\eta)]}{\eta}, \ \psi_{8} = \frac{1}{\eta}, \end{split}$$

5. Reliability analysis

The reliability of the system when it starts operation from $S_{\rm i}$ $\epsilon\,E$ is given by

 $R_i(t) = P[T_i > t]$

By probabilistic arguments, we have the following recursion relations:

$$\begin{aligned} R_{0}(t) &= Z_{0}(t) + q_{01} \odot R_{1}(t) \\ R_{1}(t) &= Z_{1}(t) + \Sigma q_{1,2c}(t) \odot R_{2c}(t) + q_{13}(t) \odot R_{3}(t) \\ R_{2c}(t) &= Z_{2c}(t) + q_{2c,0}(t) \odot R_{0}(t) \\ R_{3}(t) &= Z_{3}(t) + q_{30} \odot R_{0}(t) \\ \text{where,} \\ &= Z_{2}(t) - e^{-(2\alpha + \beta)t} \end{aligned}$$

$$Z_{0}(t) = e^{-(\alpha + \beta + \gamma)t},$$
$$Z_{1}(t) = e^{-(\alpha + \beta + \gamma)t},$$
$$Z_{2c}(t) = \overline{G}(t) e^{-(\alpha + \beta)t},$$
$$Z_{3}(t) = e^{-(\alpha + \beta + \eta)t}.$$

Taking Laplace transform of relations (1-4) and simplifying for $R^*_{\Omega}(s)$, we obtain,

$$R_{o}^{*}(s) = \frac{Z_{0}^{*} + q_{01}^{*}Z_{1}^{*} + q_{01}^{*}\Sigma q_{1,2c}Z_{2c}^{*} + q_{01}^{*}q_{30}^{*}Z_{3}^{*}}{1 - q_{01}^{*}q_{20}^{*}\Sigma q_{1,2c}^{*} - q_{01}^{*}q_{13}^{*}q_{30}^{*}}$$
(5)

¹ The Σ is extended from o to c, whenever used.

For brevity, the argument 's' is omitted from $q_{is}^{*}(s)$ and

$$Z_i^*(s)$$

By taking inverse Laplace transform of (5), we can obtain the expression for R(t). Using the usual formula, the MTSF is given by

$$E(T_{0}) = \lim_{s \to 0} R_{0}^{*}(s) = \frac{\psi_{0} + p_{01}\psi_{1} + p_{01}\Sigma p_{1,2c}\psi_{2c} + p_{01}p_{30}\psi_{3}}{1 - p_{01}p_{20}\Sigma p_{12c} - p_{01}p_{13}p_{30}}$$
(6)

6. Availability analysis

From the theory of regenerative process, the pointwise availabilities $A_i(t)$ (i = 0 \square 9) of the system are seen to satisfy the following recursion relations

$$\begin{split} &A_0(t) = Z_0(t) + q_{01}\left(t\right) \circledcirc A_1(t) + q_{09}\left(t\right) \circledcirc A_9\left(t\right) \\ &A_1(t) = Z_1(t) + \Sigma q_{1,2c}(t) \And A_{2c}(t) + q_{13}(t) \circledcirc A_3(t) + q_{16}\left(t\right) \circledcirc A_6(t) \\ &+ q_{19}\left(t\right) \circledcirc A_9(t) \end{split}$$

$$A_{2c}(t) = Z_{2c}(t) + q_{2c,0}(t) \odot A_0(t) + q_{2c,1}^{(4c)}(t) \odot A_1(t) + q_{2c,9}(t) \odot A_0(t)$$

$$\begin{aligned} A_{3}(t) &= Z_{3}(t) + q_{30}(t) \odot A_{0}(t) + q_{35}(t) \odot A_{5}(t) + q_{39}(t) \odot A_{9}(t) \\ A_{4c}(t) &= q_{4c,1}(t) \odot A_{1}(t) \\ A_{5}(t) &= q_{51}(t) \odot A_{1}(t) + \Sigma q_{5,7c}(t) \odot A_{7c}(t) + q_{58}(t) \odot A_{8}(t) \\ A_{6}(t) &= \Sigma q_{6,4c}(t) \odot A_{4c}(t) + q_{65}(t) \odot A_{5}(t) \end{aligned}$$

$$A_{7c}(t) = q_{7c,2c}(t) \odot A_{2c} + q_{7c,3}(t) \odot A_{3}(t)$$

 $A_8(t) = q_{83}(t) \odot A_3(t)$

$$A_{9}(t) = q_{90}(t) \odot A_{0}(t)$$
(7-16)

$$A_0^*(s) = N_2(s)/D_2(s)$$
 (17)

where,

$$\begin{split} \mathbf{N}_{2}(s) &= [(1 - \Sigma q_{1,2c}^{*} q_{2c,1}^{*(4c)}) (1 - q_{35}^{*} \Sigma q_{5,7c}^{*} q_{7c,3}^{*} - q_{58}^{*} q_{83}^{*} q_{35}^{*}) \\ &- (q_{16}^{*} q_{65}^{*} + q_{13}^{*} q_{35}^{*}) (q_{51}^{*} + \Sigma q_{5,7c}^{*} q_{7c,2c}^{*} q_{2c,1}^{*(4c)}) - q_{16}^{*} \Sigma q_{6,4c}^{*} q_{4c,1}^{*} \\ &\{ 1 - q_{35}^{*} (q_{58}^{*} q_{83}^{*} + \Sigma q_{5,7c}^{*} q_{7c,3}^{*}) \}] Z_{0}^{*} + q_{01}^{*} [1 - q_{35}^{*} (q_{58}^{*} q_{83}^{*} + \Sigma q_{5,7c}^{*} q_{7c,3}^{*})] Z_{1}^{*} \\ &+ q_{01}^{*} \Sigma q_{5,7c}^{*} q_{7c,2c}^{*} (q_{16}^{*} q_{65}^{*} + q_{13}^{*} q_{35}^{*}] Z_{2c}^{*} + q_{65}^{*} q_{01}^{*} q_{16}^{*} (q_{58}^{*} q_{83}^{*}) \\ &+ \Sigma q_{5,7c}^{*} q_{7c,3}^{*}) Z_{3}^{*} \end{split} \tag{18}$$

$$-(1-q_{09}^{*}q_{90}^{*})(1-\Sigma q_{1,2c}^{*}q_{2c,1}^{*(4c)})] - (q_{13}^{*}q_{35}^{*} + q_{16}^{*}q_{65}^{*})$$

$$[q_{01}^{*}\Sigma q_{5,7c}^{*}q_{7c,2c}^{*}(q_{2c,0}^{*} + q_{2c,9}^{*}q_{90}^{*}) + (1-q_{09}^{*}q_{90}^{*})(q_{51}^{*} + q_{90}^{*}(q_{09+}^{*}q_{01}^{*}q_{19}^{*}) + (1-q_{09}^{*}q_{90}^{*})[(1-\Sigma q_{1,2c}^{*}q_{2c,1}^{*(4c)}) - q_{16}^{*}\Sigma q_{6,4c}^{*}q_{4c,1}^{*}]$$

$$-q_{01}^{*}\Sigma q_{1,2c}^{*}(q_{2c,0}^{*} + q_{2c,9}^{*}q_{90}^{*})[1-q_{35}^{*}(q_{58}^{*}q_{83}^{*} + \Sigma q_{5,7c}^{*}q_{7c,3}^{*})]$$
For brevity, the argument's' is omitted from

 $q_{ij}^{*}(s)$ and $Z_{i}^{*}(s)$. Now, the steady state availability is given by

$$A_0 = N_2 / D_2 \tag{20}$$
 where,

$$\begin{split} N_2 &= [(1 - \Sigma p_{1,2c} p_{2c,1}^{(4c)}) \{1 - p_{35} (\Sigma p_{5,7c} p_{7c,3} - p_{58})\} \{1 - p_{35} + (p_{58} + \Sigma p_{5,7c} p_{7c,3})\}] \psi_0 \\ &\quad (p_{51} + \Sigma p_{5,7c} p_{7c,2c} p_{2c,1}^{(4c)}) - p_{16} \Sigma p_{6,4c} \ \{1 - p_{35} + (p_{58} + \Sigma p_{5,7c} p_{7c,3})\}] \psi_0 \\ &\quad + p_{01} [1 - p_{35} (p_{58} + \Sigma p_{5,7c} p_{7c,3}] \psi_1 + p_{01} \Sigma p_{5,7c} p_{7c,2c} \ (p_{16} p_{65} + p_{13} p_{35})] \psi_{2c} \\ &\quad + \ p_{65} p_{01} p_{16} (p_{58} + \Sigma p_{5,7c} p_{7c,3}) \psi_3 \end{split}$$

$$\begin{split} D_2 = & [(1 - \Sigma p_{1,2c} p_{2c,1}^{(4c)}) + p_{35} p_{2c,1}^{(4c)} \{\Sigma p_{1,2c} p_{58} + \Sigma p_{5,7c} (\Sigma p_{1,2c} p_{7c,3} - p_{13} p_{7c,2c})\} \\ &- p_{16} p_{65} (1 + \Sigma p_{5,7c} p_{7c,2c} p_{2c,1}^{(4c)}) - p_{16} \Sigma p_{6,4c} \{1 - p_{35} (p_{58} + \Sigma p_{5,7c} p_{7c,3})\} \\ &- p_{35} (p_{58} + \Sigma p_{5,7c} p_{7c,3} + p_{13} p_{51})] \psi_0 + [p_{01} - p_{01} p_{35} (p_{58} + \Sigma p_{5,7c} p_{7c,3})] \psi_1 \\ &+ [p_{01} \Sigma p_{1,2c} \{1 - p_{35} (p_{58+} \Sigma p_{5,7c} p_{7c,3})\} + p_{01} \Sigma p_{5,7c} p_{7c,2c} (p_{13} p_{35} + p_{16} p_{55})] \psi_5 + p_{01} p_{16} [1 - p_{35} (p_{58} + \Sigma p_{5,7c} p_{7c,3})] \psi_4 \\ &+ [p_{01} \Sigma p_{5,7c} p_{7c,3})] \psi_6 + + [p_{01} \Sigma p_{5,7c} (p_{13} p_{35} + p_{16} p_{65})] \psi_7 \\ &+ [p_{01} p_{58} (p_{13} p_{35} + p_{16} p_{65})] \psi_8 + [(p_{01} p_{1,2c} - p_{09} p_{2c,1}^{(4c)}) \{\Sigma p_{1,2c} + \Sigma p_{5,7c} p_{7c,2c} (p_{13} p_{35} + p_{16} p_{65})] \psi_7 \\ &+ [p_{01} p_{16} p_{39} p_{65} - p_{35} p_{09} - (p_{19} + \Sigma p_{1,2c} p_{2c,9}) p_{35} p_{01}] \} \\ &- p_{09} p_{51} (p_{13} p_{35} + p_{16} p_{65}) + p_{01} (p_{19} + p_{13} p_{39}) + p_{09}] \psi_9$$

Let $B_i^r(t)/B_i^{cc}(t)$ be Theires paper depresentations (7repairman is busy in repair of the failed cth component/failed components due to common cause respectively at time 't'. Similarly, $B^R(t)/B^D(t)$ be the respected probabilities that the replacement facility/detector cum inspection person is busy in replacement/detection or inspection of the failed unit or component respectively at time 't'. When system initially starts from state $S_i \in E$. Using simple probabilistic arguments as in availability analysis, the system of integral equations for $B_i^{cc}(t), B_i^r(t), B_i^R(t)$ and $B_i^D(t)$ in terms of L.T. can be found. (¹⁸The steady state probabilities

 $B_{O}^{cc}, B_{O}^{r}, B_{O}^{R} \text{ and } B_{O}^{D} \text{ are given respectively as follows:} \\B_{O}^{cc} = N_{3}/D_{2}, B_{O}^{r} = N_{4}/D_{2}, B_{O}^{R} = N_{5}/D_{2} \text{ and } B_{O}^{D} = N_{6}/D_{2} \\ \text{where,} \\N_{3} = [p_{13}p_{35} + p_{16} p_{65}) \{ p5,7cp7c,2c} q_{7c}^{*} q_{7c}^{*} q_{2c} q_{2c}^{*} q_{140}^{*} q_{1} \} - q_{01}^{*} (q_{30}^{*} + q_{150}^{*} q_{130}^{*} + q_{150}^{*} q_{130}^{*} + q_{150}^{*} q_{130}^{*} q_{130}^{*} + q_{150}^{*} q_{130}^{*} q_{130}^{*} + q_{150}^{*} q_{130}^{*} q_{130}^{*} + q_{150}^{*} q_{120}^{*} q_{120}^{*} q_{120}^{*} q_{130}^{*} + q_{110}^{*} q_{110}^{*$

 $N_{5} = p_{01} [p_{13} + \Sigma p_{5,7c} p_{7c,3} (p_{13} p_{35} + p_{16} p_{65}) + p_{16} p_{65} (p_{58} + \Sigma p_{5,7} p_{7c,3})] \psi_{3}$ + $p_{01} (p_{13} p_{35} + p_{16} p_{65}) \psi_{5} + p_{01} \Sigma p_{5,7c} (p_{13} p_{35} + p_{16} p_{65}) \psi_{7c}$ + $p_{01} p_{58} (p_{13} p_{35} + p_{16} p_{65}) \psi_{8}$

$$\begin{split} N_6 &= p_{01} \left[1 - p_{35} (p_{58} + \Sigma p_{5,7c} p_{7c,3}) \right] \psi_1 + \left[\Sigma p_{5,7c} p_{7c,2c} \left(p_{13} p_{35} + p_{16} p_{65} \right) \right. \\ &+ \left. \Sigma p_{1,2c} \left\{ 1 + p_{35} \left(p_{58} + \Sigma p_{5,7c} p_{7c,3} \right) \right\} \right] \psi_{2c} + p_{01} \left(p_{13} p_{35} + p_{16} p_{65} \right) \psi_5 \end{split}$$

 $+ p_{01}p_{16} [1 - p_{35} (p_{58} + \sum p_{5,7c}p_{7c,3}] \psi_6 + p_{01}\sum p_{5,7c} (p_{13}p_{35} + p_{16}p_{65}) \psi_7 c$ and D₂ is the same as in availability analysis.

8. Profit function analysis

The net expected profit incurred during (o, t) is given by P (t) = Expected total revenue during (o, t) – Expected total expenditure during (o, t)

$$= K_{0} \ \mu_{up}(t) - K_{1} \ \mu_{b}^{cc}(t) - K_{2} \ \mu_{ub}^{r}(t) - K_{3} \ \mu_{b}^{R}(t) \ - K_{4} \ \mu_{b}^{D}(t)$$

where K_o be the revenue per unit up time by the system and K_1/K_2 be the amount spent per unit of time in repair of the components failed due to common cause/ cth component and K_3/K_4 respectively be the amount spent per unit of time in replacement/detection or inspection of failed components.

Also,
$$\mu_{up}(t) = \int_{0}^{t} A_{o}(u) du \text{ s.t. } \mu_{up}^{*}(s) = A_{o}^{*}(s)/s$$

In the similar way, $\mu_b^{c.c}(t)$, $\mu_b^r(t)$, $\mu_b^R(t)$ and $\mu_b^D(t)$ can be defined. Now the expected profit per unit time in steady state is given by

$$P = \lim_{t \to \infty} \frac{P(t)}{t} = \lim_{s \to 0} s^2 P^* (s)$$
$$= K_0 A_0 - K_1 B_0^{cc} - K_2 B_0^r - K_3 B_0^R - K_4 B_0^D$$

9. Particular case

When the repair time distributions for common cause failure and failed cth component are taken as exponential with parameters $\Box \Box \Box$ and $\Box \Box$ respectively, the changes are as follows:

$$\begin{split} p_{2c,0} &= \frac{\lambda_2}{(\lambda_2 + \alpha + \beta)}, \qquad p_{2c,1}^{(4c)} = \frac{\lambda_2}{(\lambda_2 + \alpha + \beta)}, \qquad p_{2c,9} = \frac{\beta}{(\lambda_2 + \alpha + \beta)}\\ p_{7c,2c} &= \frac{\eta}{(\lambda_2 + \eta)}, \qquad p_{7c,3,} = \frac{\lambda_2}{(\lambda_2 + \eta)} \end{split}$$

10. Graphical analysis

For study of the system behaviour graphically, we plot curves for two important measures of system effectiveness of MTSF and profit function w.r.t. failure rate of the operative unit (α).

Figure 2 shows the variation in MTSF w.r.t. α for different values of λ_1 0.01, 0.02, 0.03 when other parameters are kept fixed as $\gamma = 0.01$, $\eta = 0.02$, $\beta = 0.001$, $\lambda_c = 0.025$ and p =. From graph, it is obvious that the MTSF rapidly decreases, initially and uniformity decreases for large values of α . It is further observed that the values of MTSF increase as the value of repair rate λ_1 increases.

Behavior of Profit function w.r.t. failure rate of operative unit (α) for different values of γ and λ_1



Fig. 3 represents the change in profit function w.r.t. α for different values of γ and λ_1 when the other parameters are kept fixed as C_0 = 5000, C_1 = 600, C_2 = 350 C_3 = 400 and C_4 = 250 while other parameters take some values as in graphical study of MTSF except the values of γ and λ_1 . It is clear from the graph that the profit function decreases as the value of failure rate α increases while the value of profit function increases as the value of γ and λ_1 increase.

References

1.L.R. Goyal, Rakesh Gupta and Praveen Gupta, "A Single Unit Multicomponent System Subject to Various Type of Failures". Microelectron Reliab., 23, 813-816 (1983).

2.M. Yamashiro, "A Reparable System with Partial and Catastrophic Failure Model". Microelectron Reliab., 21, 97-101 (1981).

3.Rakesh Gupta and Gaurav Varshney, "A Two Identical Unit Parallel System with Geometric failure and Repair Time Distribution." Vol. 32, No. 1-4, 127-136 (2007).

4.S.H. Sim and J. Endrenyi, "A Failure-Repair Model With Minimal and Major Maintenance". IEEETR on Reliability, Vol. 55, No. 1, pp. 134-140 (2005).

5.V.K. Gupta and Jai Singh, "Behaviour and Profit Analysis of a Redundant System with Imperfect Switch". JOMASS, Vol. 3, No. 1, June -2007.