



A geometric process model for two different components of warm standby system with priority

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ABSTRACT

In this paper, two different components and one repairman of with warm standby system priority is studied. Assume that two components of working time, standby time, the repair time after fault and the repair time after standby fault are subject to the different exponential distribution and the repair of the component 1 is geometry repair and the repair of the component 2 is as good as new. The component 1 has priority in use and repair. By using geometric process theory, the supplementary variable method and the Laplace transform, the reliability indices of the system are obtained.

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Introduction

The warm standby repairable system model is one of the more important in the reliability theory and application. In the reliability study, the assumption that components are as good as new. In practical problems, the component after repair is often to be worse in performance than before. With the growth in using, the life of the component will be shorter and shorter, and the repair time after fault will be longer and longer. Therefore, the study the "repair of non-new" component of the system has some theoretical and practical significance. Literature [3] considers a deteriorating cold standby repairable system consisting of two dissimilar components and one repairman.

This paper is the application of the geometric process theory to consider a warm standby system consisting of two different components and one repairman. Assume that two components of working time, standby time, the repair time after fault and the repair time after standby fault are subject to different exponential distribution and the repair of the component 1 is a geometry repair and the repair of the component 2 is as good as new, and component 1 has priority in use and repair. By using geometric process theory, the supplementary variable method and Laplace transform, the reliability indices of model are obtained.

Assumption Model

We studied a warm standby repairable system of two components and one repairman with priority by making the following assumptions.

Assumption 1. Initially, the two components are both new, and component 1 is in working state while component 2 is in warm standby state.

Assumption 2. Assume that component 1 after repair are not "as good as new" and follow a geometric process repair. When both components are good, component 1 has the higher use priority than component 2. Even if component 2 is working, it must be switched into the warm standby state as soon as component 1 after failure has been repaired, and it becomes the working state immediately. When both components fail (i.e. the

system is down), component 1 has the higher repair priority than component 2. Even if the repairman is repairing component 2 at this state, he must switch to component 1. He will work on the repair of component 2 after completing the repair on component 1.

Assumption 3. Assume that the time interval between the completion of the $(n-1)$ th repair and the completion of the n th repair of component 1 is called the n th cycle (i.e. the n th repair cycle) of component i , where $i = 1, 2, n = 1, 2, \dots$. Let

$X_n^{(i)}, A_n^{(i)}, Y_n^{(i)}$ and $B_n^{(i)}$ ($i = 1, 2$) be, respectively, working time, standby time, the repair time after fault and the repair time after standby fault of component i in the n th cycle, their distributions are

$$F_n^{(1)}(t) = F(a^{n-1}t) = 1 - \exp\{-a^{n-1}\lambda_1 t\}$$

$$V_n^{(1)}(t) = 1 - \exp\{-v_1 t\}$$

$$G_n^{(1)}(t) = G(b^{n-1}t) = 1 - \exp\{-b^{n-1}\mu_1 t\}$$

$$W_n^{(1)}(t) = 1 - \exp\{-w_1 t\}$$

$$F_n^{(2)}(t) = 1 - \exp\{-\lambda_2 t\}$$

$$V_n^{(2)}(t) = 1 - \exp\{-v_2 t\}$$

$$G_n^{(2)}(t) = 1 - \exp\{-\mu_2 t\}$$

$$W_n^{(2)}(t) = 1 - \exp\{-w_2 t\}$$

All the random variables $X_n^{(i)}, A_n^{(i)}, Y_n^{(i)}$ and $B_n^{(i)}$ ($i = 1, 2, n = 1, 2, \dots$) are mutually independent.

System Analysis

Let $N(t)$ denote the state of the system at time t th, the system states are as following:

$N(t) = 0$, the component 1 is working and the component 2 is in warm standby;

$N(t) = 1$, the component 1 is working and the component 2 is due to failure in the repair standby;

$N(t) = 2$, the component 1 is working and the component 2 is due to failure of the repair work;

$N(t) = 3$, the component 2 is working and the component 1 is due to failure of the repair work;

$N(t) = 4$, the component 1 is due to failure in the repair work and the component 2 is due to failure in the suspending repair work;

$N(t) = 5$, the component 2 is due to failure in the repair standby and the component 1 is failure;

$N(t) = 6$, the component 2 is due to failure of the repair work and the component 1 is failure.

Obviously, $\{N(t), t \geq 0\}$ is a random process, $\Omega = \{0, 1, 2, 3, 4, 5, 6\}$ is a state space, $W = \{0, 1, 2, 3\}$ is the working state set, $F = \{4, 5, 6\}$ is the fault state set. So it is not a Markov process, but it can be extended to Markov process by using the supplementary variable method. Let $S(t)$ be the number of the cycle of component 1 at time t th, $\{N(t), S(t), t \geq 0\}$ constitute two-dimensional Markov process, the state probability denote

$$p_{jk}(t) = P\{N(t) = j, S(t) = k\}, j \in \Omega, k = 1, 2, \dots$$

Through analysis, the state probability can be solved from the following differential equation

$$\left(\frac{d}{dt} + a^{k-1}\lambda_1 + v_2\right)p_{0k}(t) = w_2p_{1k}(t) + \mu_2p_{2k}(t) + b^{k-2}\mu_1p_{3k-1}(t), (k \geq 2)$$

$$\left(\frac{d}{dt} + w_2 + a^{k-1}\lambda_1\right)p_{1k}(t) = v_2p_{0k}(t) + b^{k-2}\mu_1p_{5k}(t), (k \geq 2)$$

$$\left(\frac{d}{dt} + \mu_2 + a^{k-1}\lambda_1\right)p_{2k}(t) = b^{k-2}\mu_1p_{4k-1}(t) + b^{k-2}\mu_1p_{6k}(t), (k \geq 2)$$

$$\left(\frac{d}{dt} + \lambda_2 + b^{k-1}\mu_1\right)p_{3k}(t) = a^{k-1}\lambda_1p_{0k}(t), (k \geq 2)$$

$$\left(\frac{d}{dt} + b^{k-1}\mu_1\right)p_{4k}(t) = \lambda_2p_{3k}(t), (k \geq 2)$$

$$\left(\frac{d}{dt} + b^{k-1}\mu_1\right)p_{5k}(t) = a^{k-1}\lambda_1p_{1k}(t), (k \geq 2)$$

$$\left(\frac{d}{dt} + b^{k-1}\mu_1\right)p_{6k}(t) = a^{k-1}\lambda_1p_{2k}(t), (k \geq 2)$$

$$\left(\frac{d}{dt} + \lambda_1 + v_2\right)p_{01}(t) = 0$$

$$\left(\frac{d}{dt} + w_2 + \lambda_1\right)p_{11}(t) = v_2p_{01}(t)$$

$$p_{21}(t) = 0$$

$$\left(\frac{d}{dt} + \mu_1 + \lambda_2\right)p_{31}(t) = \lambda_2p_{01}(t)$$

$$\left(\frac{d}{dt} + \mu_1\right)p_{41}(t) = \lambda_2p_{31}(t)$$

$$\left(\frac{d}{dt} + \mu_1\right)p_{51}(t) = \lambda_1p_{11}(t)$$

$$\left(\frac{d}{dt} + \mu_1\right)p_{61}(t) = \lambda_2p_{21}(t)$$

Initial conditions

$$p_{01}(0) = 1, p_{0k}(0) = 0 (k \geq 2), p_{jk}(0) = 0, j = 1, 2, 3, 4, 5, 6; k = 1, 2, \dots$$

Let $p_{jk}(t)$ denote to the Laplace transform

of $p_{jk}^*(s) = \int_0^\infty e^{-st} p_{jk}(t) dt$, and on the differential equation of both sides for the Laplace transform respectively,

$$(s + a^{k-1}\lambda_1 + v_2)p_{0k}^*(s) = w_2p_{1k}^*(s) + \mu_2p_{2k}^*(s) + b^{k-2}\mu_1p_{3k-1}^*(s), (k \geq 2) \tag{1}$$

$$(s + w_2 + a^{k-1}\lambda_1)p_{1k}^*(s) = v_2p_{0k}^*(s) + b^{k-2}\mu_1p_{5k}^*(s), (k \geq 2) \tag{2}$$

$$(s + \mu_2 + a^{k-1}\lambda_1)p_{2k}^*(s) = b^{k-2}\mu_1p_{4k-1}^*(s) + b^{k-2}\mu_1p_{6k}^*(s), (k \geq 2) \tag{3}$$

$$(s + \lambda_2 + b^{k-1}\mu_1)p_{3k}^*(s) = a^{k-1}\lambda_1p_{0k}^*(s), (k \geq 2) \tag{4}$$

$$(s + b^{k-1}\mu_1)p_{4k}^*(s) = \lambda_2p_{3k}^*(s), (k \geq 2) \tag{5}$$

$$(s + b^{k-1}\mu_1)p_{5k}^*(s) = a^{k-1}\lambda_1p_{1k}^*(s), (k \geq 2) \tag{6}$$

$$(s + b^{k-1}\mu_1)p_{6k}^*(s) = a^{k-1}\lambda_1p_{2k}^*(s), (k \geq 2) \tag{7}$$

$$(s + \lambda_1 + v_2)p_{01}^*(s) = 1 \tag{8}$$

$$(s + \lambda_1 + w_2)p_{11}^*(s) = v_2p_{01}^*(s) \tag{9}$$

$$p_{21}^*(s) = 0 \tag{10}$$

$$(s + \mu_2 + \lambda_1)p_{31}^*(s) = \lambda_1p_{01}^*(s) \tag{11}$$

$$(s + \mu_1)p_{41}^*(s) = \lambda_2p_{31}^*(s) \tag{12}$$

$$(s + \mu_1)p_{51}^*(s) = \lambda_1p_{11}^*(s) \tag{13}$$

$$(s + \mu_1)p_{61}^*(s) = \lambda_2p_{21}^*(s) \tag{14}$$

Substituting Eqs.(2)- (7) into Eq.(1), then we have

$$p_{0k}^*(s) = \frac{\beta b^{k-2}\mu_1 a^{k-2}\lambda_1(s + b^{k-1}\mu_1)(\lambda_2\mu_2 + \alpha) \cdot p_{0k-1}^*(s)}{\alpha(s + \lambda_2 + b^{k-2}\mu_1)(s + b^{k-2}\mu_1)[(s + a^{k-1}\lambda_1 + v_2)\beta - w_2v_2(s + b^{k-1}\mu_1)]} \tag{15}$$

and we make a mark:

$$\alpha = (s^2 + s\mu_2 + sa^{k-1}\lambda_1 + sb^{k-1}\mu_1 + \mu_2b^{k-1}\mu_1),$$

$$\beta = (s^2 + sw_2 + sa^{k-1}\lambda_1 + sb^{k-1}\mu_1 + w_2b^{k-1}\mu_1)$$

According to Eqs.(1)-(14), we have,

$$p_{02}^*(s) = \frac{\{\lambda_1\lambda_2\mu_1\mu_2(s + b\mu_1) + \lambda_1\mu_1(s + \mu_1)\delta_1\}\delta_2}{(s + a\lambda_1 + v_2)(s + \lambda_2 + \mu_1)(s + \lambda_1 + v_2)(s + \mu_1)\delta_1[\delta_2 - v_2(s + b\mu_1)]} \tag{16}$$

and

$$\delta_1 = (s + a\lambda_1 + \mu_2)(s + b\mu_1) - a\lambda_1\mu_1,$$

$$\delta_2 = (s + a\lambda_1 + w_2)(s + b\mu_1) - a\lambda_1\mu_1$$

Substituting Eqs.(15) - (16) into Eqs.(1) - (7), then we have

$$p_{0k}^*(s) = p_{02}^*(s) \cdot A_k, (k \geq 3)$$

$$p_{1k}^*(s) = \frac{v_2(s + a^{k-1}\lambda_1 + w_2)(s + b^{k-1}\mu_1)}{(s + a^{k-1}\lambda_1 + w_2)\beta} \cdot p_{02}^*(s) \cdot A_k$$

$$p_{2k}^*(s) = \frac{b^{k-2}\mu_1\lambda_2a^{k-2}\lambda_1(s + b^{k-1}\mu_1)}{\alpha(s + \lambda_2 + b^{k-2}\mu_1)(s + b^{k-2}\mu_1)} \cdot p_{02}^*(s) \cdot A_{k-1}$$

$$p_{3k}^*(s) = \frac{a^{k-1}\lambda_1}{(s + \lambda_2 + b^{k-1}\mu_1)} \cdot p_{02}^*(s) \cdot A_k$$

$$p_{4k}^*(s) = \frac{a^{k-1}\lambda_1\lambda_2}{(s + b^{k-1}\mu_1)(s + \lambda_2 + b^{k-1}\mu_1)} \cdot p_{02}^*(s) \cdot A_k$$

$$P_{5k}^*(s) = \frac{a^{k-1}\lambda_1 v_2}{\beta} \cdot P_{02}^*(s) \cdot A_k$$

$$P_{6k}^*(s) = \frac{a^{k-1}\lambda_1 a^{k-2}\lambda_2 b^{k-2}\mu_1}{(s+b^{k-1}\mu_1)\alpha(s+\lambda_2+b^{k-2}\mu_1)(s+b^{k-2}\mu_1)} \cdot P_{02}^*(s) \cdot A_{k-1}$$

and

$$A_k = \prod_{i=3}^k \frac{\beta b^{i-2}\mu_1 a^{i-2}\lambda_1 (s+b^{i-1}\mu_1)(\lambda_2\mu_2+\alpha)}{\alpha(s+\lambda_2+b^{i-1}\mu_1)(s+b^{i-2}\mu_1)[(s+a^{i-1}\lambda_1+v_2)\beta-w_2v_2(s+b^{i-1}\mu_1)]}$$

Important Result and Proof

Theorem 1. Let $A(t)$ be the availability of the system in time t th,

$$A(t) = P\{N(t) \in W\} = \sum_{k=1}^{\infty} [p_{0k}(t) + p_{1k}(t) + p_{2k}(t) + p_{3k}(t)].$$

So the system's steady state availability is

$$A = \lim_{t \rightarrow +\infty} A(t) = \lim_{s \rightarrow 0} sA^*(s) = 0 \tag{17}$$

Proof . The Laplace transform of $A(t)$ is

$$A^*(s) = \sum_{k=1}^{\infty} [p_{0k}^*(t) + p_{1k}^*(t) + p_{2k}^*(t) + p_{3k}^*(t)] = [p_{01}^*(t) + p_{11}^*(t) + p_{12}^*(t) + p_{13}^*(t) + p_{02}^*(t) + p_{12}^*(t) + p_{22}^*(t) + p_{32}^*(t)] + \sum_{k=3}^{\infty} [p_{0k}^*(t) + p_{1k}^*(t) + p_{2k}^*(t) + p_{3k}^*(t)]$$

According to Eqs. (8)-(11), we have

$$p_{01}^*(s) = \frac{1}{s + \lambda_1 + v_2}, \quad p_{11}^*(s) = \frac{v_2}{(s + \lambda_1 + v_2)(s + \lambda_1 + w_2)}$$

$$p_{21}^*(s) = 0, \quad p_{31}^*(s) = \frac{\lambda_1}{(s + \lambda_1 + v_2)(s + \lambda_2 + \mu_1)}$$

According to Eqs. (2) - (7), when $k = 2$, we have

$$p_{12}^*(s) = \frac{v_2(s + b\mu_1)}{(s + a\lambda_1 + w_2)(s + b\mu_1) - a\lambda_1\mu_1} \cdot P_{02}^*(s)$$

$$p_{22}^*(s) = \frac{\mu_1\lambda_2\lambda_1(s + b\mu_1)}{(s + \mu_1)(s + \lambda_2 + \mu_1)(s + \lambda_1 + \mu_2)[(s + a\lambda_1 + \mu_2)(s + b\mu_1) - a\lambda_1\mu_1]}$$

$$p_{32}^*(s) = \frac{a^{k-1}\lambda_1}{s + \lambda_2 + b\mu_1} \cdot P_{02}^*(s)$$

By using the Tauberian theorem, we get (17). This completes the proof. The steady state availability is 0, and it is consistent with physical intuition. In fact, as component 1 "repair of non-new", the work time is shorter and shorter, and the repair work will be longer and longer, which means that the time ($t \rightarrow +\infty$) limit availability tends to 0.

Theorem 2. Let $R(t)$ be system reliability and the Laplace transform of $R(t)$ be

$$R^*(s) = \left[\frac{1}{s + \lambda_1 + v_2} + \frac{v_2}{(s + \lambda_1 + w_2)(s + \lambda_1 + v_2)} + \frac{\lambda_1}{(s + \mu_1 + \lambda_2)(s + \lambda_1 + v_2)} \right]$$

$$+ \sum_{k=2}^{\infty} \left[1 + \frac{v_2}{s + a^{k-1}\lambda_1 + w_2} + \frac{a^{k-1}\lambda_1}{s + b^{k-1}\mu_1 + \lambda_2} \right] \cdot B_k \cdot q_{01}^*(s) \tag{18}$$

Proof. In order to work out the system reliability $R(t)$, we consider the states with absorption followed by two-dimensional continuous Markov process $\{N^0(t), s(t), t \geq 0\}$. The system of

failure states in $\{N(t), S(t), t \geq 0\}$ are absorbing states, we get $\{N^0(t), s(t), t \geq 0\}$. The state probability denote

$$q_{ij} = P\{N^0(t) = j, s(t) = k\}, j \in \Omega, k = 1, 2L$$

Then

$$R(t) = P(T \geq t) = \sum_{k=1}^{\infty} [q_{0k}(t) + q_{1k}(t) + q_{3k}(t)]$$

Similar to the preceding discussion, the corresponding differential equations are

$$\left(\frac{d}{dt} + a^{k-1}\lambda_1 + v_2\right)q_{0k}(t) = w_2q_{1k}(t) + b^{k-2}\mu_1q_{3k-1}(t), (k \geq 2)$$

$$\left(\frac{d}{dt} + a^{k-1}\lambda_1 + w_2\right)q_{1k}(t) = v_2q_{0k}(t), (k \geq 2)$$

$$\left(\frac{d}{dt} + \lambda_2 + b^{k-1}\mu_1\right)q_{3k}(t) = a^{k-1}\lambda_1q_{0k}(t), (k \geq 2)$$

$$\left(\frac{d}{dt} + \lambda_1 + v_2\right)q_{01}(t) = 0$$

$$\left(\frac{d}{dt} + \lambda_2 + w_2\right)q_{11}(t) = v_2q_{01}(t)$$

$$\left(\frac{d}{dt} + \lambda_2 + \mu_1\right)q_{31}(t) = \lambda_1q_{01}(t)$$

Initial conditions

$$q_{01}(0) = 1, q_{11}(0) = 0, q_{31}(0) = 0, q_{0k}(0) = 0, q_{1k}(0) = 0, q_{3k}(0) = 0, k = 2, 3, \dots$$

Laplace transform of the above equations, we have

$$(s + a^{k-1}\lambda_1 + v_2)q_{0k}^*(s) = w_2q_{1k}^*(s) + b^{k-2}\mu_1q_{3k-1}^*(s), (k \geq 2)$$

$$(s + a^{k-1}\lambda_1 + w_2)q_{1k}^*(s) = v_2q_{0k}^*(s), (k \geq 2)$$

$$(s + \lambda_2 + b^{k-1}\mu_1)q_{3k}^*(s) = a^{k-1}\lambda_1q_{0k}^*(s), (k \geq 2)$$

$$(s + \lambda_1 + v_2)q_{01}^*(s) = 1$$

$$(s + \lambda_1 + w_2)q_{11}^*(s) = v_2q_{01}^*(s)$$

$$(s + \lambda_2 + \mu_1)q_{31}^*(s) = \lambda_1q_{01}^*(s)$$

Then we have

$$q_{0k}^*(s) = \frac{a^{k-2}\lambda_1 b^{k-2}\mu_1 (s + a^{k-1}\lambda_1 + w_2)}{(s + \lambda_2 + b^{k-2}\mu_1)(s + a^{k-1}\lambda_1)(s + a^{k-1}\lambda_1 + v_2 + w_2)} \cdot q_{0k-1}^*(s) = B_k \cdot q_{01}^*(s)$$

$$q_{1k}^*(s) = \frac{v_2}{s + w_2 + a^{k-1}\lambda_1} \cdot B_k \cdot q_{01}^*(s),$$

$$q_{3k}^*(s) = \frac{a^{k-1}\lambda_1}{s + \lambda_2 + b^{k-1}\mu_1} \cdot B_k \cdot q_{01}^*(s)$$

and

$$B_n = \prod_{k=2}^n \frac{a^{k-2}\lambda_1 b^{k-2}\mu_1 (s + a^{k-1}\lambda_1 + w_2)}{(s + \lambda_2 + b^{k-2}\mu_1)(s + a^{k-1}\lambda_1)(s + a^{k-1}\lambda_1 + v_2 + w_2)}$$

According to the initial conditions, we have

$$q_{01}^*(s) = \frac{1}{s + \lambda_1 + v_2}, \quad q_{11}^*(s) = \frac{v_2}{s + \lambda_1 + w_2} \cdot \frac{1}{s + \lambda_1 + v_2},$$

$$q_{31}^*(s) = \frac{\lambda_1}{s + \mu_1 + \lambda_2} \cdot \frac{1}{s + \lambda_1 + v_2}.$$

Then we get (18). This completes the proof.

Theorem 3. The mean time to the first failure

$$\begin{aligned}
 MTTF = & \left[\frac{1}{\lambda_1 + v_2} + \frac{v_2}{(\lambda_1 + v_2)(\lambda_1 + w_2)} + \frac{\lambda_1}{(\mu_1 + \lambda_2)(\lambda_1 + v_2)} \right] \\
 & + \sum_{k=2}^{\infty} \frac{1}{\lambda_1 + v_2} \left[1 + \frac{v_2}{w_2 + a^{k-1}\lambda_1} + \frac{a^{k-1}\lambda_1}{\lambda_2 + b^{k-1}\mu_1} \right] \cdot \prod_{i=2}^k \frac{a^{i-2}\lambda_1 b^{i-2}\mu_1 (a^{i-1}\lambda_1 + w_2)}{a^{i-1}\lambda_1 (\lambda_2 + b^{i-1}\mu_1) (a^{i-1}\lambda_1 + v_2 + w_2)}
 \end{aligned}
 \tag{19}$$

Proof. We can easily complete the proof by use the nature of Laplace transform

$$MTTF = \int_0^{+\infty} R(t)dt = \lim_{s \rightarrow 0} R^*(s).$$

Theorem 4. Let $I(t)$ to be the probability of the repairman idle, and the repairman of the steady state idle probability is

$$I = \lim_{t \rightarrow +\infty} I(t) = \lim_{s \rightarrow 0} sI^*(s) = 0 \tag{20}$$

Proof. When the component 1 is working and component 2 is warm standby, the repairman will not be idle, so the probability of repairman idle at time t is

$$I(t) = P\{N(t) = 0\} = \sum_{k=1}^{\infty} p_{0k}(t).$$

Laplace transform of $I(t)$ is

$$I^*(s) = \sum_{k=1}^{\infty} p_{0k}^*(s) = p_{01}^*(s) + p_{02}^*(s) + \sum_{k=3}^{\infty} p_{0k}^*(s)$$

By the Tauberian theorem, we get (20). This completes the proof. The probability of repairman idle tends to 0. This result is also consistent with intuition, because component 2 can not repair as good as new, and its successive repair time is increasing random. Finally, it almost can not repair and repairman had to go on forever. This also means when $t \rightarrow +\infty$ that the probability of repairman idle is 0.

Conclusion

For the "repair of non-new" component of the warm standby system with priority, the model has some theoretical and practical significance. In this paper, by using the geometric process theory, a warm standby repairable system of two different components and one repairman with priority is studied. It has some practical value. Especially,

When $w_1 = w_2 = 0$, the system considers a geometric process model for two different components of the warm standby system with priority's other situation, which the repair time of the component after fault and the standby time after fault are subject to the same exponential distribution.

When $v_1 = v_2 = 0, w_1 = w_2 = 0$, the system considers a geometric process repair model for a repairable cold standby system with priority in use and repair. Literature [3] has been discussed.

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