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On product of intuitionistic L – fuzzy H-ideals of BF-algebras

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ABSTRACT

This paper introduces the notion of the Generalization of Cartesian Product on Intuitionistic L-fuzzy H-ideals of BF-algebra and deals some simple but interesting results.

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Introduction

Lofti A. Zadeh[9] introduced the notion of a fuzzy subset of a set as a method for representing uncertainty in real physical world. As a generalization of this, Intuitionistic Fuzzy Subset was defined by K.T.Atanassov [2] in 1986. BF-Algebras were introduced by Andrzej Walendziak[1] introduced in 2007. The Fuzzy BF_subalgebras of were developed by A. Borumand Saeid and M. A. Rezvani[3] in 2009. Motivated by this, we have introduced the notions Intuitionistic L-fuzzy BF-subalgebras[5] and Intuitionistic L-fuzzy ideals of BF-algebras[6]. Intuitionistic L-fuzzy ideals of H-ideals of BF-algebras[8] and the Product[7] on Intuitionistic L-fuzzy ideals of a BF-algebras. In this paper, we discuss the Product on Intuitionistic L-fuzzy H-ideals of two BF-algebras and study some results.

Preliminaries

Definition 2.1.[2] Let $L=(L,\leq)$ be complete lattice with an involutive order reversing operation $N : L \rightarrow L$. Then an Intuitionistic L-fuzzy Subset (ILFS) A in a non-empty set X is defined as an object of the form $A=\{< x, \mu_A(x), \nu_A(x) > / \ x \in X\}$ where $\mu_A : X \rightarrow L$ is the degree membership and $\nu_A : X \rightarrow L$ is the degree non-membership of the element $x \in X$ satisfying $\mu_A(x) \leq N(\nu_A(x))$.

Definition 2.2 [1] A BF-algebra is a non-empty set *X* with a consonant 0 and a binary operation * satisfying the following axioms:

i. $x * x = 0$

ii. $x * 0 = x$

iii. $0 * (x * y) = y * x \quad \forall\, x, y \in X$

Definition 2.3.

(1) A non-empty subset S of a BF-algebra X is said to be a BF-subalgebra if

$$x * y \in S \quad \forall x, y \in S.$$

(2) A non-empty subset I of a BF-algebra X is said to be a Ideal of X if

$$(i)\ 0 \in I$$

$$(ii)\ x * y \in I \ \ and \ \ y \in I \ \ imply\ that\ x \in I\ \forall\ x, y \in I.$$

(3) An ideal I of X is called closed

$$if \quad 0 * x \in I \quad \forall x \in I.$$

(4) A non-empty subset I of a BF-algebra X is said to be a H-ideal of X

$$(i)\ 0 \in I$$

$$(ii)\ x * (y * z) \in I\ and\ y \in I\ \ imply\ that\ x * z \in I\ \forall\ x, y z \in I.$$

(5) A H-ideal I of X is called closed if

$$0 * x \in I \quad \forall\ x \in I$$

Definition.2.4. *Let* $(X, *_X, 0_X), (Y, *_Y, 0_Y)$ be two BF-algebras. The Cartesian product of X ×Y is defined to be the set $X \times Y = \{(x, y) / x \in X, y \in Y\}$

In X ×Y we define the product x × y as follows: $(x_1, y_1) *_{X \times Y} (x_2, y_2) = (x_1 *_X x_2\ , y_1 *_Y y_2)$

One can easily verify that the Cartesian product of two BF-algebras is again a BF-algebra.

Definition.2.5.[6] An Intuitionistic L-fuzzy Subset A in a BF-algebra X is said to be an Intuitionistic L-fuzzy BF-ideal of X if

1) $\mu_A(0) \geq \mu_A(x)$

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2) $\nu_A(0) \le \nu_A(x)$

3) $\mu_A(x) \ge \mu_A(x*y) \wedge \mu_A(y)$.

4) $\nu_A(x) \le \nu_A(x*y) \vee \nu_A(y) \quad \forall\, x, y \in X$

Definition.2.6.[6] An Intuitionistic L-fuzzy Subset A in a BF-algebra X is said to be an Intuitionistic L-fuzzy closed BF-ideal of X if

1) $\mu_A(0*x) \ge \mu_A(x)$

2) $\nu_A(0*x) \le \nu_A(x)$

3) $\mu_A(x) \ge \mu_A(x*y) \wedge \mu_A(y)$

4) $\nu_A(x) \le \nu_A(x*y) \vee \nu_A(y) \quad \forall\, x, y \in X$.

Definition.2.7. [8] An intuitionistic L-fuzzy subset A of a BF-algebra X is said to be an intuitionistic L-fuzzy H-ideal of X if

1) $\mu_A(0) \ge \mu_A(x)$

2) $\nu_A(0) \le \nu_A(x)$

3) $\mu_A(x*z) \ge \mu_A(x*(y*z)) \wedge \mu_A(y)$

4) $\nu_A(x*z) \le \nu_A(x*(y*z)) \vee \nu_A(y)$

$\forall\, x, y, z \in X$

Definition.2.8. [8] An Intuitionistic L-fuzzy subset A of a BF - algebra X is said to be an Intuitionistic L-fuzzy Closed H-ideal of X if

1) $\mu_A(0*x) \ge \mu_A(x)$

2) $\nu_A(0*x) \le \nu_A(x)$

3) $\mu_A(x*z) \ge \mu_A(x*(y*z)) \wedge \mu_A(y)$

4) $\nu_A(x*z) \le \nu_A(x*(y*z)) \vee \nu_A(y) \quad \forall\, x, y, z \in X$

Product on intuitionistic L-fuzzy H-ideals of BF-algebras

In this section we introduce the notion of Cartesian Product of two Intuitionistic L-fuzzy H-ideals of two BF-algebras X and Y. We start with the following definition.

Definition.3.1.[7] For any two Intuitionistic L-fuzzy sets A and B of X, their Cartesian Product is defined to be the set $A \times B = (X \times X,\ \mu_A \times \mu_B,\ \nu_A \times \nu_B)$ with the membership and non-membership functions $\mu_A \times \mu_B : X \times X \to L$ and $\nu_A \times \nu_B : X \times X \to L$ such that $(\mu_A \times \mu_B)(x, y) = \mu_A(x) \wedge \mu_B(y)$ and $(\nu_A \times \nu_B)(x, y) = \nu_A(x) \vee \nu_B(y)$ where $\forall\, x, y \in X$.

In the following we extend the above definition to the Cartesian product of two Intuitionistic L-fuzzy sets of any two BF-algebras X and Y.

Definition.3.2.[7] For any two Intuitionistic L-fuzzy sets A and B of X and Y, their Cartesian Product is defined to be the set $A \times B = (X \times Y,\ \mu_A \times \mu_B,\ \nu_A \times \nu_B)$ with the membership and non-membership functions $\mu_A \times \mu_B : X \times Y \to L$ and $\nu_A \times \nu_B : X \times Y \to L$ such that $(\mu_A \times \mu_B)(x, y) = \mu_A(x) \wedge \mu_B(y)$ and $(\nu_A \times \nu_B)(x, y) = \nu_A(x) \vee \nu_B(y)$ where $\forall\, x \in X$ *and* $y \in Y$.

Theorem.3.3. Let A and B be any two Intuitionistic L-fuzzy H-ideals of X and Y. Then $A \times B$ is an Intuitionistic L-fuzzy H-ideal of $X \times Y$.

Proof. Take $(x, y) \in X \times Y$.

Then
$$\begin{aligned}(\mu_A \times \mu_B)(0, 0) &= \mu_A(0) \wedge \mu_B(0)\\ &\ge \mu_A(x) \wedge \mu_B(y)\\ &= (\mu_A \times \mu_B)(x, y)\\ &\forall\, x \in X \text{ and } y \in Y\end{aligned}$$

And
$$\begin{aligned}(\nu_A \times \nu_B)(0, 0) &= \nu_A(0) \vee \nu_B(0)\\ &\le \nu_A(x) \vee \nu_B(y)\\ &= (\nu_A \times \nu_B)(x, y)\\ &\forall\, x \in X \text{ and } y \in Y\end{aligned}$$

Now take $(x_1, y_1), (x_2, y_2)$ *and* $(x_3, y_3) \in X \times Y$.

Then $(\mu_A \times \mu_B)[(x_1, y_1) * (x_3, y_3)]$

$= (\mu_A \times \mu_B)[(x_1 * x_3), (y_1 * y_3)]$

$= \mu_A(x_1 * x_3) \wedge \mu_B(y_1 * y_3)$

$\ge (\mu_A(x_1 * (x_2 * x_3)) \wedge \mu_A(x_2)) \wedge (\mu_B(y_1 * (y_2 * y_3)) \wedge \mu_B(y_2))$

$= (\mu_A(x_1 * (x_2 * x_3)) \wedge \mu_B(y_1 * (y_2 * y_3))) \wedge (\mu_A(x_2) \wedge \mu_B(y_2))$

$= (\mu_A \times \mu_B)[(x_1 * (x_2 * x_3)), (y_1 * (y_2 * y_3))] \wedge (\mu_A \times \mu_B)(x_2, y_2)$

$= (\mu_A \times \mu_B)[(x_1, y_1) * (x_2, y_2) * (x_3, y_3)] \wedge (\mu_A \times \mu_B)(x_2, y_2)$

Also $(\nu_A \times \nu_B)[(x_1, y_1) * (x_3, y_3)]$

$= (\nu_A \times \nu_B)[(x_1 * x_3), (y_1 * y_3)]$

$= \nu_A(x_1 * x_3) \vee \nu_B(y_1 * y_3)$

$\le (\nu_A(x_1 * (x_2 * x_3)) \vee \nu_A(x_2)) \vee (\nu_B(y_1 * (y_2 * y_3)) \vee \nu_B(y_2))$

$= (\nu_A(x_1 * (x_2 * x_3)) \vee \nu_B(y_1 * (y_2 * y_3))) \vee (\nu_A(x_2) \vee \nu_B(y_2))$

$= (\nu_A \times \nu_B)[(x_1 * (x_2 * x_3)), (y_1 * (y_2 * y_3))] \vee (\nu_A \times \nu_B)(x_2, y_2)$

$= (\nu_A \times \nu_B)[(x_1, y_1) * (x_2, y_2) * (x_3, y_3)] \vee (\nu_A \times \nu_B)(x_2, y_2)$ Thus we have proving A $\times$ B is an intuitionistic L-fuzzy H-ideal of X $\times$ Y.

And it can be extended for any n BF-algebras.

Lemma 3.4. Let A and B be two intuitionistic L-fuzzy subsets of X and Y. If A $\times$ B is an intuitionistic L-fuzzy H-ideal of X $\times$ Y then following are true.

(i) $\mu_A(0) \ge \mu_B(y)$ *and* $\mu_B(0) \ge \mu_A(x)$ for all $x \in X,\ y \in Y$.

(ii) $\nu_A(0) \le \nu_B(y)$ *and* $\nu_B(0) \le \nu_A(x)$ for all $x \in X,\ y \in Y$.

Proof Assume $\mu_B(y) > \mu_A(0)$ *and* $\mu_A(x) > \mu_B(0)$ for some $x \in X,\ y \in Y$.

Then $(\mu_A \times \mu_B)(x, y) = \mu_A(x) \wedge \mu_B(y) \ge \mu_B(0) \wedge \mu_A(0) = (\mu_A \times \mu_B)(0,0)$ which is a contradiction.

Similarly, assume $\nu_A(x) < \nu_B(0)$ *and* $\nu_B(y) < \nu_A(0)$ for some $x \in X,\ y \in Y$.

Then $(\nu_A \times \nu_B)(x, y) = \nu_A(x) \vee \nu_B(y) \leq \nu_B(0) \vee \nu_A(0) = (\nu_A \times \nu_B)(0,0)$ which is also a contradiction, thus proving the result.

Theorem.3.5. Let A and B be any two Intuitionistic L-fuzzy subsets of X and Y such that $A \times B$ is an intuitionistic L-fuzzy H-ideal of $X \times Y$. Then either A is an intuitionistic L-fuzzy H-ideal of X or B is an intuitionistic L-fuzzy H-ideal of Y.

Proof. Now by lemma 3.4 if we take $\mu_A(0) \geq \mu_B(y)$ *and* $\nu_A(0) \leq \nu_B(y)$ then

$$(\mu_A \times \mu_B)(0, y) = \mu_A(0) \wedge \mu_B(y) = \mu_B(y)$$

$$and\ (\nu_A \times \nu_B)(0, y) = \nu_A(0) \vee \nu_B(y) = \nu_B(y)$$

…………………….. (1)

Since $A \times B$ is an intuitionistic L-fuzzy H-ideal of $X \times Y$.

$$(\mu_A \times \mu_B)((x_1, y_1) * (x_3, y_3))$$
$$\geq (\mu_A \times \mu_B)[(x_1, y_1) * ((x_2, y_2) * (x_3, y_3))]$$
$$\wedge (\mu_A \times \mu_B)(x_2, y_2)$$

…………………. (2)

Putting $x_1 = x_2 = x_3 = 0$ in (2) we get,

$$(\mu_A \times \mu_B)((0, y_1) * (0, y_3))$$
$$\geq (\mu_A \times \mu_B)[(0, y_1) * ((0, y_2) * (0, y_3))] \text{ i.e.}$$
$$\wedge (\mu_A \times \mu_B)(0, y_2)$$

$$(\mu_A \times \mu_B)(0, y_1 * y_3)$$
$$\geq (\mu_A \times \mu_B)[0, (y_1 * (y_2 * y_3))] \wedge (\mu_A \times \mu_B)(0, y_2)$$

Using equation (1) in (3) we have $\mu_B(y_1 * y_3) \geq \mu_B(y_1 * (y_2 * y_3)) \wedge \mu_B(y_2)$

In the similar way we can prove $\nu_B(y_1 * y_3) \leq \nu_B(y_1 * (y_2 * y_3)) \vee \nu_B(y_2)$

This proves that B is an Intuitionistic L- fuzzy H-ideal of Y.

Theorem.3.6. Let A and B be any two Intuitionistic L-fuzzy H-ideals X and Y. Then $A \times B$ is an Intuitionistic L-fuzzy H-ideal of $X \times Y$ if and only if $(\mu_A \times \mu_B)(x, y)$ and $(\overline{\nu_A \times \nu_B})(x, y)$ are L-fuzzy H-ideals of $X \times Y$.

Proof. Let $A \times B$ is an Intuitionistic L-fuzzy H-ideal of $X \times Y$.

Clearly $(\mu_A \times \mu_B)(x, y) = \mu_A(x) \wedge \mu_B(y)$ is L-fuzzy H-ideal of $X \times Y$.

We have

$$(\nu_A \times \nu_B)(x, y) = \nu_A(x) \vee \nu_B(y)$$
$$\Rightarrow 1 - (\overline{\nu_A} \times \overline{\nu_B})(x, y) = (1 - \overline{\nu_A(x)}) \vee (1 - \overline{\nu_B(y)})$$

$$\Rightarrow 1 - \{(1 - \overline{\nu_A(x)}) \vee (1 - \overline{\nu_B(y)})\} = (\overline{\nu_A} \times \overline{\nu_B})(x, y)$$
$$\Rightarrow (\overline{\nu_A} \times \overline{\nu_B})(x, y) = \overline{\nu_A(x)} \wedge \overline{\nu_B(y)}.$$

Thus $(\overline{\nu_A} \times \overline{\nu_B})(x, y) = \overline{\nu_A(x)} \wedge \overline{\nu_B(y)}$ is L-fuzzy H-ideal of $X \times Y$.

Conversely, assume $(\mu_A \times \mu_B)(x, y)$ and $(\overline{\nu_A \times \nu_B})(x, y)$ are L-fuzzy H-ideal of $X \times Y$.

Now $A \times B = (X \times Y, \mu_A \times \mu_B, \nu_A \times \nu_B)$.

Since

$$(\overline{\nu_A} \times \overline{\nu_B})(x, y) = \overline{\nu_A(x)} \wedge \overline{\nu_B(y)}$$
$$\Rightarrow (\nu_A \times \nu_B)(x, y) = \nu_A(x) \vee \nu_B(y),$$

we can easily observe that $A \times B$ is an Intuitionistic L-fuzzy H-ideal of $X \times Y$.

Theorem.3.7. Let A and B be any ILFS of X and Y. A and B are Intuitionistic L-fuzzy H-ideals of X and Y if and only if $\Box(A \times B) = (X \times Y, \mu_A \times \mu_B, \overline{\mu_A} \times \overline{\mu_B})$ and $\Diamond(A \times B) = (X \times Y, \overline{\nu_A} \times \overline{\nu_B}, \nu_A \times \nu_B)$ are Intuitionistic L-fuzzy H-ideals of $X \times Y$.

Proof. Since

$$(\mu_A \times \mu_B)(x, y) = \mu_A(x) \wedge \mu_B(y)$$
$$\Rightarrow (\overline{\mu_A} \times \overline{\mu_B})(x, y) = \overline{\mu_A(x)} \vee \overline{\mu_B(y)}$$

and

$$(\nu_A \times \nu_B)(x, y) = \nu_A(x) \vee \nu_B(y)$$
$$\Rightarrow (\overline{\nu_A} \times \overline{\nu_B})(x, y) = \overline{\nu_A(x)} \wedge \overline{\nu_B(y)}$$

the proof is clear.

Theorem.3.8. Let A and B be any two Intuitionistic L-fuzzy Closed H-ideals of X and Y. Then $A \times B$ is an Intuitionistic L-fuzzy Closed H-ideal of $X \times Y$.

Proof. Take $(x, y) \in X \times Y$.

Then $(\mu_A \times \mu_B)((0,0) * (x, y)) = (\mu_A \times \mu_B)(0 * x, 0 * y)$

$$= \mu_A(0 * x) \wedge \mu_B(0 * y)$$
$$\geq \mu_A(x) \wedge \mu_B(y)$$
$$= (\mu_A \times \mu_B)(x, y)$$
$$\forall\ x \in X \text{ and } y \in Y$$

Now

$$(\nu_A \times \nu_B)((0,0) * (x, y)) = (\nu_A \times \nu_B)(0 * x, 0 * y)$$
$$= \nu_A(0 * x) \vee \nu_B(0 * y)$$
$$\leq \nu_A(x) \vee \nu_B(y)$$
$$= (\nu_A \times \nu_B)(x, y)$$
$$\forall\ x \in X \text{ and } y \in Y$$

Thus $A \times B$ is an Intuitionistic L-fuzzy Closed H-ideal of $X \times Y$.

And it can be extended for any n BF-algebras.

One can easily prove theorem 3.6 and 3.7 for Intuitionistic L-fuzzy Closed H-ideals.

Conclusion

In this paper, we have extended the notion of the Cartesian product of Intuitionistic L-fuzzy sets to the notion of the generalized Cartesian product of Intuitionistic L-fuzzy H-ideals of BF-algebras. In [4] we have discussed the concept of Intuitionistic L-fuzzy subalgebras of BG-algebras. We expect that all the results proved in this paper can be proved for BG-algebras.

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