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Independent, perfect and connected neighborhood number of an M-strong fuzzy graph

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ABSTRACT

A neighborhood set $S \subseteq V$ of an *M*-strong fuzzy graph G is said to be independent neighborhood set if S is independent. S is said to be perfect neighborhood set if all $u, v \in S, u \neq v$, the full fuzzy sub graphs $\langle\langle N[u]\rangle\rangle$ and $\langle\langle N[v]\rangle\rangle$ are edge disjoint. Also S is said to be connected neighborhood set if full fuzzy sub graph $\langle\langle S\rangle\rangle$ is connected. The minimum scalar cardinality taken over all independent neighborhood set (perfect neighborhood set and connected neighborhood set) is called independent neighborhood number (perfect neighborhood number and connected neighborhood number). In this paper, these numbers are determined for various known fuzzy graphs and its relationship with some other known parameters of G is investigated.

Introduction

Rosenfeld introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretical concepts such as path, cycle, connectedness etc., in the year 1975[4]. M-strong fuzzy graph and some oprations are defined on M-strong fuzzy graph are introduced by K.R. Bhurani and A. Battou in the year 2003[1]. *Mordeson and Peng* introduced the concept of fuzzy line graph and developed its basic properties in the year 1993[3]. The neighborhood numbers (n_0) of various known fuzzy graphs are introduced in by *S. Ismail Mohideen and A. Mohamed Ismayil* in the year 2010[2]. Independent neighborhood number, perfect neighborhood number and connected neighborhood number in crisp graph are introduced by *E. Sambathkumar and Prabha S. Neeralagi* in the year 1995[6]. In this paper, Independent neighborhood number, perfect neighborhood number and connected neighborhood number of various fuzzy graphs are discussed. The relationship between these numbers and other well known parameters vertex independent number and cut vertex are derived.

Preliminaries

Definition: Let V be a finite non empty set and E be the collection of two element subsets of V. A *fuzzy graph* $G = (\sigma, \mu)$ is a set with two functions $\sigma: V \to [0,1]$ and $\mu: E \to [0,1]$ such that $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$.

Definition: Let $G = (\sigma, \mu)$ be a fuzzy graph on *V* and $S \subseteq V$. Then the *scalar cardinality* of S is defined by $\sum_{u \in S} \sigma(u)$. The *order (p)* and *size (q)* of a fuzzy graph $G = (\sigma, \mu)$ are the scalar cardinality of *V* and *E* respectively.

Definition: A fuzzy graph $G_1 = (\sigma_1, \mu_1)$ is called the *fuzzy sub graph* of G if $\sigma_1(u) \leq \sigma(u)$ for all $u \in V$ and $\mu_1(u,v) \leq \sigma_1(u) \wedge \sigma_1(v) \wedge \mu(u,v)$ for all $u, v \in V$. A fuzzy graph $G_1 = (\sigma_1, \mu_1)$ is called the *full fuzzy sub graph* of G if $\sigma_1(u) = 0$ or $\sigma_1(u) = \sigma(u)$ for all $u \in V$ and $\mu_1(u,v) = 0$ *or* $\mu_1(u,v) = \sigma_1(u) \wedge \sigma_1(v) \wedge \mu(u,v)$ for all $u, v \in V$.

Definition: A fuzzy graph $G_1 = (\sigma_1, \mu_1)$ is called the *fuzzy sub graph induced by V_1* if $\sigma_1(u) \leq \sigma(u)$ for all $u \in V_1$ and $\mu_1(u,v) \leq \sigma_1(u) \wedge \sigma_1(v) \wedge \mu(u,v)$ for all $u, v \in V_1$ and is denoted by $\langle V_1\rangle$.

A fuzzy graph $G_1 = (\sigma_1, \mu_1)$ is called the *full fuzzy sub graph induced by V_1* if $\sigma_1(u) = \sigma(u)$ for all $u \in V_1$ and $\mu_1(u,v) = \sigma_1(u) \wedge \sigma_1(v) \wedge \mu(u,v)$ for all $u, v \in V_1$ and is denoted by $\langle\langle V_1\rangle\rangle$.

Definition: A fuzzy graph $G_1 = (\sigma_1, \mu_1)$ is called the *spanning fuzzy sub graph* of G if $\sigma_1(u) \neq 0$ and $\sigma_1(u) \leq \sigma(u)$ for all $u \in V_1$ and $\mu_1(u,v) \leq \sigma_1(u) \wedge \sigma_1(v) \wedge \mu(u,v)$ for all $u, v \in V$. A fuzzy graph $G_1 = (\sigma_1, \mu_1)$ is called the *full spanning fuzzy sub graph* of G if $\sigma_1(u) \neq 0$ and $\sigma_1(u) = \sigma(u)$ for all $u \in V$ and $\mu_1(u,v) = 0$ or $\mu_1(u,v) = \sigma_1(u) \wedge \sigma_1(v) \wedge \mu(u,v)$ for all $u, v \in V$.

Definition: Underlying crisp graph is a fuzzy graph $G = (\sigma, \mu)$ is denoted by $G^* = (\sigma^*, \mu^*)$ where $\sigma^* = \{u \in V / \sigma(u) > 0\}$ and $\mu^* = \{(u,v) \in E / \mu(u,v) > 0\}$.

Definition: A fuzzy path in a fuzzy graph G is a sequence of distinct vertices u_0 , u_1, $u_2, \ldots u_n$ such that

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$\mu(u_{i-1},u_i) = \sigma(u_{i-1}) \wedge \sigma(u_i)$, $1 \le i \le n$, $n>0$ is called the length of the path. The path is called a fuzzy cycle if $u_0=u_n$, $n \ge 0$.

Definition: Two vertices $u, v \in V$ of a fuzzy graph are said to be *fuzzy independent* if $\mu(u,v) < \sigma(u) \wedge \sigma(v)$. A set $S \subseteq V$ is said to be *fuzzy independent set* of G if any two vertices of S is independent.

Definition: A fuzzy graph $G = (\sigma, \mu)$ is said to be *complete fuzzy graph* if $\mu(u,v) = \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$. The *complement* of a fuzzy graph $G = (\sigma, \mu)$ is a fuzzy graph $\bar{G} = (\bar{\sigma}, \bar{\mu})$, where $\bar{\sigma}(u) = \sigma(u)$ for all $u \in \bar{V}$ and $\mu(u,v) = \sigma(u) \wedge \sigma(v) - \mu(u,v)$ for all $u, v \in \bar{V}$.

Definition: A fuzzy graph $G = (\sigma, \mu)$ is said to be bipartite if the vertex set V can be partitioned into two non-empty sets V_1 and V_2 such that $\mu(u,v) < \sigma(u) \wedge \sigma(v)$ if $u, v \in V_1$ or $u, v \in V_2$ and if $\mu(u,v) \le \sigma(u) \wedge \sigma(v)$ for all $u \in V_1$ and $v \in V_2$. Then G is called fuzzy bipartite fuzzy graph. A fuzzy graph $G = (\sigma, \mu)$ is said to be complete bipartite if the vertex set V can be partitioned into two non-empty sets V_1 and V_2 such that $\mu(u,v) = \sigma(u) \wedge \sigma(v)$ if $u, v \in V_1$ or $u, v \in V_2$ and if $\mu(u,v) = \sigma(u) \wedge \sigma(v)$ for all $u \in V_1$ and $u \in V_2$. Then G is called fuzzy bipartite fuzzy graph.

Definition: A fuzzy graph $G = (\sigma, \mu)$ is said to be *connected* if every pair of vertices has at least one fuzzy path between them, otherwise it is *disconnected*.

Definition: A *cut vertex* of a fuzzy graph *G* is one which whose removal disconnect the fuzzy graph.

Definition: A vertex *u* of a fuzzy graph $G = (\sigma, \mu)$ is said to be *isolated vertex* if $\mu(u,v) < \sigma(u) \wedge \sigma(v)$ for all $v \in V \backslash u$. An edge *e* = *(u, v)* of a fuzzy graph is called an *effective edge* if $\mu(u,v) = \sigma(u) \wedge \sigma(v)$. Here the vertex *u* is adjacent to *v* and the edge *e* is incident to *u* and *v*. A fuzzy graph $G = (\sigma, \mu)$ is said to be *M-strong fuzzy graph* [6] if $\mu(u,v) = \sigma(u) \wedge \sigma(v)$ for all $(u,v) \in E$.

Definition: Let $u, v \in V$ and *e* = *(u, v)* then $N(u) = \{v \in V : \mu(u,v) = \sigma(u) \wedge \sigma(v)\}$ is called *open neighborhood of u* and $N[u] = N(u) \cup \{u\}$ is called *closed neighborhood of u*.

Definition: A set $S \subseteq V$ is a *neighborhood set (n-set)* of an *M*-strong fuzzy graph $G = (\sigma, \mu)$ if $G = \bigcup_{u \in S} \langle\langle N[u] \rangle\rangle$, where $\langle\langle N[u] \rangle\rangle$ is a full induced fuzzy sub graph of G. The *neighborhood number* of an *M*-strong fuzzy graph G is the minimum scalar cardinality taken over all n-set of G and is denoted by n_0.

Independent and Perfect neighborhood number

Definition: A neighborhood set $S \subseteq V$ of an *M*-strong fuzzy graph *G* is said to be an *independent neighborhood set* if *S* is independent and is denoted by *in-set*. The minimum scalar cardinality taken over all *in*-set is called *independent neighborhood number* of *G* and is denoted by n_i.

Definition: A neighborhood set $S \subseteq V$ of an *M*-strong fuzzy graph *G* is said to be a *perfect neighborhood set (pn-set)* if all $u, v \in S, u \ne v$, the full induced fuzzy sub graphs $\langle\langle N[u] \rangle\rangle$ and $\langle\langle N[v] \rangle\rangle$ are edge disjoint and is denoted by *pn*-set. The minimum scalar cardinality taken over all *pn*-set is called *perfect neighborhood number* of *G* and is denoted by n_p.

Throughout this paper *M*-strong fuzzy graph $G = (\sigma, \mu)$ alone are considered. *M*-strong fuzzy graph $G = (\sigma, \mu)$ is simply denoted by *G* in the following sections.

Note: n_i-set and n_p-set are denoted by independent neighborhood set and perfect neighborhood set with minimum scalar cardinality respectively.

Remarks:

1)There are fuzzy graph which has neither an *in*-set nor a *pn*-set. For example, any fuzzy cycle of odd length $n \ge 5$ has neither an *in*-set nor a *pn*-set.

2) Every *pn*-set is an in-set but converse is not true. Example: For example, from the fuzzy graphs G_1 and G_2 given in Fig.3.1, the neighborhood set *{u,v}* of G_1 is a *pn*-set and an in-set whereas the neighborhood set *{u,v}* of G_2 is an *in*-set but not a *pn*-set.

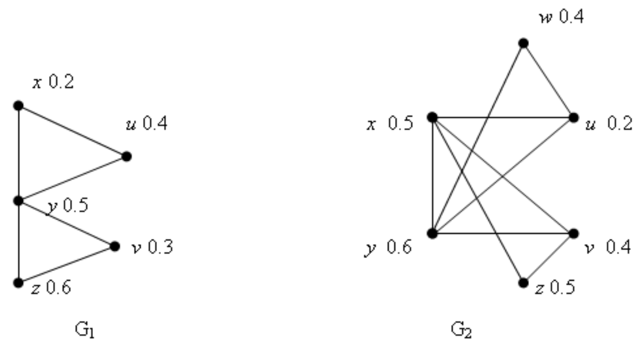


Fig.3.1

Definition A fuzzy graph *G* is an *independent neighborhood fuzzy graph(inf*-graph) if *G* has an *in*-set. A fuzzy graph *G* is a *perfect neighborhood fuzzy graph* (*pnf*-graph) if G has a *pn*-set.

Remark: Every *pnf*-graph is an *inf*-graph but not conversely. For example, from the Fig.3.1 G_1 is a *pnf*-graph and also *inf*-graph. G_2 is an *inf*-graph but not a *pnf*-graph.

Observation: *In a fuzzy graph* G, $n_0 = n_i = n_p = min_{u \in V}\, \sigma(u)$ *if G is* K_σ.

Observation: *In a fuzzy graph G,* $n_0 = n_i = n_p = \min\{|\sigma_1|, |\sigma_2|\}$ *if G is* K_{σ_1,σ_2}.

Theorem: *A fuzzy graph G is an inf-graph if and only if there exists a neighborhood set S such that V-S is a vertex cover*.

Proof: For a neighborhood set *S* of a fuzzy graph *G*, the set *V-S* is a vertex cover if and only if *S* is independent.

Theorem: *If G is a pnf-graph of order p, then* $0 < n_0 \le n_i \le n_p \le \beta_0 < p$.

Proof: Every vertex independent set is a perfect neighborhood set, every perfect neighborhood set is an independent neighborhood set and every independent neighborhood set is a vertex neighborhood set.

Hence, $0 < n_0 \le n_i \le n_p \le \beta_0 \le p$.

Example: Consider the following pnf-graph.

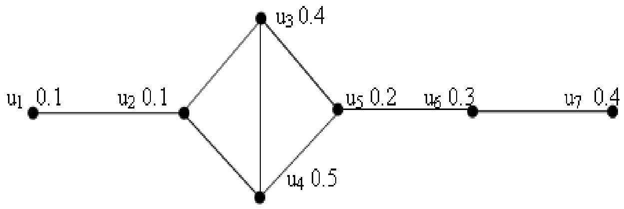


fig.3.2

Here, n_0-set={u_2, u_5, u_6}, n_i-set={u_2, u_5, u_7}, n_p-set={u_1, u_3, u_6}and β_0-set={u_1, u_3, u_7}

$0 < n_0 = 0.6 \leq n_i = 0.7 \leq n_p = 0.8 \leq \beta_0 = 0.9 \leq p = 2.0$.

Theorem: *Given real numbers a, b, c, d and p with* $0 < a \leq b \leq c \leq d \leq p$ *where* $d = b + c - a$ *and* $p \leq b + c + 1$, *then there exists a fuzzy graph G of order p such that* $n_0=a$, $n_i=b$, $n_p=c$ *and* $\beta_0 = d$.

Proof: Consider a fuzzy path $G_1 = (\sigma_1, \mu_1)$ with $2l$ vertices $u_1, u_2, u_3, \ldots, u_{2l}$ as given in the Fig .3.3.

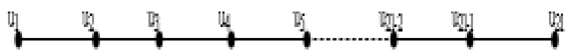


Fig 3.3 : G_1

Let | {$u_1, u_3, u_5, \ldots, u_{2l-3}, u_{2l}$} | = | {$u_2, u_4, u_6, \ldots, u_{2l-2}, u_{2l-1}$} | and $\sigma(u_{2l-1}) = \sigma(u_{2l})$.

Let S= {$u_2, u_4, u_6, \ldots, u_{2l-2}, u_{2l-1}$} is a minimum neighborhood set with scalar cardinality a. Hence $n_0(G_1)=a$, where $a \in R$.

In G_1, Replace the vertex u_3 by a complete fuzzy graph with 4vertices and delete an edge from it and add an effective edge with pendent vertex u at u_{2l-2} such that $\sigma(u) = \sigma(u_1)$ and $\sigma(v) = \sigma(w) = \sigma(u_3)$. Then the fuzzy graph $G_2 = (\sigma_2, \mu_2)$ is obtained (as in Fig. 3.4).

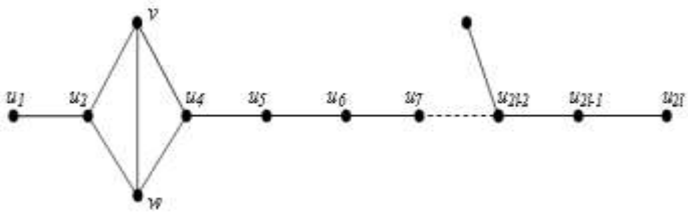


Fig 3.4 : G_2

Now, S is also a minimum n-set of G_2. Suppose $b - a = t_1$ and $c - b = t_2$.

Add pendent effective edges, with scalar cardinality of the pendent vertices as t_1, at each of the vertices u_{2l-2} and u_{2l-1}. Also, add pendent effective edges with scalar cardinality of the pendent vertices as $t_2 - \sigma(u_1)$ at u_2. Then the fuzzy graph $G_3 = (\sigma_3, \mu_3)$ is obtained (as in Fig. 3.5)

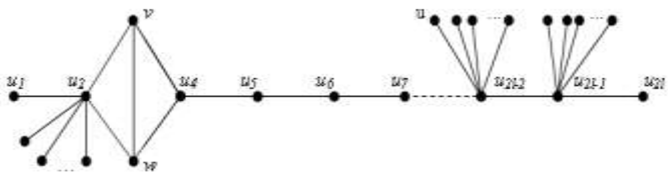


Fig 3.5 : G_3

Now, we show that G_3 is the required fuzzy graph. Let S_1, S_2, S_3 be the set of all pendent vertices adjacent to u_2, u_{2l-2} and u_{2l-1} respectively. Then $|S_1| = t_2$, $|S_2| = t_1 + \sigma(u)$, $|S_3| = t_1 + \sigma(u_{2l})$. Clearly, the set S is a minimum n-set of G_3 and $n_0(G_3)=a$.

Also {$u_2, u_4, u_6, \ldots, u_{2l-2}$}$\cup S_3$ is a minimum in-set with scalar cardinality is

$a - \sigma(u_{2l-1}) + t_1 + \sigma(u_{2l}) = a + b - a = b.$

Further the set

$S_1 \cup \{v\} \cup \{u_5, u_7, \ldots u_{2l-3}\} \cup S_2 \cup \{u_{2l-1}\}$ is a minimum pn-set with scalar cardinality

$t_2 + \sigma(v) + n_0 + \sigma(u_1) - \sigma(u_3) - \sigma(u_{2l}) + \sigma(u) + \sigma(u_{2l-1}) = t_2 + a + t_1 = c - b + a + b - a = c$

. Thus $n_0 = a, n_i = b, n_p = c$.

The set $S_1 \cup \{v\} \cup \{u_5, u_7, \ldots u_{2l-3}\} \cup S_2 \cup S_3$ is a maximum independent set with scalar cardinality is

$t_2 + \sigma(v) + n_0 - \sigma(u_1) - \sigma(u_3) - \sigma(u_{2l}) + t_1 + \sigma(u) + t_1 + \sigma(u_{2l})$

$= t_2 + a + t_1 + t_1 = c - b + a + b - a + b - a = b + c - a$

.The order of G_3 is the scalar cardinality of $S_1 \cup \{u_2, v, w\} \cup \{u_4, u_5, u_6, \ldots, u_{2l}\} \cup S_2 \cup S_3$.

$p =$

$t_2 + \sigma(u_2) + \sigma(v) + \sigma(w) + 2n_0 - \sigma(u_1) - \sigma(u_2) - \sigma(u_3) - \sigma(u_{2l}) + t_1 + \sigma(u) + t_1 + \sigma(u_{2l})$

$= t_2 + \sigma(w) + 2n_0 + t_1 + t_1$

$= c - b + \sigma(w) + 2a + b - a + b - a$

$= c + b + \sigma(w) \leq b + c + 1$.

$(\because 0 < \sigma(w) \leq 1)$

Connected neighborhood set

Definition: A neighborhood set $S \subseteq V$ of a fuzzy graph G is a *connected neighborhood set* if full induced fuzzy sub graph $\langle\langle S \rangle\rangle$ is connected and is denoted by *cn*-set. The minimum scalar cardinality taken over all *cn*-set is called connected neighborhood number of G and is denoted by n_c.

Example: Consider the fuzzy graph in Fig.4.1.

Fig.4.1 : G_1

n_0 –set ={v_1, v_3}$\Longrightarrow n_0 = 0.3$, and n_c-set ={ v_2, v_3} $\Longrightarrow n_c = 0.6$.

Therefore $n_0 \leq n_c$.

Remarks:

1)Every connected neighborhood set is a neighborhood set, ie. $n_0(G) \leq n_c(G)$.

2)Every independent neighborhood set is a neighborhood set, ie. $n_0(G) \leq n_i(G)$.

3)Every connected neighborhood set is a connected dominating set, ie. $\gamma_c(G) \leq n_c(G)$.

4)There is no relation between n_i and n_c. For example, consider the fuzzy graph in Fig.4.2.

Fig.4.2 : G_2

n_i –set ={ v_1, v_3}$\Longrightarrow$ n_i =0.8, and n_c-set ={ v_2, v_3} $\Longrightarrow$ n_c = 0.5. Therefore $n_i \geq n_c$. In example 4.2, we obtain that $n_i \leq n_c$. Hence no relation exist between n_i and n_c.

Observation: *In a complete fuzzy graph* K_σ, $n_c = min_{u \in V}\, \sigma(u)$.

Observation: *In a fuzzy cycle* C_σ *of order p with n vertices , n≥4 ,* $n_c = p - min_{u \in V}\, \sigma(u)$.

Theorem: *In a complete bipartite fuzzy graph* K_{σ_1,σ_2} *with* $|\sigma_1| \leq |\sigma_2|$,

$$n_c = \begin{cases} |\sigma_1| & if\ |\sigma_1^*| = 1 \\ |\sigma_2| & if\ |\sigma_2^*| = 1\ and\ |\sigma_1^*| \neq 1 \\ |\sigma_1| + \min\limits_{u \in \sigma_2} \sigma_2(u) & ,otherwise \end{cases}$$

Proof: *Case (i):* If $|\sigma_1^*| = 1$, then every vertex of V_2 is a neighborhood of $u \in V_1$. Hence $n_c = \sigma_1(u) = |\sigma_1|$. *Case (ii):* If $|\sigma_2^*| = 1\ and\ |\sigma_1^*| \neq 1$, then every vertex of V_1 is a neighborhood of $v \in V_2$. Hence $n_c = \sigma_2(v) = |\sigma_2|$. *Case (iii):* Let K_{σ_1,σ_2} be the complete bipartite fuzzy graph, σ_1 defined on V_1 and σ_2 defined on V_2 with $|\sigma_1| \leq |\sigma_2|$ and $v \in V_2$ such that $\sigma(v)$ is a minimum weight. Since V_1 is a neighborhood set, since K_{σ_1,σ_2} is a complete bipartite and $V_1 \cup \{v\}$ is also a neighborhood set and its full fuzzy induced sub graph is connected. Hence, $n_c = |\sigma_1| + \min_{v \in \sigma_2} \sigma_2(v)$.

Observation: *For a fuzzy tree* T_σ *of order p,* $n_c = p - t$, *where t is the scalar cardinality of the set of all the pendent vertices in* T_σ.

Note: Let $\in_T$ denote the maximum scalar cardinality of the set of all pendent vertices in any full spanning fuzzy tree of *G*.

Theorem: *In any connected fuzzy graph G of order p, any cut vertex of G belongs to each connected neighborhood set and* $k \leq n_c \leq p - \frac{\in_T - 1}{2}$, *where k is the minimum scalar cardinality of cut vertices of G.*

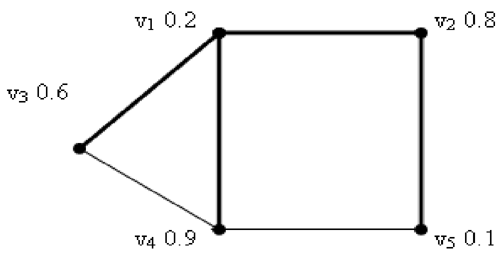
Proof: Let S be a connected neighborhood set, and v be a cut vertex of G. Suppose $v \notin S$, clearly *v* lies on two blocks B_1 and B_2 (say). There exists a vertex $u \in B_1$ and $w \in B_2$ such that $v, w \in$ S and both of them different from *v*. Also *u-w* fuzzy path contains *v* since $v \notin S$, the full fuzzy induced sub graph $\langle\langle S \rangle\rangle$ is disconnected, which is a contradiction. Hence $v \in S$ and $k \leq n_c$.

Now, consider a full spanning fuzzy tree T_σ of *G* with maximum scalar cardinality of pendent vertices. Let *W* be the set of all non-pendent vertices of T_σ. Delete all the vertices of *G* which belongs to *W*. Let G' be the resultant graph with $V' (= V - W)$ vertices and E' edges. Let *S* be the minimum neighborhood set of G' after deleting the isolated vertices in G. Then $|S| \leq \frac{\in_T - 1}{2}$ and clearly $W \cup S$ is a connected neighborhood set of *G*. Hence, $n_c \leq p - \in_T + \frac{\in_T - 1}{2} = p - \frac{\in_T - 1}{2}$.

Corollary: If *G* is complete fuzzy graph, then G' is also complete. Hence, $n_c = |W| = \min_{u \in V} \sigma(u)$.

Corollary: If *G* is tree, then G' is a null graph. Hence $n_c = |W| = p - \in_T$.

Example: Consider the fuzzy graph



Bold lines indicates the spanning tree T_σ of G. v_1 and v_2 are the non pendent vertices of T_σ and {v_3, v_5} is a minimum n-set of G'. Therefore {v_1, v_2, v_3, v_5} is a cn-set. The n_c-set is {v_1, v_2, v_5}. Hence $n_c \leq p - \frac{\in_T - 1}{2}$ $\Longrightarrow$1.1≤2.3.

Observation: *Let G be a connected fuzzy graph of order p with not less than two vertices which is neither complete nor a fuzzy cycle. Then G has at least one pair of non adjacent vertex u,v such that the fuzzy graph G-{u, v} is connected neighborhood set.*

Theorem: *Let G be a connected fuzzy graph of order p. Then* $n_c = p - max_{u \in V}\, \sigma(u)$ *if and only if G is complete fuzzy graph with 2 vertices or G is a fuzzy cycle with four or more vertices.*

Proof: Clearly, $n_c = p - \max_{u \in V} \sigma(u)$ if *G* is complete fuzzy graph with two vertices or *G* is a fuzzy cycle with four or more vertices.

Conversely, suppose $n_c = p - \max_{u \in V} \sigma(u)$ and G is not a complete fuzzy graph with vertices 2. Then $G \neq K_\sigma$ with vertices more that 3, for otherwise $n_c = \min_{u \in V} \sigma(u) < p - \max_{u \in V} \sigma(u)$ which is a contradiction. If G is not a fuzzy cycle, then the observation

, there exists two non-adjacent vertices u, v in G such that at least one of them have maximum weight. Then the fuzzy graph $G' = G - \{u, v\}$ is connected. Clearly V(G') is a connected neighborhood set of G of scalar cardinality $p - \sigma(u) - \sigma(v)$. Therefore $n_c \leq p - \sigma(u) - \sigma(v)$, $\sigma(u) \text{ or } \sigma(v)$ maximum weight, which is a contradiction. Hence a must be a fuzzy cycle of length $\geq$4.

4.15 Corollary: Let G be a connected fuzzy graph of order p and sixe q except complete fuzzy graph with 2 vertices. If G is not a fuzzy cycle, then

$$n_c \leq p - (\sigma_1 + \sigma_2)$$

Where $\sigma_1 \text{ and } \sigma_2$ are the successive minimum weights of the vertices in *G*.

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