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A new reinforcement learning optimisation approach for capacitor placement in distribution systems

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ABSTRACT

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Keywords Reactive power planning, Reinforcement Learning, Radial distribution feeder. The problem of capacitor allocation in electric distribution systems involves maximizing energy and peak power loss reductions by means of capacitors installation. This paper presents a novel approach using reinforcement learning (RL) algorithm to determine suitable candidates' nodes in a distribution system for capacitor placement. The problem formulation considers two distinct objectives related to total cost of power loss and total cost of capacitors including the purchase and installation costs. The proposed method of this article uses RL for sizing and placing of capacitors in radial distribution feeders. The proposed method has been implemented in a software package and its effectiveness has been verified through a 9-bus radial distribution feeder, 34-bus radial distribution feeder along with 33-bus and 66-bus distribution systems. A comparison has been done among the proposed method of this paper and similar methods in other research works that shows the effectiveness of the proposed method of this paper for solving optimum capacitor planning problem.

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Introduction

Reactive currents in an electrical utility distribution system produce losses and result in increased ratings for distribution components. Shunt capacitors can be installed in a distribution system to reduce energy and peak demand losses, release the KVA capacities of distribution apparatus, improve the system voltage profile in the power distribution systems. Capacitor planning must determine the optimal site and size of capacitors to be installed on the buses of a radial distribution system. There have been analytical approaches, numerical programming methods and AI-based techniques devised to solve this capacitor problem. For instance, ref. [1] formulated the problem as a mixed integer programming problem that incorporated power flows and voltage constraints. The problem was decomposed into a master problem and a slave problem to determine the place of the capacitors, and the types as well as size of the capacitors placed on the system. A heuristic approach to identify the sensitive nodes by the levels of effect on the system losses is proposed in [2, 3]. Ref. [4] adopted an equivalent circuit of a lateral branch to simplify the distribution loss analysis, which obtained the capacitor operational strategies according to the reactive load duration curve and sensitivity index. Moreover, optimal capacitor planning based on the fuzzy algorithm was implemented to present the imprecise nature of its parameters or solutions in practical distribution systems [5]-[7]. Several investigations have recently applied artificial intelligence (AI) techniques to resolve the optimal capacitor planning problem due to the growing popularity of AI. A solution methodology based on a simulated annealing (SA) technique presented in [8, 9]. Ref. [10] applied the tabu search (TS) technique to determine the optimal capacitor planning in Chiang et al's [8] distribution system, and compared the results of the TS with the SA. Genetic algorithm method (GA) is implemented to obtain the optimal selection of capacitors in [11, 12]. The capacitor planning problem is formulated as an objective problem. The formulation proposed in this paper considers total cost of power loss and total cost of capacitors as an objective function and also considers load flow restrictions security and operational constraints like loading of feeders, maximum voltage profile and maximum reactive compensation.

It should be noticed that in this new paper a major change was made in selecting and valuing actions in Q-learning method other than what we did in ref [15] for example we used ε -greedy method. In the paper of ref [15] we only introduced the RL method but in the recent paper we explored our method and verified its behaviour over more networks so that we can use the method in future studies. In order to test our new RL method more case study networks were analyses. The 33 buses and the 66 busses networks are added more than the 9 busses and 34 buses for testing our method.

The rest of this article is organized as follows: Section 2 describes formulation of the capacitor planning problem. A solution algorithm based on the RL method is developed in section 3. Section 4 describes the implementation of RL method and in section 5 effectiveness of the RL algorithm on four distribution case study is demonstrated. Finally conclusions are presented in section 6.

Mathematical Model of the Problem

3.7

The objective function (OF) in the capacitor planning problem for radial distribution feeders is considered as follow [19]:

$$OF = K_P \times P_{loss} + \sum_{i=1}^{N_C} (C^{Q_i}_{inst} + C^{Q_i}_{purc})$$
(1)

Such that:

$$P_{gi} - P_{di} - V_i \sum_{j=1}^{N} V_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij}) = 0$$
(2)

$$Q_{gi} - Q_{di} - V_i \sum_{j=1}^{N} V_j Y_{ij} \sin (\delta_i - \delta_j - \theta_{ij}) = 0$$
 (3)

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$$\mathbf{V}_{i}^{\min} \mathbf{O}_{i} \mathbf{V}_{i} \mathbf{O}_{i}^{\max} \quad i = 1 \dots N \tag{4}$$

$$P_{ij}^{\min} \le P_{ij} \le P_{ij}^{\max} \tag{5}$$

$$Q_C^{Total} \le Q_L^{Total} \tag{6}$$

Where:

 K_p , Cost per power loss, kW/year

N, Total Number of buses in radial distribution network

 P_{loss} , Total Active power loss

Nc, Total number of possible capacitor sizes

 $C^{Q_i}_{inst}$, The cost of installation of a capacitor bank of Q (Var) on bus *i*, \$/KVAR/year

 $C^{Q_i}{}_{purc}$, The cost of purchasing of a capacitor bank of Q (Var) for bus *i*, KVAR/year

 P_{oi}, Q_{oi} , Active and reactive power generations at bus *i*.

 P_{di}, Q_{di} , Active and reactive power load at bus *i*.

V ,s , δ ,s System bus voltages magnitudes and phase angles.

 Y_{ij}, θ_{ij} , Bus admittance matrix elements

 P_{ii} , Active loading between buses i and j

 Q_C^{Total} , Total connected Var by capacitor banks for radial distribution network

 Q_L^{Total} , Total Var of connected loads in radial distribution network

This objective function considered here in equation (1), consists of two terms. The first term denotes the cost of power loss and the second term includes the total cost of capacitors that consist of the purchase and installation costs. Regarding the equations constraints, (2)and (3) point to well-known load flow restrictions while security and operational constraints like voltage profile and loading of feeders have been formulated in inequalities (4) and (5). As a general rule, for reactive-power compensation, the maximum capacitor size should not exceed the connected reactive load. This results in a limited number of available capacitor sizes for installing on the radial distribution network. This concept has been formulated by equation (6) in the set of constraints of introduced objective function.

Reinforcement Learning

Reinforcement learning is defined by Kaelbling, Littman and Moore (1996) as "the problem faced by an agent that must learn behavior through trial-and-error interactions with a dynamic environment". Mathematically, the reinforcement learning problem has been formalized as a Markov Decision Process (a process where the probability of the agent moving from one state to another, given its choice of action, is independent of the history of the system prior to reaching that state). The mathematics of Markov processes has been extensively studied, one significant result, Bellman (1957), being that an algorithm based on dynamic programming can be shown to converge to an optimal policy if the Markov process is stationary (a stationary Markov process is one in which the state transition probabilities, given the agent's choice of action, do not change over time). In the standard reinforcement-learning model, an agent is connected to its environment via perception and action. On each step of interaction the agent receives as

input some indication of the current state s of the environment; then the agent chooses an action a, to generate as output. The action changes the state of the environment and the value of this state transition is communicated to the agent through a scalar reinforcement signal [13].

Formally, the RL problem consists of:

- A discrete set of environment states, *S*;
- A discrete set of agent actions, *A*;
- A set of scalar reinforcement signals, *R*;
- ${\ }$ ${\ }$ Policy, ${\ }{\ }\pi$, which chooses the actions that has to be taken.
- Value Function, which maps each state to a measure of the value of that state, $V^{\pi}(s)$.

In the RL problem the goal of agent is to maximize the reward it receives in the long run. In general, we seek to maximize the expected return, where the return, R_t , is defined as some

specific function of the reward (r_i) sequence. In the simplest case the return is the sum of rewards:

$$R_t = r_{t+1} + r_{t+2} + \dots + r_T$$

Where t denote the time steps and T is the final time step. We have this notion of final time step when the agent– environment interaction breaks naturally into subsequences called *Episodes*. If we use discount factor γ , $(0 \le \gamma \le 1)$, in the equation (8) we have:

$$R_{t} = r_{t+1} + \gamma r_{t+2} + \gamma^{2} r_{t+3} \dots = \sum_{k=0}^{\infty} \gamma^{k} r_{t+k+1}$$
(8)

Almost all reinforcement learning algorithms are based on estimating value functions. Value functions are functions of states that estimate how good it is for the agent to be in a given state. We have the policy \mathcal{T} which is mapping from each state named $s \square S$, and action, $a \square \square A$, to the probability p(s,a), of taking action a when in state s. The value of a state s under a policy \mathcal{T} , denoted $V^{\mathcal{T}}(s)$, is the expected return when starting in s and following policy \mathcal{T} thereafter.

$$V^{\pi}(s) = E_{\pi} \{ R_t \, \middle| \, s_t = s \} = E\{ \sum_{k=0}^{\infty} \gamma^k r_{t+k+1} \, \middle| \, s_t = s \}$$
(9)

Where E_{π} { } denote the expected value given that the agent follows policy π . We call the function $V^{\pi}(s)$ the state-value function for policy.

Similarly, we define the value of taking action a in state s under a policy π , denoted $Q^{\pi}(s,a)$, as the expected return starting from s, taking the action a, and thereafter following policy π :

$$Q^{\pi}(s,a) = E_{\pi}\{R_{t} | s_{t} = s, a_{t} = a\} = E\{\sum_{k=0}^{\infty} \gamma^{k} r_{t+k+1} | s_{t} = s, a_{t} = a\}$$
(10)

 Q^π is calling the action-value function for policy π .

Action-Value Method

Unlike the supervised learning methods in RL the environment is explicitly on the trade-off between exploration and exploitation. The agent must learn which actions maximize reward function in the time, but also how to act to reach this maximization, looking for actions still not selected or regions not considered in a state space. The exploration and exploitation processes are usually mixed. Action-value methods are used to estimating the values of actions and for using the estimates to make action selection decisions. The simplest action selection rule is to select the action (or one of the actions) with highest estimated action value, that is, to select on play t one of the

greedy actions,
$$a^*$$
, for which:
 $Q_t(a^*) = \max_a Q_t(a)$ (11)

 $Q_t(a)$ Estimated value of action a at the t play

This method always exploits current knowledge to maximize immediate reward; it spends no time at all sampling apparently inferior actions to see if they might really be better. A simple alternative is to behave greedily most of the time, but every once in a while, say with small probability \mathcal{E} ; instead select an action at random, uniformly, independently of the action-value estimates. We call methods using this near-greedy action selection rule $\mathcal{E} - greedy$ methods. An advantage of these methods is that, in the limit as the number of plays increases, every action will be sampled an infinite number of

times, guaranteeing that ${}^{k_a \to \infty}$ for all a , and thus ensuring that all the ${}^{Q_t(a)}$ converge to ${}^{Q*(a)}$.

Q – Learning

One of the most important breakthroughs in reinforcement learning was development of an off-policy temporal-difference (TD) control algorithm known as Q-learning (Watkins, 1989). The one-step Q-Learning form is defined by:

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha[r_{t+1} + \gamma \max_{a} Q(s_{t+1}, a) - Q(s_t, a_t)]$$
(12)

Where α , $(0 \le \alpha \le 1)$ is the learning step.

It is important to note that the new value for $Q(s_t, a_t)$

memory is based both on the current value of $Q(s_t, a_t)$, and the values of immediate rewards obtained by next searches. So, the α parameter plays a critical role representing the amount of the updated Q-memory and affects the number of iterations. This method is identical to Sarsa learning except that when considering the next state action transition, the action is chosen that will maximize the next Q-value. Q-learning is shown to converge to an optimal policy under the usual assumptions (Watkins and Dayan, 1992), and it remains the most popular reinforcement learning algorithm because no model of the environment is required, it is intuitive, easy to implement, and can be run interactively with updates made immediately, as and when states are visited [14].

Rewards

Application of the Q-learning algorithm to reactive power planning problem is linked to the choice of an immediate reward (r), such that the iterative value of Q-function (12) is maximized for the whole planning period. The reward function is calculated by equation (13) as follow:

$$r = \frac{1}{\cos t(P_{loss})} + \frac{1}{\cos t(C_{total})}$$
(13)

The first term in (13) is the reverse of power losses cost and the second term is the reverse of cost of used capacitors. So as the agent maximizes the reward function it will follow our goal and minimizes the objective function (1).

Problem Formulation and Implementation

For solving the capacitor allocation problem using the reinforcement learning method states Vector and actions Vector, reward function and the optimized solution with a fast convergence are should be determined. For the purpose of our analysis, the Q-learning algorithm is the "agent", the state vectors (S) are the number of available buses for capacitor placement in the distribution system and the action vectors (A) are the discrete values of possible capacitor banks [15]. The policy is based on Q-learning algorithm and ε -greedy action value method is used for choosing the actions.

The algorithm proceeds as follows:

1- Read input data (consist of number of system buses (n), lines impedances, complex power of each of them, initial values, voltages boundaries and ...).

2- Put
$$i = n$$

3- While

3-1 state(s) == bus n

3-2 choose greedy action a by using ε – greedy method.

 $i \ge 1$

3-3 performs load flow and calculates real power loss.

3-4 Using equation (13) and calculate the reward of next episode.

3-5 Update Q-function using equation (12) 3-6 i = i - 1

4- If the voltage constraint of equation (4) is guaranteed for all system buses go next step otherwise return step 3

5- Save the results of chosen action, results and load flow

6- Calculate the Objective function equation (1)

7- Print the result.

Discount (γ) and Learning Step (α) Parameters

In addition to the above definitions, two parameters γ and α for implementing the Q-learning algorithm need to be chosen. RL theory gives generic guidelines for these parameters [13]–[14].

Parameter γ used in equations (8) & (12), is the control factor by which later rewards are discounted and it must be between 0 and 1. In our application, later rewards are not important because there is no interdependence among load flow solutions, therefore, the value of γ is initially should be zero.

The critical parameter α used in equation (12), expresses the amount of the updated Q-function, in other words the rate of learning. A large enough parameter (close to 1) allows fast convergence of the Q-learning algorithm, while a small value (close to 0) avoids instability of Q-learning. Since the Q-learning enforced in constrained load flow problem does not depend on previous Q-learning steps as stated above, this parameter will work well close to 1.

In our application, initially we set $\gamma = 0 \& \alpha = 1$, but by these values the agent is so myopic and the effect of future actions will not take into account. Therefore by using a dynamic approach $\gamma \& \alpha$ are changing slightly between $0 \le \gamma \le 0.5$ and $0.5 \le \alpha \le 1$. By previous experiment we had in [15] the best value for α is 0.995 and for γ is 0.005.

Test Cases and Simulation Results

The Q-learning algorithm is applied to four distribution systems. The test cases are a small network (with 9 buses), two medium-size networks (with 33 and 34 buses) and a large-scale network (with 66 buses). On each test case a comparison has

been done among the proposed method of this paper and similar methods in other research works. In all test cases it is assumed that all buses are available for placement of capacitor banks. In order to compare the proposed RL method four papers [16, 17, 18 & 19] are considered. In [16] by using a fuzzy theory and heuristic strategy five methods and an exact solution for reactive power compensation is proposed. In the presented methods different membership function shapes for real losses, reactive losses and bus voltages are suggested, then the candidate bus with lowest membership value selected and the capacitor with lowest cost without violating the constraints is installed. In table 4 and figures 2&3 the results of methods in [16] are presented. In [17] a robust searching hybrid differential evolution (RSHDE) method is used to solve the capacitor placement problem in distribution systems. In this method two new schemes, the multi-direction search scheme and the search space reduction scheme, are embedded into the hybrid algorithm of differential evolution (HDE). These two schemes are used to enhance the search ability before performing the initialization step of the solution process. In this study a comparison among DE, HDE, SA and GA methods is done too. In tables 5&7 the results of [17] and RL method are presented. In paper [18] two new heuristic techniques for reactive power compensation is introduced and the best result of this paper is compared with RL method. In [19] a new algorithm based on a combination of fuzzy (FUZ), Dynamic Programming (DP), and Genetic Algorithm (GA) approach is presented for capacitor allocation in distribution feeders. The proposed method of [19] uses fuzzy reasoning for placing of capacitors, DP for sizing and GA for finding the optimum shape of membership functions which are used in fuzzy reasoning stage. According to [16] available threephase capacitors size and cost are selected. In table 1 possible combination of these capacitor banks with minimum cost is shown.

Case Study 1: 9-bus system

The 9-bus radial distribution feeder of figure 1 is taken as the first test feeder. The system line data and other information are described in table 2; the rated voltage is 23 kV and the total reactive load of the system is 4186 kVar that according to (6) leads to 27 practical combinations of mentioned standard capacitor banks available in table 1, so the action vectors have these 27 members plus capacitor size and cost equal zero that means no capacitor is placed and state vectors have 9 members. The cost constant K_D is selected as 168 \$/kW/year.



Fig.1 Diagram of 9-bus Distribution System, Case Study 1

Table 4 shows the results of capacitor planning in [16] and [19]. The first six columns show methods 1-5 and the exact solution that are described in [16] and column seven is the result of [19]. The last column is our purposed RL method that compared with them. In table 5 the results of methods in [17] and RL method are illustrated.

From table 4 it is obvious that the RL result is much better than all five methods and exact solution mentioned in [16] and method of [19] in term of cost saving and real power loss reduction however in the case of the minimum voltage profile this does not happened; for example in the exact solution the minimum voltage is slightly better than the RL method and have the improvement of 0.0001 p.u. According to table 5 it is seen the HDE and RSHDE methods have better results of cost and real power loss so that the improvement of real power reduction is about 1.584 kw and cost saving is 195\$. The best result in [18] for 9-bus system reactive power compensation leads to 97.7 kw power loss and \$15526 cost reduction but in the RL method the cost saving is \$16080 and the real power loss reduction is 106.82 kw that are better results compare to [18].

In this test case the RL method is getting better than the fuzzy approach of [16] and heuristic methods of [18] but it is slightly weaker compared to complicated methods described in [17], we think the main purpose of this may be because of the inherent nature of Q-learning method. This method needs to interact between actions and states and find the optimal policy by exploration and exploitation, so because the number of states (buses) is to some how low the exploration and exploitation processes may not performed well enough and the policy cannot be good enough.

Case Study 2: 34-bus system

A radial distribution network with 34 load points is used to simulate the proposed RL Method. The data of this test system has been taken from reference [16].The system voltage is 11 kV. Before compensation, the cost is U.S. \$ 37212, this is based on the previously defined cost parameters, the active and reactive losses are 221.5 kW and 65.04 KVar, respectively, and the voltage limits in per unit are 0.9417 and 1.0. Considering total connected reactive load of 2873.5 KVar of this system, 19 capacitor bank combination of table 1 can be used.

The result of RL method is illustrated in table 6, while the comparison between results of the methods 1-5 of [16] and methods 6 are presented in figures 2&3.



Fig. 2 Cost function for the 6 methods applied to 34-bus test feeder, Case Study 2



Fig. 3 Active power losses for the 6 methods applied to 34bus test feeder, Case Study 2

According to figures 2 and 3 we can conclude the RL result is much better than the five methods of [16] in term of cost and real power loss reduction. The best result for this test case in [18] yields to 61.8 kw real power loss and \$9842 cost reduction while the corresponding result in RL method are 60.93 kw and \$9645.In 34-bus system the RL method is still better than [16] but have slightly weaker result according to [18]. This is because the test feeder is not heavy loaded so the voltage constraint of equation (4) is satisfied and it makes the algorithm to quit the exploration and exploitation process and abrupt the searching.

Case Study 3:33-bus and 66-bus systems

In order to compare the performance of our method with the RSHDE method the second case study of [17] is used, which is an 11 kV, 4-lateral, and 33-section feeder in which used as a medium-size system and by doubling this system we have a large-size system with 66 buses. The equivalents annual cost per unit of power loss (K_p) is selected to be \$ 150/ kW/year. Table

7 illustrates the results of methods of [17] and our proposed RL method.

Table 7 shows that as the number of buses increased the RL method result is much better than the methods of [16] and make a great improvement in total loss, cost and net saving in these test feeders.

Conclusions

This article presents a new optimization method for optimum capacitor planning problem. The proposed method uses reinforcement learning method for reactive power compensation in distribution system.

The method developed herein is tested on small, medium and large distribution systems and the results have been compared with similar research works. The comparison shows the effectiveness of proposed method in case of investment and improving the performance of the distribution network. Although in small distribution system the RSHDE method perform a little better but as the number of busses increase and so the number of states increase, the searching space and the number of iteration for finding optimum policy by Q-learning agent is increased and the RL method performs better. That is why the results in table 7 for medium and large size distribution systems have a great improvement in total loss and cost by using RL method compare to RSHDE method.

It should be mention that the proposed RL method has a novel formulation of optimization that is so easy in formulation and suitable to use in new approaches for future work in similar fields.

Mathematical Symbols

 δ , System bus voltages phase angle.

- θ , Bus admittance matrix elements
- π , Reinforcement learning policy
- γ , Discount factor
- α , Learning rate

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Κ	Qc Size(KVar)	Cost(\$/KVar)	Κ	Qc Size(KVar)	Cost(\$/KVar)	k	Qc Size(KVar)	Cost(\$/KVar)
1	150	0.500	10	1500	0.201	19	2850	0.183
2	300	0.35	11	1650	0.193	20	3000	0.180
3	450	0.253	12	1800	0.187	21	3150	0.195
4	600	0.220	13	1950	0.211	22	3300	0.174
5	750	0.276	14	2100	0.176	23	3450	0.188
6	900	0.183	15	2250	0.197	24	3600	0.170
7	1050	0.228	16	2400	0.170	25	3750	0.183
8	1200	0.170	17	2550	0.189	26	3900	0.182
9	1350	0.207	18	2700	0.187	27	4050	0.179

Table 1 Possible Choice of Capacitor Sizes and Cost/KVAr [16]

Table 2 9-buses system data

Bus i	P (Kw)	Q (Kvar)	$\frac{R_{i\text{-}l,i}}{(\Omega)}$	(Ω)
1	1840	460	0.1233	0.4127
2	980	340	0.014	0.6051
3	1790	446	0.7463	1.205
4	1598	1840	0.6984	0.6084
5	1610	600	1.9831	1.7276
6	780	110	0.9053	0.7886
7	1150	60	2.0552	1.164
8	980	130	4.7953	2.716
9	1640	200	5.3434	3.0264

Table 3 shows the system status before compensation where the voltage of the
substation (bus number 0) is assumed to be 1 p.u.Table 3 Results of 9-buses system before compensation

Cost Function (\$)	Active Loss (KW)	Reactive Loss (kVar)	Max Voltage (p.u)	Min Voltage (p.u)
131670	783.78	1036.5	0.9929	0.8375

Table 4 Results for all methods applied to the 9-bus feeder in [16], [19] and RL Method

Bus No.	Qc(KVar) Using Method 1	Qc(KVar) Using Method 2	Qc(KVar) Using Method 3	Qc(KVar) Using Method 4	Qc(KVar) Using Method 5	Exact Solution	Qc(KVar) Using Method [18]	Qc(KVar) Using RL Method 6
1								
2			3300			3600	3600	4050
3		1050	3900	3300	2850		4050	2400
4	2100	1050		1800	2100	4050	450	1200
5	2500	1950	1200	1050	1050	1650	1200	1500
6								
7								450
8						600	150	
9	900	900	900	900	900		600	450
Real Loss(KW)	707	705	689	692	691.6	686	681.28	676.96
\$ Cost	119736	119420	117330	117571	117479	117095	116320	115590
Min V (p.u.)	0.9000	0.9029	0.9006	0.90004	0.9000	0.9003	0.90014	0.9002
Max V (p.u.)	1.0000	1.0000	1.006	1.0012	1.001	1.007	1.007	1.007

Bus No.	Qc(KVar) Using SA	Qc(KVar) Using GA	Qc(KVar) Using DE	Qc(KVar) Using HDE	Qc(KVar) Using RSHDE	Qc(KVar) Using RL
1						
2	3600	3600	3600	4050	4050	4050
3	1800	1800	3450	2100	2100	2400
4	2700	2400	1800	2100	2100	1200
5	450	600	750	900	900	1500
6	450	450	600	450	450	
7	150	450	150			450
8			150	150	150	
9	600	450	300	450	450	450
Real Loss(KW)	677.222	676.550	679.531	675.376	675.376	676.96
\$ Cost	115645	115484	160479	115395	115395	115590
Min V (p.u.)	0.9001	0.9000	0.9023	0.9002	0.9002	0.9002
Max V (p.u.)	1.0064	1.0064	1.0084	1.0073	1.0073	1.0070

Table 5 Results for all methods applied to the 9-bus feeder in [17] and RL Method

Table 6 Result of RL method in 34-bus test feeder

Bus	5_1	5_5	5_9	9_3	Cost (\$)	Loss(KW)	Loss(kVar)	Min V (p.u)	Max V (p.u)
Qc(kVar)	900	300	750	450	27567	160.57	47.01	0.9503	1

Table 7 Results of [17] &RL methods in 33-bus&66-bus test cases

Test	Itoms	Before	After Compensation							
Cases	nems	Compensation	SA	GA	DE	HDE	RSHDE	RL		
22.1	Total Loss(kw)	883.240	714.037	709.653	718.504	705.585	704.557	635.673		
33-bus system	Cost(\$)	132486	120606	120410	127576	119638	119634	96356		
	Net Saving(\$)	-	11880(8.97%)	12076(9.11%)	4910(3.71%)	12848(9.70%)	12852(9.70%)	36130(27.27%)		
((h	Total Loss(kw)	1123	879.712	863.642	-	-	855.684	685.705		
system	Cost(\$)	168450	134178	133947	-	-	130645	111290		
	Net Saving(\$)	-	34270(20.34%)	34501(20.48%)	-	-	37803(22.44%)	57160(33.93%)		