Available online at www.elixirpublishers.com (Elixir International Journal)

Discrete Mathematics

Elixir Dis. Math. 44 (2012) 7477-7479

ISSN: 2229-712X

On anti Q-fuzzy normal HX groups

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ARTICLE INFO

Article history:
Received: 30 August 2011;
Received in revised form:
15 March 2012;
Accepted: 23 March 2012;

Keywords
Fuzzy group,
Q-fuzzy group, anti Q-fuzzy normal group, Anti Q-fuzzy normal HX groups,
Lower level sub HX group,
Anti Q-fuzzy normalizer.

ABSTRACT
In this paper, we define a new algebraic structure of an anti Q-fuzzy normal HX group and lower level subset of an anti Q-fuzzy normal HX group and discussed some of its properties. We also define anti Q-fuzzy normalizer and establish the relation with anti Q-fuzzy normal HX group. Characterizations of lower level subsets of an anti Q-fuzzy HX subgroup of a HX group are given.

Introduction

K.H.Kim [4] introduce the concept of intuitionistic Q-fuzzy semi prime ideals in semi groups and Osman kazanci, sultan yamark and Serife yilmaz[10] introduce the concept of intuitionistic Q-fuzzy R-subgroups of near rings and Muthuraj. R, Manikandan. K. H., Muthuraman. M. S., Sitharselvam. P. M.,[8] introduce the concept of anti Q-fuzzy HX group and its lower level sub HX groups. A.Solairaju and R.Nagarajan [13] introduce and define a new algebraic structure of Q-fuzzy groups. In this paper we define a new algebraic structure of an anti Q-fuzzy normal HX group and lower level subset of an anti Q-fuzzy normal HX group and discussed some of its properties.

Preliminaries

In this section we site the fundamental definitions that will be used in the sequel. Throughout this paper, G = (G , *) is a group, e is the identity element of G, and xy, we mean x * y.

Definition [5]

In $2^G-\{\phi\}$, a nonempty set $\vartheta \subset 2^G-\{\phi\}$ is called a HX group on G, if ϑ is a group with respect to the algebraic operation defined by AB = { ab /a ∈ A and b ∈ B}, which its unit element is denoted by E.

Definition [12]

Let X be any non empty set. A fuzzy subset λ of X is a function $\lambda : X \rightarrow [0,1]$.

Definition [6][8]

A fuzzy set λ is called fuzzy HX subgroup of a HX group ϑ if for A , B $\in\vartheta$,

- $\lambda(AB) \geq \min\{\lambda(A), \lambda(B)\}$
- $\lambda(A^{-1}) = \lambda(A)$.

Definition [8]

A fuzzy set λ is called an anti fuzzy HX subgroup of a HX group ϑ if A , B $\in\vartheta$,

- $\lambda(AB) \leq \max\{\lambda(A), \lambda(B)\}$,
- $\lambda(A^{-1}) = \lambda(A)$.

Definition [8][13]

Let Q and ϑ be any two sets. A mapping $\lambda : \vartheta \times Q \rightarrow [0,1]$ is called a Q–fuzzy set in ϑ .

Definition [8][13]

A Q-fuzzy set λ is called Q-fuzzy HX subgroup of a HX group ϑ if for A , B $\in\vartheta$ and $q\in Q$,

- $\lambda(AB, q) \geq \min\{\lambda(A,q), \lambda(B,q)\}$,
- $\lambda(A^{-1}, q) = \lambda(A, q)$.

Definition [8]

A Q-fuzzy set λ is called an anti Q-fuzzy HX subgroup of a HX group ϑ if A , B $\in\vartheta$ and $q\in Q$,

- $\lambda(AB, q) \leq \max\{\lambda(A, q), \lambda(B, q)\}$,
- $\lambda(A^{-1}, q) = \lambda(A, q)$.

Definition

Let ϑ be a HX group. A Q-fuzzy HX subgroup λ of ϑ is said to be normal if for all A, B ∈ ϑ and q ∈ Q , $\lambda(ABA^{-1}, q) = \lambda(B, q)$ or $\lambda(AB, q) = \lambda(BA, q)$.

Definition

Let ϑ be a HX group. An anti Q-fuzzy HX subgroup λ of ϑ is said to be normal if for all A, B ∈ ϑ and q ∈ Q , $\lambda(ABA^{-1}, q) = \lambda(B, q)$ or $\lambda(AB, q) = \lambda(BA, q)$.

Properties of Anti Q-fuzzy normal HX subgroup

In this section, we establish the relation between Q-fuzzy normal HX subgroup and anti Q-fuzzy normal HX subgroup of ϑ and also discuss some of the properties of anti Q-fuzzy normal HX subgroup of ϑ.

Theorem

λ is a Q- fuzzy normal HX subgroup of ϑ, iff λ^C is an anti Q-fuzzy normal HX subgroup of ϑ.

Proof

Suppose λ is a Q-fuzzy HX subgroup of ϑ. Then for all A,B ∈ ϑ and q ∈Q ,

$$\lambda(AB, q) \geq \min\{\lambda(x, q), \lambda(B, q)\}$$

$$\Leftrightarrow \quad 1-\lambda^c(AB, q) \geq \min\{(1-\lambda^c(A, q)), (1-\lambda^c(B, q))\}$$

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$\Leftrightarrow \lambda^c(AB, q) \leq 1 - \min\{(1-\lambda^c(A, q)),(1-\lambda^c(B, q))\}$

$\Leftrightarrow \lambda^c(AB, q) \leq \max\{\lambda^c(A, q), \lambda^c(B, q)\}$.

We have, $\lambda(A, q) = \lambda(A^{-1}, q)$ for all A in ϑ and $q \in Q$,

$\Leftrightarrow 1-\lambda^c(A, q) = 1-\lambda^c(A^{-1}, q)$.

Therefore, $\lambda^c(A, q) = \lambda^c(A^{-1}, q)$.

Hence λ^c is an anti Q-fuzzy HX subgroup of ϑ.

λ is a Q-fuzzy HX subgroup of ϑ iff $\lambda(ABA^{-1}, q) = \lambda(B, q)$.

iff $1-\lambda(ABA^{-1}, q) = 1-\lambda(B, q)$.

iff $\lambda^c(ABA^{-1}, q) = \lambda^c(B, q)$.

iff λ^c is an anti Q-fuzzy normal HX subgroup of ϑ.

Hence, λ is a Q- fuzzy normal HX subgroup of ϑ, iff λ^C is an anti Q-fuzzy normal HX subgroup of ϑ.

Theorem

The union of any two anti Q-fuzzy normal HX subgroups of ϑ is always an anti Q-fuzzy normal HX subgroup of ϑ.

Proof

Let λ_1 and λ_2 be any two anti Q-fuzzy normal HX subgroup of ϑ.Then for all A, B in ϑ,

$\lambda_1(AB,q) \leq \max\{\lambda_1(A,q), \lambda_1(B,q)\}$ and $\lambda_2(AB,q) \leq \max\{\lambda_2(A,q), \lambda_2(B,q)\}$.

Clearly, $(\lambda_1 \vee \lambda_2)(A,q) = \max\{\lambda_1(A,q), \lambda_2(A,q)\}$ and

$(\lambda_1 \vee \lambda_2)(AB,q) = \max\{\lambda_1(AB,q), \lambda_2(AB,q)\}$

$= \max\{\max\{\lambda_1(A,q), \lambda_1(B,q)\}, \max\{\lambda_2(A,q), \lambda_2(B,q)\}$

$\leq \max\{\max\{\lambda_1(A,q), \lambda_2(A,q)\}, \max\{\lambda_1(B,q), \lambda_2(B,q)\}$

$\leq \max\{(\lambda_1 \vee \lambda_2)(A,q), (\lambda_1 \vee \lambda_2)(B,q)\}$

Since, $\lambda_1(A,q) = \lambda_1(A^{-1},q)$ and $\lambda_2(A,q) = \lambda_2(A^{-1},q)$ for all A in ϑ,

We have, $(\lambda_1 \vee \lambda_2)(A^{-1},q) = \max\{\lambda_1(A^{-1},q), \lambda_2(A^{-1},q)\}$

$= \max\{\lambda_1(A,q), \lambda_2(A,q)\}$

$= (\lambda_1 \vee \lambda_2)(A,q)$.

That is, $(\lambda_1 \vee \lambda_2)(A^{-1},q) = (\lambda_1 \vee \lambda_2)(A,q)$ for all A in ϑ.

Hence $\lambda_1 \vee \lambda_2$ is an anti Q-fuzzy HX subgroup of ϑ.

Since λ_1 is an anti Q-fuzzy normal HX subgroup of ϑ ,then $\lambda_1(ABA^{-1}, q) = \lambda_1(B, q)$, for all A, B $\in \vartheta$ and $q \in Q$ and λ_2 is an anti Q-fuzzy normal HX subgroup of ϑ, then $\lambda_2(ABA^{-1}, q) = \lambda_2(B, q)$, for all A, B $\in \vartheta$ and $q \in Q$.

Now, $(\lambda_1 \vee \lambda_2)(ABA^{-1},q) = \max\{\lambda_1(ABA^{-1},q), \lambda_2(ABA^{-1},q)\}$

$= \max\{\lambda_1(B, q), \lambda_2(B, q)\}$

$= (\lambda_1 \vee \lambda_2)(B, q)$.

That is, $(\lambda_1 \vee \lambda_2)(ABA^{-1},q) = (\lambda_1 \vee \lambda_2)(B, q)$.

Hence $\lambda_1 \vee \lambda_2$ is an anti Q-fuzzy normal HX subgroup of ϑ.

Remark

- Arbitrary union of anti Q-fuzzy normal HX subgroups of ϑ are anti Q-fuzzy normal HX subgroup of ϑ.
- Intersection of any two anti Q-fuzzy normal HX subgroups of ϑ are not an anti Q-fuzzy normal HX subgroup of ϑ.

Properties of Lower level subsets of an anti Q-fuzzy normal HX subgroup

In this section, we introduce the concept of lower level subset of an anti Q-fuzzy normal HX subgroup and discuss some of its properties.

Definition [8]

Let λ be an anti Q-fuzzy normal HX subgroup of a HX group ϑ. For any $t \in [0,1]$, we define the set $L(\lambda; t) = \{A \in \vartheta / \lambda(A, q) \leq t\}$ is called the lower level subset of λ.

Theorem

Let λ be an anti Q-fuzzy normal HX subgroup of a HX group ϑ. Then for $t \in [0,1]$ such that $t \geq \lambda(E, q)$, $L(\lambda; t)$ is a normal sub HX group of G.

Proof

For all A, B $\in L(\lambda; t)$, we have,

$\lambda(A, q) \leq t$; $\lambda(B, q) \leq t$.

Now, $\lambda(AB^{-1}, q) \leq \max\{\lambda(A, q), \lambda(B, q)\}$.

$\lambda(AB^{-1}, q) \leq \max\{t, t\}$.

$\lambda(AB^{-1}, q) \leq t$.

$AB^{-1} \in L(\lambda; t)$.

Hence $L(\lambda; t)$ is a sub HX group of ϑ.

For all $B \in L(\lambda; t)$, $A \in \vartheta$ and $q \in Q$ and for $t \in [0,1]$ such that $t \geq \lambda(E, q)$ we have, $\lambda(B, q) \leq t$.

Since, λ be an anti Q-fuzzy normal HX subgroup of a HX group ϑ,

$\lambda(ABA^{-1},q) = \lambda(B,q) \leq t$ for all A , B $\in \vartheta$ and $q \in Q$.

$ABA^{-1} \in L(\lambda; t)$. Hence $L(\lambda; t)$ is a normal sub HX group.

Theorem

Let ϑ be a HX group and λ be a Q-fuzzy subset of ϑ such that $L(\lambda; t)$ is a normal sub HX group of ϑ. For $t \in [0,1]$ $t \geq \lambda(E, q)$, λ is an anti Q-fuzzy normal HX subgroup of ϑ.

Proof

Let A, B in ϑ and $\lambda(A,q) = t_1$ and $\lambda(B,q) = t_2$.

Suppose $t_1 < t_2$, then A, B $\in L(\lambda; t_2)$.

As $L(\lambda; t_2)$ is a subgroup of G, $AB^{-1} \in L(\lambda; t_2)$.

Hence, $\lambda(AB^{-1}, q) \leq t_2 = \max\{t_1, t_2\}$

$\leq \max\{\lambda(A, q), \lambda(B, q)\}$

That is, $\lambda(AB^{-1}, q) \leq \max\{\lambda(A, q), \lambda(B, q)\}$.

Hence λ is an anti Q-fuzzy HX subgroup of ϑ.

Conversely, for any $t \in [0,1]$, $L(\lambda; t) \neq \phi$ and $L(\lambda; t)$ is a classical normal sub HX group.

Then ,we have, $\lambda(ABA^{-1},q) = \lambda(B,q)$ for all A , B $\in \vartheta$ and $q \in Q$.

Otherwise, if there exists A_0 or $B_0 \in \vartheta$ and $q \in Q$ such that, $\lambda(A_0B_0A_0^{-1}, q) > \lambda(B_0,q)$.

Take $t_0 = 0.5 [\lambda(B_0,q) + \lambda(A_0B_0A_0^{-1}, q)]$.

Evidently, $t_0 \in [0,1]$, we can infer that,

$\lambda(B_0,q) < t_0$ and $\lambda(A_0B_0A_0^{-1}, q) > t_0$.

Consequently, we have $B_0 \in L(\lambda; t_0)$ and $A_0B_0A_0^{-1} \notin L(\lambda; t_0)$.

This contradicts that $L(\lambda; t_0)$ is a normal sub HX group.

Hence, we get,

$\lambda(ABA^{-1},q) = \lambda(B,q)$ for all A , B $\in \vartheta$ and $q \in Q$.

Hence, λ is an anti Q-fuzzy normal HX subgroup of a HX group ϑ.

Theorem

Let ϑ be a HX group and λ be an Q-fuzzy subset of ϑ. Then λ is an anti Q-fuzzy normal HX subgroup of ϑ iff the level subset $L(\lambda, t)$, $t \in [0,1]$, is a normal sub HX group of ϑ.

Proof

It is clear.

Theorem

Let λ be an anti Q-fuzzy normal HX subgroup of a HX group ϑ. If two lower level normal sub HX groups $L(\lambda; t_1)$, $L(\lambda; t_2)$, for, $t_1, t_2 \in [0,1]$ and $t_1, t_2 \geq \lambda(E, q)$ with $t_1 < t_2$ of λ are equal then there is no λ in ϑ such that $t_1 < \lambda(A, q) \leq t_2$.

Proof

Let $L(\lambda; t_1) = L(\lambda; t_2)$.

Suppose there exists $A \in \vartheta$ such that $t_1 < \lambda(A, q) \leq t_2$ then $L(\lambda; t_1) \subseteq L(\lambda; t_2)$.

Then $A \in L(\lambda; t_2)$,but $A \notin L(\lambda; t_1)$, which contradicts the assumption that, $L(\lambda; t_1) = L(\lambda; t_2)$. Hence there is no A in ϑ such that $t_1 < \lambda(A, q) \leq t_2$.

Conversely, suppose that there is no A in ϑ such that $t_1 < \lambda(A, q) \leq t_2$.
Then, by definition, $L(\lambda ; t_1) \subseteq L(\lambda ; t_2)$.
Let $A \in L(\lambda ; t_2)$ and there is no A in ϑ such that $t_1 < \lambda(A, q) \leq t_2$.
Hence $A \in L(\lambda ; t_1)$ and $L(\lambda ; t_2) \subseteq L(\lambda ; t_1)$.
Hence $L(\lambda ; t_1) = L(\lambda ; t_2)$.

Remark

As a consequence of the Theorems , the lower level normal sub HX groups of an anti Q-fuzzy normal HX subgroup λ of a HX group ϑ form a chain. Since $\lambda(E, q) \leq \lambda(A, q)$ for all A in ϑ and $q \in Q$, therefore $L(\lambda ; t_0)$, where $\lambda(E, q) = t_0$ is the smallest and we have the chain :
$\{E\} \subset L(\lambda ; t_0) \subset L(\lambda ; t_1) \subset L(\lambda ; t_2) \subset \ldots \subset L(\lambda ; t_n) = \vartheta$, where $t_0 < t_1 < t_2 < \ldots\ldots < t_n$.

Definition

Let ϑ be a HX group and λ be an anti Q-fuzzy normal HX subgroup of ϑ. Let $N(\lambda) = \{ A \in \vartheta / \lambda(ABA^{-1}, q) = \lambda(B, q)$ for all $B \in \vartheta\}$. Then $N(\lambda)$ is called the anti Q-fuzzy normalizer of λ.

Theorem

Let λ be an anti Q-fuzzy normal HX subgroup of a HX group ϑ. Then

- $N(\lambda)$ is a sub HX group of ϑ.
- λ is an anti Q-fuzzy normal $\Leftrightarrow N(\lambda) = \vartheta$.
- λ is an anti Q-fuzzy normal HX subgroup of the HX group $N(\lambda)$.

Proof

i. Let $A, B \in N(\lambda)$ then $\lambda(ACA^{-1}, q) = \lambda(C, q)$, and $\lambda(BCB^{-1}, q) = \lambda(C, q)$, for all $C \in \vartheta$.

$$\text{Now } \lambda((AB)C(AB)^{-1}, q) = \lambda(ABCB^{-1}A^{-1}, q) = \lambda(BCB^{-1}, q) = \lambda(C, q)$$

Thus we get, $\lambda((AB)C(AB)^{-1}, q) = \lambda(C, q)$.
This implies, $AB \in N(\lambda)$.
Therefore, $N(\lambda)$ is a sub HX group of ϑ.
ii. Let λ be an anti Q-fuzzy normal HX Subgroup of ϑ.
Clearly $N(\lambda) \subseteq \vartheta$, Let $A \in \vartheta$, then $\lambda(ABA^{-1}, q) = \lambda(B, q)$, Then $A \in N(\lambda)$ implies $\vartheta \subseteq N(\lambda)$.
Hence $N(\lambda) = \vartheta$.
Conversely, let $N(\lambda) = \vartheta$.
Clearly $\lambda(ABA^{-1}, q) = \lambda(B, q)$, for all $B \in \vartheta$ and $A \in \vartheta$.
Hence λ is an anti Q – fuzzy normal HX subgroup of ϑ.
iii. From (ii) , λ is an anti Q -fuzzy normal HX subgroup of a HX group $N(\lambda)$.

Conclusion

In this paper, we defined a new algebraic structure of an anti Q-fuzzy normal fuzzy HX group and studied some of its properties. Further, we defined the relation between Q-fuzzy normal HX group and anti Q-fuzzy normal HX group in HX group and also some operations on anti Q-fuzzy HX groups are investigated and the same on intuitionistic fuzzy and on some of the other groups are in progress.

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