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# A class of almost unbiased modified ratio estimators for population mean with known population parameters

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**ABSTRACT**

The present paper deals with estimation of the population mean of the study variable when the information on the auxiliary variable is known and their population parameters are known. In the past, a number of modified ratio estimators are suggested with known values for the Co-efficient of Variation, Co-efficient of Kurtosis, Co-efficient of Skewness, Population Correlation Coefficient etc. However all these modified ratio estimators are biased but with less mean squared errors compared to the usual ratio estimator. In this paper some strategies have been suggested to improve the performance of the existing modified ratio estimators, which lead to a class of almost unbiased modified ratio estimators; and their performances are better than the modified ratio estimators. These are explained with the help of numerical examples.



## Introduction

In sample surveys, auxiliary information on the finite population under study is quite often available from previous experience, census or administrative databases. The sampling theory describes a wide variety of techniques/ methods for using auxiliary information to improve the sampling design and to obtain more efficient estimators like Ratio, Product and Regression estimators. Ratio estimators, improves the precision of estimate of the population mean or total of a study variable by using prior information on auxiliary variable $X$ which is correlated with the study variable $Y$. Over the years the ratio method of estimation has been extensively used because of its intuitive appeal and the computational simplicity. Before discussing further about the modified estimators and the proposed estimators the notations to be used are described below:

- $N$ – Population size
- $n$ – Sample size
- $Y$ – Study variable
- $X$ – Auxiliary variable
- $\bar{X}, \bar{Y}$ – Population means
- $\bar{x}, \bar{y}$ – Sample means
- $S_x, S_y$ – Population standard deviations
- $C_x, C_y$ – Coefficient of variations
- $\rho$ – Coefficient of correlation
- $\beta_1 = \frac{N\sum_{i=1}^{N}(X_i-\bar{X})^3}{(N-1)(N-2)S^3}$ – Coefficient of skewness of the auxiliary variable
- $\beta_2 = \frac{N(N+1)\sum_{i=1}^{N}(X_i-\bar{X})^4}{(N-1)(N-2)(N-3)S^4} - \frac{3(N-1)^2}{(N-2)(N-3)}$ – Coefficient of kurtosis of the auxiliary variable
- $B(.)$ – Bias of the estimator
- $MSE(.)$ – Mean squared error of the estimator
- $\hat{\bar{Y}}(\hat{\bar{Y}}_p)$ – Existing (proposed) modified ratio estimator of $\bar{Y}$

The classical Ratio estimator for the population mean $\bar{Y}$ of the study variable $Y$ is defined as

$$\hat{\bar{Y}}_R = \frac{\bar{y}}{\bar{x}}\bar{X} = \hat{R}\,\bar{X}, \qquad \text{where} \qquad \hat{R} = \frac{\bar{y}}{\bar{x}} = \frac{y}{x} \tag{1.1}$$

where $\hat{R}$ is the estimate of $R = \frac{\bar{Y}}{\bar{X}} = \frac{Y}{X}$, $\bar{y}$ is the sample mean of the study variable $Y$ and $\bar{x}$ is the sample mean of auxiliary variable $X$. It is assumed that the population mean $\bar{X}$ of auxiliary variable $X$ is known. Among the modified ratio type estimators available in the literature, five of the mostly used modified ratio type estimators are considered for further improvements. For the benefit of the readers the modified ratio estimators together with their biases and their mean squared errors are given below.

When the population coefficient of variation of auxiliary variable $C_x$ is known, Sisodia and Dwivedi [2] has suggested a modified ratio estimator for $\bar{Y}$ together with its bias and mean squared error as given below:

$$\hat{\bar{Y}}_1 = \bar{y}\left[\frac{\bar{X}+C_x}{\bar{x}+C_x}\right]$$

$$B\left(\hat{\bar{Y}}_1\right) = \frac{(1-f)}{n}\,\bar{Y}\,(\theta_1^2 C_x^2 - \theta_1 C_x C_y \rho)$$

$$MSE\left(\hat{\bar{Y}}_1\right) = \frac{(1-f)}{n}\,\bar{Y}^2(C_y^2 + \theta_1^2 C_x^2 - 2\theta_1 C_x C_y \rho)$$

where $f = \frac{n}{N}$ and $\theta_1 = \frac{\bar{X}}{\bar{X}+C_x}$ (1.2)

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Motivated by Sisodia and Dwivedi [2], Singh and Kakran [3] has developed another ratio type estimator for $\bar{Y}$ using Coefficient of Kurtosis together with its bias and mean squared error as given below:

$$\hat{\bar{Y}}_2 = \bar{y}\left[\frac{\bar{X}+\beta_2}{\bar{x}+\beta_2}\right]$$

$$B\left(\hat{\bar{Y}}_2\right) = \frac{(1-f)}{n}\,\bar{Y}\,(\theta_2^2 C_x^2 - \theta_2 C_x C_y \rho)$$

$$MSE\left(\hat{\bar{Y}}_2\right) = \frac{(1-f)}{n}\,\bar{Y}^2(C_y^2 + \theta_2^2 C_x^2 - 2\theta_2 C_x C_y \rho)$$

where $f = \frac{n}{N}$ and $\theta_2 = \frac{\bar{X}}{\bar{X}+\beta_2}$ (1.3)

Yan and Tian [7] suggested a ratio type estimator using the coefficient of skewness of the auxiliary variable together with its bias and mean squared error as given below:

$$\hat{\bar{Y}}_3 = \bar{y}\left[\frac{\bar{X}+\beta_1}{\bar{x}+\beta_1}\right]$$

$$B\left(\hat{\bar{Y}}_3\right) = \frac{(1-f)}{n}\,\bar{Y}\,(\theta_3^2 C_x^2 - \theta_3 C_x C_y \rho)$$

$$MSE\left(\hat{\bar{Y}}_3\right) = \frac{(1-f)}{n}\,\bar{Y}^2(C_y^2 + \theta_3^2 C_x^2 - 2\theta_3 C_x C_y \rho)$$

where $f = \frac{n}{N}$ and $\theta_3 = \frac{\bar{X}}{\bar{X}+\beta_1}$ (1.4)

Using the population correlation coefficient between $X$ and $Y$, Singh and Tailor [5] proposed another ratio type estimator for $\bar{Y}$ together with its bias and mean squared error as given below:

$$\hat{\bar{Y}}_4 = \bar{y}\left[\frac{\bar{X}+\rho}{\bar{x}+\rho}\right]$$

$$B\left(\hat{\bar{Y}}_4\right) = \frac{(1-f)}{n}\,\bar{Y}\,(\theta_4^2 C_x^2 - \theta_4 C_x C_y \rho)$$

$$MSE\left(\hat{\bar{Y}}_4\right) = \frac{(1-f)}{n}\,\bar{Y}^2(C_y^2 + \theta_4^2 C_x^2 - 2\theta_4 C_x C_y \rho)$$

where $f = \frac{n}{N}$ and $\theta_4 = \frac{\bar{X}}{\bar{X}+\rho}$ (1.5)

By using the population variance of auxiliary variable $X$, Singh [4] proposed a modified ratio estimator for $\bar{Y}$ together with its bias and mean squared error as given below:

$$\hat{\bar{Y}}_5 = \bar{y}\left[\frac{\bar{X}+S_x}{\bar{x}+S_x}\right]$$

$$B\left(\hat{\bar{Y}}_5\right) = \frac{(1-f)}{n}\,\bar{Y}\,(\theta_5^2 C_x^2 - \theta_5 C_x C_y \rho)$$

$$MSE\left(\hat{\bar{Y}}_5\right) = \frac{(1-f)}{n}\,\bar{Y}^2(C_y^2 + \theta_5^2 C_x^2 - 2\theta_5 C_x C_y \rho)$$

where $f = \frac{n}{N}$ and $\theta_5 = \frac{\bar{X}}{\bar{X}+S_x}$ (1.6)

For want of space and for the convenience of the readers, the estimators, biases, and mean squared errors discussed in equations (1.2) to (1.6) are represented in a class of modified ratio estimators as given below:

$$\hat{\bar{Y}}_i = \bar{y}\left[\frac{\bar{X}+\alpha_i}{\bar{x}+\alpha_i}\right] \quad i = 1,2,3,4,5$$

$$B\left(\hat{\bar{Y}}_i\right) = \frac{(1-f)}{n}\,\bar{Y}\,(\theta_i^2 C_x^2 - \theta_i C_x C_y \rho) \quad i = 1,2,3,4,5$$

$$MSE\left(\hat{\bar{Y}}_i\right) = \frac{(1-f)}{n}\,\bar{Y}^2\left(C_y^2 + \theta_i^2 C_x^2 - 2\theta_i C_x C_y \rho\right) \quad i = 1,2,3,4,5$$

where $\alpha_1 = C_x$, $\alpha_2 = \beta_2$, $\alpha_3 = \beta_1$, $\alpha_4 = \rho$ and $\alpha_5 = S_x$ (1.7)

$$f = \frac{n}{N}, \theta_1 = \frac{\bar{X}}{\bar{X}+C_x}, \theta_2 = \frac{\bar{X}}{\bar{X}+\beta_2}, \theta_3 = \frac{\bar{X}}{\bar{X}+\beta_1}, \theta_4 = \frac{\bar{X}}{\bar{X}+\rho} \text{ and } \theta_5 = \frac{\bar{X}}{\bar{X}+S_x}$$

The estimators discussed above are biased but having minimum mean squared error compared to ratio estimator. These points have motivated us to introduce a class of almost unbiased modified ratio estimators based on the above estimators. In fact the proposed estimators are all unbiased if the known population parameters are the true values. However in practical problems the known values are replaced by the values estimated from the previous studies or from another sample. Hence these values are not exactly equal to the true value of the population parameters. That is why the proposed estimators are called as a class of almost unbiased modified ratio estimators. Further, the corresponding mean squared errors are lesser than the mean squared errors of the above estimators defined in (1.7). Further the efficiencies of the proposed estimators with that of the modified ratio estimators are assessed for a certain known populations.

**Proposed Modified Ratio Estimators**

New estimators are generally proposed or constructed by modifying the structure of the sampling designs or the structure of the estimators itself with reasonable and convincing motivations. Moving along this direction, we intend in this paper to show how the problem of estimating the unknown population mean of a study variable can be treated in a unified way by defining a class of estimators which will be (almost) unbiased and more efficient estimators.

The proposed modified ratio estimator for population mean $\bar{Y}$ is

$$\hat{\bar{Y}}_{p_i} = k_i\,\bar{y}\left[\frac{\bar{X}+\alpha_i}{\bar{x}+\alpha_i}\right] \quad i = 1,2,3,4,5 \qquad (2.1)$$

where $k_i = \frac{S_y}{S_y + \lambda_i C_y}$, $\alpha_1 = C_x$, $\alpha_2 = \beta_2$, $\alpha_3 = \beta_1$, $\alpha_4 = \rho$ and $\alpha_5 = S_x$

and $\lambda_i$'s are constants. $S_y$ and $C_y$ are the Population variance and Coefficient of variation of study variable $Y$ respectively. It is reasonable to assume that the values of $S_y$ and $C_y$ are known from the previous studies. Then the expected values of the proposed estimators are obtained as

$$E\left(\hat{\bar{Y}}_{p_i}\right) = E\left(k_i\,\bar{y}\left[\frac{\bar{X}+\alpha_i}{\bar{x}+\alpha_i}\right]\right) \qquad (2.2)$$

where $k_i$ and $\alpha_i$ are as defined above. If we assume that $\lambda_i = 0$ then the proposed estimators are exactly equal to the estimators given in (1.2) to (1.6). If we assume that $\lambda_i = B\left(\hat{\bar{Y}}_i\right)$ then the proposed estimators are almost unbiased ratio estimators corresponding to the estimators given in (1.2) to (1.6). The corresponding mean squared errors of the proposed estimators are as given below:

$$MSE\left(\hat{\bar{Y}}_{p_i}\right) = k_i^2\, MSE\left(\hat{\bar{Y}}_i\right) \; i = 1,2,3,4,5 \quad (2.3)$$

**Efficiency Comparison**

To the first degree of approximation, the biases of $\hat{\bar{Y}}_i$ (i=1, 2, 3, 4, 5) are respectively given by

$$B\left(\hat{\bar{Y}}_i\right) = \frac{(1-f)}{n}\,\bar{Y}\left(\theta_i^2 C_x^2 - \theta_i C_x C_y \rho\right) \; i = 1,2.3,4,5 \quad (3.1)$$

Similarly the mean squared error of $\hat{\bar{Y}}_i$ (i=1, 2, 3, 4, 5) are respectively given by

$$MSE\left(\hat{\bar{Y}}_i\right) = \frac{(1-f)}{n}\,\bar{Y}^2\left(C_y^2 + \theta_i^2 C_x^2 - 2\theta_i C_x C_y \rho\right) i = 1,2,3,4,5 \quad (3.2)$$

where

$$f = \frac{n}{N}, \theta_1 = \frac{\bar{X}}{\bar{X}+C_x}, \theta_2 = \frac{\bar{X}}{\bar{X}+\beta_2}, \theta_3 = \frac{\bar{X}}{\bar{X}+\beta_1}, \theta_4 = \frac{\bar{X}}{\bar{X}+\rho} \text{ and } \theta_5 = \frac{\bar{X}}{\bar{X}+S_x}$$

As stated earlier, if $\lambda_i$=0 then the proposed estimator $\hat{\bar{Y}}_{p_i}$ reduces to the estimator $\hat{\bar{Y}}_i$. By choosing different values for the constants $\lambda_i$ one can get a class of estimators for the estimators $\hat{\bar{Y}}_{p_i}$. We have the following two situations:

***Case -1:*** When $\lambda_i = B\left(\hat{\bar{Y}}_i\right)$

In this case $k_i = \frac{S_y}{S_y + B(\hat{\bar{Y}}_i) C_y}$

By substituting the value of $k_i$ in the proposed estimator and taking expectation we can show that $E\left[\hat{\bar{Y}}_{p_i}\right] = \bar{Y}$. That is, $\hat{\bar{Y}}_{p_i}$ is an unbiased estimator for $\bar{Y}$.

It is true only when the values of the known population parameters are true and exact otherwise the proposed estimators are called as almost unbiased. That is, $\hat{\bar{Y}}_{p_i}$ is an almost unbiased estimator of population mean $\bar{Y}$ if the known population parameters are not the true values. When $\lambda_i = B\left(\hat{\bar{Y}}_i\right)$ and $B\left(\hat{\bar{Y}}_i\right)$ >0 the value of $k_i$ will be less than one, the mean squared errors of the proposed estimator $\hat{\bar{Y}}_{p_i}$ will be less than mean squared errors of $\hat{\bar{Y}}_i$.

That is, $MSE\left(\hat{\bar{Y}}_{p_i}\right) < MSE\left(\hat{\bar{Y}}_i\right)$ (3.3)

***Case- 2:*** When $0 < \lambda_i < B\left(\hat{\bar{Y}}_i\right)$

The proposed estimator $\hat{\bar{Y}}_{p_i}$ is an almost unbiased estimator with lesser bias and lesser mean squared error compared to $\hat{\bar{Y}}_i$.

That is, $MSE\left(\hat{\bar{Y}}_{p_i}\right) < MSE\left(\hat{\bar{Y}}_i\right)$ (3.4)

**Numerical Illustration**

The proposed estimators are computed for three populations to demonstrate what we have discussed above. The population 1 is the data given in Khoshnevisan et.al., [6] and the populations 2 and 3 are taken from Murthy [1] in page 228. The population constants obtained from the above data are given below:

**Population-1: Khoshnevisan et.al., [6]**

| | | | |
|---|---|---|---|
| N = 20 | n = 8 | $\bar{Y}$ = 19.55 | $\bar{X}$ = 18.8 |
| ρ = -0.9199 | $S_y$ = 6.9441 | $C_y$ = 0.3552 | $S_x$ = 7.4128 |
| $C_x$= 0.3943 | $\beta_2$=3.0613 | $\beta_1$ = 0.5473 | $\theta_1$=0.9794 |
| $\theta_2$=0.8599 | $\theta_3$=0.9717 | $\theta_4$=1.0514 | $\theta_5$=0.7172 |

**Population-2: Murthy [1]**

$X$= Fixed Capital and $Y$= Output for 80 factories in a region

| | | | |
|---|---|---|---|
| N = 80 | n = 20 | $\bar{Y}$ = 51.8264 | $\bar{X}$ = 11.2646 |
| ρ = 0.9413 | $S_y$ = 18.3569 | $C_y$ = 0.3542 | $S_x$ = 8.4563 |
| $C_x$= 0.7507 | $\beta_2$=-0.06339 | $\beta_1$ = 1.05 | $\theta_1$=0.9375 |
| $\theta_2$=1.0056 | $\theta_3$=0.91473 | $\theta_4$=0.9228 | $\theta_5$=0.5712 |

**Population-3: Murthy [1]**

$X$ = Data on number of workers and $Y$= Output for 80 factories in a region

| | | | |
|---|---|---|---|
| N = 80 | n = 20 | $\bar{Y}$ = 51.8264 | $\bar{X}$ = 2.8513 |
| ρ = 0.9150 | $S_y$ = 18.3569 | $C_y$ = 0.3542 | $S_x$ = 2.7042 |
| $C_x$= 0.9484 | $\beta_2$=1.3005 | $\beta_1$ = 0.6978 | $\theta_1$=0.7504 |
| $\theta_2$=0.6867 | $\theta_3$=0.8033 | $\theta_4$=0.7570 | $\theta_5$=0.5132 |

The existing modified ratio estimators with their biases and mean squared errors are given in the following tables:

**Conclusion**

The present paper has discussed about the strategies for improving the performance of the existing biased ratio type estimators. As a result we have proposed a class of almost unbiased modified ratio estimators based on the available modified ratio estimators and also obtained their mean squared errors. In fact, the proposed estimators are unbiased and more efficient than the existing modified ratio estimators. However the known values of the population parameters are not true and exact to the population parameter values, since more often the known values of the parameters are taken from the previous studies or from another samples obtained for other studies. We support this theoretical result with numerical examples and shown that the proposed class of almost unbiased modified ratio estimators performs better than the existing modified ratio estimators.

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**Table 4.1: Biases and mean squared errors of the existing modified ratio estimators**

| Existing Estimators | Population 1 | | Population 2 | | Population 3 | |
|---|---|---|---|---|---|---|
| | Bias | MSE | Bias | MSE | Bias | MSE |
| $\hat{\bar{Y}}_1$ | 0.4037 | 15.1265 | 0.5066 | 15.2581 | 0.5360 | 17.1881 |
| $\hat{\bar{Y}}_2$ | 0.3310 | 13.2644 | 0.6184 | 19.3382 | 0.4142 | 12.8425 |
| $\hat{\bar{Y}}_3$ | 0.3988 | 15.0020 | 0.4714 | 14.0112 | 0.6483 | 21.3660 |
| $\hat{\bar{Y}}_4$ | 0.4506 | 16.3099 | 0.4839 | 14.4502 | 0.5496 | 17.6849 |
| $\hat{\bar{Y}}_5$ | 0.2527 | 11.2065 | 0.0794 | 2.3565 | 0.1538 | 4.7220 |

The proposed modified ratio estimators together with the values of $k_i$ and the mean squared errors are given below:

Case- 1: When $\lambda_i = B\left(\hat{\bar{Y}}_i\right)$

**Table 4.2: Values of $k_i$ and mean squared errors of the proposed modified ratio estimators**

| Proposed Estimators | Population 1 | | Population 2 | | Population 3 | |
|---|---|---|---|---|---|---|
| | $k_i$ | MSE | $k_i$ | MSE | $k_i$ | MSE |
| $\hat{\bar{Y}}_{p1}$ | 0.9797 | 14.5206 | 0.9903 | 14.9641 | 0.9897 | 16.8379 |
| $\hat{\bar{Y}}_{p2}$ | 0.9833 | 12.8263 | 0.9882 | 18.8848 | 0.9920 | 12.6397 |
| $\hat{\bar{Y}}_{p3}$ | 0.9800 | 14.4081 | 0.9909 | 13.7597 | 0.9876 | 20.8413 |
| $\hat{\bar{Y}}_{p4}$ | 0.9774 | 15.5832 | 0.9907 | 14.1841 | 0.9895 | 17.3156 |
| $\hat{\bar{Y}}_{p5}$ | 0.9872 | 10.9222 | 0.9984 | 2.3493 | 0.9970 | 4.6940 |

*Case- 2:* When $0 < \lambda_i < B\left(\hat{\bar{Y}}_i\right)$ and at $\lambda_i = B\left(\hat{\bar{Y}}_i\right)/2$

**Table 4.3: Values of $k_i$ and mean squared errors of the proposed modified ratio estimators**

| Proposed Estimators | Population 1 | | Population 2 | | Population 3 | |
|---|---|---|---|---|---|---|
| | $k_i$ | MSE | $k_i$ | MSE | $k_i$ | MSE |
| $\hat{\bar{Y}}_{p1}$ | 0.9897 | 14.818 | 0.99513 | 15.1100 | 0.9948 | 17.0116 |
| $\hat{\bar{Y}}_{p2}$ | 0.9916 | 13.042 | 0.99406 | 19.1095 | 0.9960 | 12.7405 |
| $\hat{\bar{Y}}_{p3}$ | 0.9899 | 14.700 | 0.99547 | 13.8846 | 0.9937 | 21.1012 |
| $\hat{\bar{Y}}_{p4}$ | 0.9886 | 15.940 | 0.99535 | 14.3162 | 0.9947 | 17.4988 |
| $\hat{\bar{Y}}_{p5}$ | 0.9935 | 11.063 | 0.99923 | 2.3529 | 0.9985 | 4.7080 |

From Tables 4.1, 4.2, and 4.3, we summarize the mean squared errors of the existing ( $\lambda_i = 0$ ) and the proposed estimators for different values of $\lambda_i$ in the tables given below:

**Table 4.4: Comparison of Estimators for different values of $\lambda_i$ for Population-1**

| Proposed Estimators | $MSE\left(\hat{\hat{Y}}_{p_i}\right)$ | | $MSE\left(\hat{\hat{Y}}_{p_i}\right)$ |
|---|---|---|---|
| | $\lambda_i = 0$ (Existing estimators) | $\lambda_i = B\left(\hat{\hat{Y}}_i\right)/2$ | $\lambda_i = B\left(\hat{\hat{Y}}_i\right)$ |
| $\hat{\hat{Y}}_{p1}$ | 15.1265 | 14.8189 | 14.5206 |
| $\hat{\hat{Y}}_{p2}$ | 13.2644 | 13.0426 | 12.8263 |
| $\hat{\hat{Y}}_{p3}$ | 15.0020 | 14.7005 | 14.4081 |
| $\hat{\hat{Y}}_{p4}$ | 16.3099 | 15.9403 | 15.5832 |
| $\hat{\hat{Y}}_{p5}$ | 11.2065 | 11.0630 | 10.9222 |

**Table 4.5: Comparison of Estimators for different values of $\lambda_i$ for Population-2**

| Proposed Estimators | $MSE\left(\hat{\hat{Y}}_{p_i}\right)$ | | $MSE\left(\hat{\hat{Y}}_{p_i}\right)$ |
|---|---|---|---|
| | $\lambda_i = 0$ (Existing estimators) | $\lambda_i = B\left(\hat{\hat{Y}}_i\right)/2$ | $\lambda_i = B\left(\hat{\hat{Y}}_i\right)$ |
| $\hat{\hat{Y}}_{p1}$ | 15.2581 | 15.1100 | 14.9641 |
| $\hat{\hat{Y}}_{p2}$ | 19.3382 | 19.1095 | 18.8848 |
| $\hat{\hat{Y}}_{p3}$ | 14.0112 | 13.8846 | 13.7597 |
| $\hat{\hat{Y}}_{p4}$ | 14.4502 | 14.3162 | 14.1841 |
| $\hat{\hat{Y}}_{p5}$ | 2.3565 | 2.3529 | 2.3493 |

**Table 4.6: Comparison of Estimators for different values of $\lambda_i$ for Population-3**

| Proposed Estimators | $MSE\left(\hat{\hat{Y}}_{p_i}\right)$ | | $MSE\left(\hat{\hat{Y}}_{p_i}\right)$ |
|---|---|---|---|
| | $\lambda_i = 0$ (Existing estimators) | $\lambda_i = B\left(\hat{\hat{Y}}_i\right)/2$ | $\lambda_i = B\left(\hat{\hat{Y}}_i\right)$ |
| $\hat{\hat{Y}}_{p1}$ | 17.1881 | 17.0116 | 16.8379 |
| $\hat{\hat{Y}}_{p2}$ | 12.8425 | 12.7405 | 12.6397 |
| $\hat{\hat{Y}}_{p3}$ | 21.3660 | 21.1012 | 20.8413 |
| $\hat{\hat{Y}}_{p4}$ | 17.6849 | 17.4988 | 17.3156 |
| $\hat{\hat{Y}}_{p5}$ | 4.7220 | 4.7080 | 4.6940 |