Available online at www.elixirpublishers.com (Elixir International Journal)

Chemical Engineering

Elixir Chem. Engg. 44 (2012) 7218-7222

Elixir ISSN: 2229-712X

Tuning of PID controllers for isothermal continuous stirred tank reactor

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ARTICLE INFO

Article history:

Received: 2 January 2012;

Received in revised form:

25 February 2012;

Accepted: 6 March 2012;

Keywords

Internal model controller,

Stability analysis method,

Direct synthesis method,

CSTR,

PID controller,

Second Order Plus Time Delay system with a Zero (SOPTDZ).

ABSTRACT

Tuning proportional integral and derivative (PID) controllers for Isothermal CSTR system is proposed. The PID controller is designed based on internal model control and stability analysis principles. The proposed controllers are applied to stable transfer function models of isothermal CSTR carrying out Van de Vusse reaction. Simulation results on non-linear model equations of isothermal CSTR carrying out Van de Vusse reaction is given to show the effectiveness of the proposed PID controllers. The performance under model uncertainty is also studied considering perturbation in one parameter at a time. The performance of proposed controllers is compared with the direct synthesis method (Chien et al., 2003).

Introduction

PID controllers give satisfactory performance for many of the control processes. Due to their simplicity and usefulness, PID controller has become a powerful solution to the control of a large number of industrial processes. The control systems performance is complicated by the numerator dynamics (presence of a zero) of the process. Several processes exhibit second order plus time delay system with a zero transfer function model. Examples for such processes are transfer function model relating concentration of the product to the feed flow rate in isothermal CSTR carrying out Van de Vusse reaction [1], transfer function model relating the reactor temperature to jacket temperature in jacketed CSTR [1], transfer function model relating concentration of the cells to the dilution rate in bioreactors [2] and transfer function relating temperature of incinerator to the inlet load rate in municipal waste incinerator. Many recycle processes where energy and mass recycle takes place are represented by SOPTDZ transfer function model.

Many methods of designing PID controllers are available in the literature. There are methods based on stability analysis [3,4,6], extension of Halmann method [8], modified gain and phase margin method [9], optimization method using artificial neural networks [11], gain and phase margin methods [10], extension of Smith predictor using modern H_∞ control theory [16], simple method [17] etc.

Chien et al. (2003) [12] have proposed a direct synthesis method of designing PID controllers for SOPTD systems with a zero. They have equated the integral time and derivative time of the series type of controller to two time constants of the system and resulting closedloop characteristic equation is compared with the desired characteristic equation to calculate the controller gain. In the present work, the performance comparison of the proposed controllers is made with direct synthesis method [12].

In the present work, a simple method for designing PID controllers based on internal model control principles and stability analysis is proposed. The PID tuning parameters are given as function of process model parameters. Simulation results on isothermal CSTR and non-linear models are given to show the efficiency of the proposed controllers.

Proposed methods

Imc method

The process transfer function for SOPTD with a positive zero is given by

$$G_p(s) = \frac{K_p(1-ps)e^{-Ls}}{a_1 s^2 + a_2 s + 1} \qquad (1)$$

Now, let $G_p(s) = G_p{}^{+}(s) G_P{}^{-}(s)$ (2)

Where $G_p{}^{+}(s)$ is invertible part and $G_p{}^{-}(s)$ is non-invertible part. Therefore, the invertible part of the process transfer function in Eq (14) is

$$G_P{}^{+}(s) = \frac{K_p}{(a_1 s^2 + a_2 s + 1)} \qquad (3)$$

The IMC controller for this system is given as,

$$Q = [G_P{}^{+}(s)]^{-1}.f \qquad (4)$$

Where f is IMC filter and is given by $f = \frac{1}{(1+\lambda s)^n}$. The order (n) of denominator is selected in such a way to make controller realizable (the controller numerator order should be less than or equal to that of the order of denominator). Therefore IMC filter f is given by

$$f = \frac{1}{(\lambda s + 1)^2} \qquad (5)$$

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Using Eq (16) and Eq (18) in Eq (17), IMC controller is given by

$$Q = \frac{(a_1 s^2 + a_2 s + 1)}{K_P(\lambda s+1)^2} \tag{6}$$

Where λ is IMC filter time constant which is a tuning parameter and λ is selected by trial and error procedure in such a way that the maximum magnitude of complementary sensitivity function is between 1 to 1.5 [16]. The equivalent PID controller is obtained by using Eq (7).

$$G_c(s) = \frac{Q}{(1 - G_P(s)Q)} \tag{7}$$

Substituting Eq (1) and Eq (6) in the Eq (7) and using Pade's approximation for time delay, $e^{-Ls} = \frac{1-0.5Ls}{1+0.5Ls}$

$$G_c(s) = \frac{(1+0.5Ls)(a_1 s^2 + a_2 s + 1)}{K_p[(\lambda s+1)^2(1+0.5Ls) - (1-ps)(1-0.5Ls)]} \tag{8}$$

Eq (8) can be rearranged into the following form,

$$G_c(s) = K_c[1 + \frac{1}{\tau_I s} + \tau_d s]\frac{(1+\alpha_0 s)}{(\alpha_1 s^2 + \alpha_2 s + 1)} \tag{9}$$

This is a PID controller with a lead lag filter

Where, $$K_c = \frac{a_2}{K_p(2\lambda + L + p)} \tag{10}$$

$$\tau_I = a_2 \tag{11}$$

$$\tau_D = \frac{a_1}{a_2} \tag{12}$$

$$\alpha_0 = 0.5L \tag{13}$$

$$\alpha_1 = \frac{0.5L\lambda^2}{(2\lambda + L + p)} \tag{14}$$

$$\alpha_2 = \frac{\lambda^2 + L\lambda - 0.5Lp}{(2\lambda + L + p)} \tag{15}$$

IMC-PID controller settings for stable SOPTD with a positive zero are calculated using the Eq (10) to Eq (15).

(II) Stability analysis method

The process transfer function for SOPTD with a positive zero is given by

$$G_p(s) = \frac{K_p(1-ps)e^{-Ls}}{a_1 s^2 + a_2 s + 1}$$

The transfer function of the controller is given by

$$G_c(s) = K_c(1 + \frac{1}{\tau_I s} + \tau_D s) \tag{16}$$

The combined transfer function model of the process and the controller is given by

$$G_c(s)G_p(s) = [\frac{K_P(1-ps)e^{-Ls}}{a_1 s^2 + a_2 s + 1}][K_c(1 + \frac{1}{\tau_I s} + \tau_D s)] \tag{17}$$

let

$$\tau_I = a_2 \tag{18}$$

$$\tau_D = \frac{a_1}{a_2} \tag{19}$$

With these assumptions, the Eq (17) becomes

$$G_c(s)G_p(s) = \frac{K_c K_p(1-ps)e^{-Ls}}{\tau_I s} \tag{20}$$

Phase angle criteria for the given system is as

$$-\frac{\pi}{2} - L\omega_c - \tan^{-1}(p\omega_c) + \pi = 0 \tag{21}$$

Eq (21) is solved by using *fsolve* of MATLAB for ω_c

Using amplude criteria, $K_{c,max}$ is given by

$$K_{c,\max} = \frac{\omega_c \tau_I}{K_p\sqrt{1+p^2\omega^2}} \tag{22}$$

The design value of K_c is calculated by using Eq (23). The PID parameters are given by

$$K_c = \frac{K_{c,\max}}{G_m} \tag{23}$$

$$\tau_I = a_2 \tag{24}$$

$$\tau_D = \frac{a_1}{a_2} \tag{25}$$

Simulation results

In this section, proposed IMC-PID controller (PID controller designed by IMC method) and proposed SA-PID controller (PID controller designed by stability analysis method) are applied to various CSTR transfer function models and non-linear models to show the efficiency of the proposed controllers. The performance of the proposed controllers is compared with DS-PID controller (PID controller designed by direct synthesis method [12]).

Case study-1

Consider a continuous stirred tank reactor where in the following isothermal series-parallel (Van de Vusse) reactions takes place. Here the product B is the desired one.

$$A \xrightarrow{K_1} B \xrightarrow{K_2} C$$

$$2A \xrightarrow{K_3} D$$

The non-linear model equations for the species A and B are,

$$\frac{dC_A}{dt} = \frac{F}{V}(C_{Af} - C_A) - K_1 C_A - K_3 C_A{}^2 \tag{26}$$

$$\frac{dC_B}{dt} = -\frac{F}{V}C_B + K_1 C_A - K_2 C_B \tag{27}$$

The above non-linear equations are linearised around stable operating point C_A=3 g mole/l and C_B=1.1170 g mole/l. The process parameters are given in Table 1. The transfer function relating the desired concentration (C_B) to the ratio flow rate and volume of the reactor along with a measurement time delay of 0.1 min is given by

$$\frac{C_B(s)}{F/V(s)} = \frac{-1.117s + 3.1472}{s^2 + 4.6429s + 5.3821}e^{-0.1s} \tag{28}$$

The IMC-PID controller, SA-PID controller and DS-PID [12] controller parameters are given in Table 2. The servo and regulatory problem of the process transfer function model with IMC-PID controller, SA-PID controller, and DS-PID [12] controller is shown in Fig 1. The performance of servo and regulatory problems is almost same for all the controllers. The performance in terms of ISE, IAE, and ITAE of the process transfer function model with IMC-PID controller, SA-PID controller and DS-PID [12] controller is given in Table 3. The simulation of non-linear model equations with IMC-PID controller, SA-PID controller and DS-PID [12] controller for servo problem (change in product concentration C_B from 1.117 to 1.2 g mole/l) is shown in Fig 2. The regulatory problem of constant volume CSTR for the change in flow rate (F/V) from 0.5714 to 0.6 min^{-1} is shown in Fig 3. For servo problem, all the controllers give almost similar response. But for regulatory problem SA-PID controller is superior compared to the IMC-PID controller and DS-PID [12] controller. The performance comparison of the proposed controllers for nonlinear models in terms of ISE, IAE, and ITAE for both servo and regulatory problems are given in Table 4. The controllers are designed based on nominal model parameters and applied to the process with perturbed parameters. The proposed IMC-PID controller is robust for model uncertainty considering one parameter at a time compared to SA-PID controller and DS-PID [12] controller. Maximum magnitude of complimentary sensitivity function of these three controllers is given in table 5.

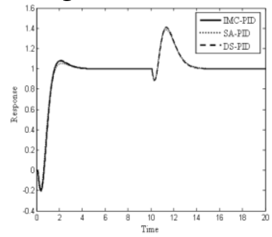

Fig 1. Comparison of servo problem and regulatory response of constant volume CSTR carrying CSTR out Van de Vusse reaction for linear model

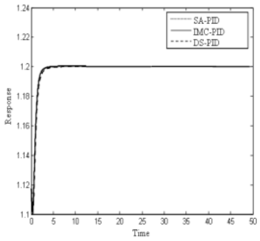

Fig 2. Comparison of servo problem of constant volume CSTR carrying out Van de Vusse reaction for change in product concentration CB from 1.117 to 1.2 gmole/l for non-linear model

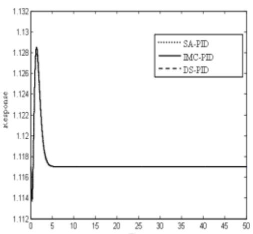

Fig 3. Comparison of regulatory problem of constant volume CSTR carrying out Van de Vusse reaction for change in ratio of feed flow rate to volume of reactor (F/V=0.5714 to 0.6 min^{-1}) for non-linear model in case study-5.

Conclusions

Two PID controller design methods based on internal model control principles and stability analysis method for Isothermal CSTR is proposed.The proposed IMC-PID controller and SA-PID controller are applied to control isothermal constant volume CSTR carrying out Van de Vusse reaction. The performance of the proposed IMC-PID controller and SA-PID controller is compared with DS-PID [11] controller and the IMC-PID controller gives the best performance for both servo and regulatory problems. The performance of proposed IMC-PID controller and SA-PID controller for uncertainty in parameters considering one parameter at a time is studied and compared with the performance of PID controller designed by DS-PID [11] controller. The IMC-PID controller and SA-PID controller gives robust performance compared to DS-PID [11] controller. The performance comparison is given in terms of ISE, IAE and ITAE for both linear model and nonlinear linear model for both servo and regulatory problems. Simulation results on non-linear model equations of constant volume CSTR carrying out Van de Vusse reaction show that the proposed IMC-PID controller and SA-PID controller performs better than controller designed by DS-PID [11] controller.

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Nomenclature

G_c Control transfer function
G_p Process transfer function
G_m Gain margin
K_c Controller gain
$K_{C,max}$ Ultimate controller gain
τ_I Integral time
K_p Process gain
L Time delay
P Numerator time constant
t Time
τ Time constant
τ_D Derivative time
$\alpha_0, \alpha_1, \alpha_2$ Filter time constants
$a_1.\ a_2$ Denominator constants
ω_c Cross over frequency
λ IMC filter time constant
ISE Integral square error
IAE Integral absolute error
ITAE Integral time absolute error
DS-PID Proportional Integral and Derivative Controller designed by Direct synthesis Method (Chien et al., 2003)
SA-PID Proportional Integral and Derivative Controller designed by Stability analysis method
IMC-PIDProportional Integral and Derivative Controller designed by Internal model control.

Table 1. Parameter values of Isothermal CSTR.

Parameter	Case study-1
$\frac{F}{V}$	0.5714 min^{-1}
K_1	$\frac{5}{6}$ min^{-1}
K_2	$\frac{5}{3}$ min^{-1}
K_3	$\frac{1}{6}$ min^{-1}
C_{Af}	10 gmole/l

Table 2. PID settings for different methods.

Controller	K_C	τ_I	τ_D	α_0	α_1	α_2
IMC-PID	1.4685	0.8627	0.2154	0.05	0.0038	0.0850
SA-PID	1.4685	0.8627	0.2154	---	---	----
DS-PID	1.4588	0.8627	0.2154	---	---	0.0415

Table 3. Performance comparison in terms of ISE, IAE, ITAE for linear models.

Controller	Servo problem			Regulatory problem		
	ISE	IAE	ITAE	ISE	IAE	ITAE
IMC-PID	1.0634	1.2171	0.8732	0.1808	0.6582	1.1067
SA-PID	1.0131	1.1622	0.7933	0.1714	0.6582	1.1267
DA-PID	1.0656	1.2316	0.8967	0.1829	0.6613	1.1136

Table 4. Performance comparison for non-linear model

Performance	Controller	Change in Concentration of B(From:1.117 to 1.2 mole/l)	Change in F/V(From:0.5471 to 0.6 min^{-1})	Change in feed concentration of A(from:10 to 12 mole/l)
		Servo problem	Regulatory Problem	
ISE	IMC-PID	0.0079	1.456×10^{-4}	0.0076
	SA-PID	0.0076	1.382×10^{-4}	0.0071
	DS-PID	0.0079	1.472×10^{-4}	0.0077
IAE	IMC-PID	0.1119	0.0188	0.1103
	SA-PID	0.1119	0.0188	0.1103
	DS-PID	0.1126	0.0189	0.1111
ITAE	IMC-PID	0.087	0.0320	0.1274
	SA-PID	0.0922	0.0325	0.1309
	DS-PID	0.0881	0.0322	0.1280

Table 5. Maximum magnitude of complimentary sensitivity function

Transfer function model	Controller	Complimentary sensitivity function
Case study-1	IMC-PID	1
	SA-PID	1
	DS-PID	1