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Radiation and mass transfer effects on MHD free convective flow past a semi-infinite vertical porous plate

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ABSTRACT

In this article, we studied the effects of variable viscosity and thermal conductivity on an unsteady two-dimensional laminar flow of a viscous incompressible electrically conducting fluid past a semi-infinite vertical plate taking into account the mass transfer. The fluid viscosity is assumed to vary as a linear function of temperature. The governing equations for the flow are transformed into a system of non-linear ordinary differential equations are solved by a closed analytical form. The effects of the various parameters on the velocity, temperature, concentration and skin-friction profiles are presented graphically.

Introduction

Free convection flow involving coupled heat and mass transfer occurs frequently in several areas of chemical engineering and manufacturing process areas. A few representative fields of interest in which combined heat and mass transfer plays an important role are designing of chemical processing equipment, formation and dispersion of fog, distribution of temperature and moisture over agricultural fields and fruit trees, crop damage due to freezing and environmental disorders. Thermal radiation has become a significant branch of engineering sciences and is an essential aspect of various scenarios in mechanical, aerospace, chemical and solar power engineering.

Extensive research work has been published on semi-infinite vertical plate with different boundary conditions. An exact solution to the Navier-Stokes equation of the flow of a viscous incompressible fluid past an impulsively started infinite horizontal plate moving in its own plane was first examined by Stokes [1] which is being often referred as Rayleigh's problem in the literature. Later, Stewartson [2] presented analytic solution to the viscous flow past an impulsively started semi-infinite horizontal plate. Subsequently, the problem of Stewartson was examined by Hall [3] by using finite difference method of a mixed explicit-implicit type, which was tested for convergence and its stability. Soundalgekar [4] for the first time obtained the exact solution of the Stokes problem for the case of infinite vertical plate. One of the first models for combined radiative hydromagnetic heat transfer considering the case of free convective channel flows with an axial temperature gradient was analysed and discussed by Moss [5]. Chang *et al.* [6] examined the effect of radiation heat transfer on free convection regimes in enclosures, with applications in geophysics and geothermal reservoirs. Later, Soundalgekar *et al.* [7] studied the finite difference analysis of mass transfer effects on flow past an impulsively started infinite isothermal vertical plate in a dissipative fluid. Thereafter, Mahajan *et al.* [8] reported the influence of viscous heating dissipation effects in natural convective flows, showing that the heat transfer rates are reduced by an increase in the dissipation parameter. The mass transfer effects on the flow past an impulsively started infinite vertical plate with constant mass flux and chemical reaction was studied by Das *et al.* [9]. Thereafter, Sacheti *et al.* [10] obtained an exact solution for unsteady MHD free convection flow on an impulsively started vertical plate with constant heat flux while, Hossain *et al.* [11] studied the radiation effects on mixed convection along a vertical plate with uniform surface temperature using the Rosseland flux model. The coupled magnetic field and thermal radiation effects in non-gray fluid boundary layer heat transfer, using a Runge-Kutta Merson quadrature was analysed by Takhar *et al.* [12]. Subsequently, Shankar *et al.* [13] discussed the effect of mass transfer on the MHD flow past an impulsively started infinite vertical plate with variable temperature or constant heat flux. Muthucumaraswamy *et al.* [14] studied the problem of unsteady flow past an impulsively started isothermal vertical plate with mass transfer by an implicit finite difference method. While, Abd EI-Naby *et al* [15] studied the radiation effects on MHD unsteady free-convection flow over vertical plate with variable surface temperature. The influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time dependent suction was investigated by Isreal-Cookey *et al.* [16]. Muthucumaraswamy [17] studied the natural convection on flow past an impulsively started vertical plate with variable surface heat flux. Radiation and mass transfer effects on two dimensional flows past an impulsively started isothermal vertical plate was studied by Ramachandra Prasad *et al.* [18]. In all the investigations mentioned above, viscous mechanical dissipation is neglected. Such effects are important in geophysical flows and also in certain industrial operations and are usually characterized by the Eckret number. A number of authors have considered viscous heating effects on Newtonian flows. Very recently, the network simulation method [NSM] to study the effects of viscous dissipation and radiation on unsteady MHD

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free convection flow past a vertical porous plate was used by Zueco [19].

In all above investigations and analysis, the fluid to consider is to be electrically non-conducting. However, the flow of Newtonian electrically conducting fluids is also of great interest in high speed aerodynamics, astronautical plasma flows; MHD boundary layer control, MHD accelerator technologies and the applications are many from the view of science and technology.

However, the interaction of radiation with mass transfer of an electrically conducting dissipative fluid past an impulsively started isothermal vertical plate has received a little attention. Hence, the present study is attempted.

Mathematical Formulation

In a situation of two dimensional unsteady laminar natural convection flows of a viscous, incompressible, electrically conducting, radiating fluid past an impulsively started semi-infinite vertical plate in the presence of transverse magnetic field with viscous dissipation is considered. The fluid is assumed to be gray, absorbing-emitting but non-scattering. The x - axis is taken along the plate in the upward direction and the y - axis is taken normal to it. The fluid is assumed to be slightly conducting and hence the magnetic Reynolds number is much less than unity and the induced magnetic field is negligible in comparison with the transverse applied magnetic field. Initially, it is assumed that the plate and the fluid are at the same temperature T'_∞ and concentration level C'_∞ everywhere in the fluid. At time $t' > 0$, the plate starts moving impulsively in the vertical direction with constant velocity u_0 against the gravitational field. Also, the temperature of the plate and the concentration level near the plate are raised to T'_w and C'_w, respectively and are maintained constantly thereafter. It is assumed that the concentration C' of the diffusing species in the binary mixture is very less in the comparison to the other chemical species, which are present and hence the Soret and Dufour effects are negligible. It is also assumed that there is no chemical reaction between the diffusing species and the fluid. Then, under the above assumptions, in the absence of an input electric field, the governing boundary layer equations with Boussinesq's approximation are

Continuous equation

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \quad (1)$$

Momentum conservation

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta\left(T-T_\infty\right)+g\beta^*\left(C-C_\infty\right)+v\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u \quad (2)$$

Energy conservation

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}+\frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \quad (3)$$

Species conservation

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2} \quad (4)$$

The initial and boundary conditions are as follows:

$$\left.\begin{array}{lllll} t\le 0, & u=0, & v=0, & T=T_\infty, & C=C_\infty \\ t>0 & u=u_0, & v=0, & T=T_w, & C=C_w \text{ at } y=0 \\ & u\to 0, & T\to T_\infty, & C'\to C_\infty & \text{as } y\to\infty \end{array}\right\} \quad (5)$$

Thermal radiation is assumed to be present in the form of a unidirectional flux in the y - direction i.e., q_r (transverse to the vertical surface). By using the Rosseland approximation, the radiative heat flux q_r is given by

$$q_r=-\frac{4\sigma_s}{3k_c}\frac{\partial T'^4}{\partial y} \quad (6)$$

where σ_s is the Stefan-Boltzmann constant and K_c - the mean absorption coefficient. In the Rosseland approximation, the present analysis is limited to optically thick fluids. If temperature differences within the flow are sufficiently small, then equation (6) can be linearized by expanding T'^4 into the Taylor series about T'_∞, which after neglecting higher order terms takes the form:

$$T'^4 \cong 4T'^3_\infty T'-3T'^4_\infty \quad (7)$$

In view of equations (6) and (7), equation (3) reduces to

$$\frac{\partial T'}{\partial t'}+u\frac{\partial T'}{\partial x}+v\frac{\partial T'}{\partial y}=\alpha\frac{\partial^2 T'}{\partial y^2}+\frac{16\sigma_s T'^3_\infty}{3k_c\sigma c_p}\frac{\partial^2 T'}{\partial y^2}+\frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \quad (8)$$

The nondimensionless quantities introduced in these equations are defined as

$$\left.\begin{array}{llll} X=\frac{xu_0}{v}, \quad Y=\frac{yu^0}{v}, \quad t=\frac{t'u_0^2}{v}, & & U=\frac{u}{u_0}, & V=\frac{v}{u_0} \\ Gr=\frac{vg\beta\left(T'_w-T'_\infty\right)}{u_0^3}, & Gm=\frac{vg\beta^*\left(C'_w-C'_\infty\right)}{u_0^3}, & N=\frac{k_c k}{4\sigma_s T'^3_\infty}, & M=\frac{\sigma\beta_0^2 v}{u_0^2} \\ T=\frac{T'-T'_\infty}{T'_w-T'_\infty}, & C=\frac{C'-C'_\infty}{C'_w-C'_\infty}, & \Pr=\frac{\nu}{\alpha}, & Sc=\frac{\nu}{D}, \end{array}\right\} \quad (9)$$

In a situation where only one dimensional flow is considered, the above set of equations (1), (2), (8) and (4) are reduced to the following non-dimensional form:

$$\frac{\partial V}{\partial Y}=0\Rightarrow V=-V_0 \text{ (where } V_0=1) \quad (10)$$

$$\frac{\partial U}{\partial t}-\frac{\partial U}{\partial Y}=GrT+GmC+\frac{\partial^2 U}{\partial Y^2}-MU \quad (11)$$

$$\frac{\partial T}{\partial t}-\frac{\partial T}{\partial Y}=\frac{1}{\Pr}\left(1+\frac{4}{3N}\right)\frac{\partial^2 T}{\partial Y^2} \quad (12)$$

$$\frac{\partial C}{\partial t}-\frac{\partial C}{\partial Y}=\frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} \quad (13)$$

The corresponding initial and boundary conditions are as follows

$$\left.\begin{array}{lllll} t\le 0: & U=0, & T=0 & C=0 & \\ t>0: & U=1, & T=1, & C=1 & \text{at } Y=0 \\ & U\to 0, & T\to 0, & C\to 0 & \text{as } Y\to\infty \end{array}\right\} \quad (14)$$

Solution of the problem

Assuming a trial solution for the above governing equations as:

$$U(y,t) = U_0(y)e^{i\omega t}$$

$$T(y,t) = T_0(y)e^{i\omega t}$$

$$C(y,t) = C_0(y)e^{i\omega t}$$

The solution for the equations (11), (12) and (13) subject to the conditions (14), is found to be

$$U(y,t) = (m_4 + m_6 + 1)e^{-m_7 y} - \left(m_4 e^{-m_2 y} + m_6 e^{-m_1 y}\right)$$

$$T(y,t) = e^{-m_2 y}$$

$$C(y,t) = e^{-m_1 y}$$

Here the constants are not given because sake of brevity.

Skin-friction: The dimensionless shearing stress on the surface of a body, due to the fluid motion, is known as skin-friction and is defined by the Newton's law of viscosity.

The skin-friction is

$$\tau = \left(\frac{\partial U}{\partial y}\right)_{y=0} = -m_7(m_4 + m_6 + 1) + m_2 m_4 + m_1 m_6$$

Results and discussion

The effect of the frequency of excitation over the velocity field is illustrated in Fig. 1. It is noticed that as the frequency of excitation increases the velocity field decreases. Further, it is observed that as we move far away from the plate the velocity decreases rapidly initially and thereafter the decrease is found to be slow. Fig. 2 illustrates the effect of the radiation parameter on the velocity profiles. It is observed that, increase in radiation parameter contributes to the decrease in velocity field. Further, as we move away from the plate the effect of higher values of radiation parameter is not that prominent as compared to the smaller values. The effect of magnetic intensity of the velocity profiles is shown in Fig. 3. While all other participating parameters in the velocity field are held constant and magnetic intensity is increased, a drop in the velocity field is noticed. When the magnetic intensity is relatively small the velocity field increases initially and thereafter it decreases substantially. The effect of magnetic field is found to have zero effect on the velocity field as we move far away from the plate. The effect of Schmidt number on velocity field is exhibited in Fig. 4. As the Schmidt number increases the velocity field decreases. Far away from the plate it is noticed that the Schmidt number has no effect on the velocity field.

The effect of Prandtl number on the velocity field is shown in Fig. 5. As the Prandtl number increases the velocity field is found to be decreasing. As we move away from the plate it is noticed that at higher values of Prandtl number does not contribute much on the velocity field but a smaller values contributes to the increase in the velocity. The effect of the solutal Grash of number on the velocity profiles is illustrated in Fig. 6. The increase in the solutal Grash of number contributes to the raise in the velocity field. Moreover, it is observed that the effect is almost zero as we move far away from the plate. Fig. 7 illustrates the effect of thermal Grashof number on velocity field. While all other parameters in the velocity field are held constant, increase in the thermal Grashof number contributes to the raise in the velocity field. However, the effect is found to be absolutely zero as we move away from the bounding surface.

The contribution of radiation parameter on the temperature field is illustrated in Fig. 8. It is observed that for the smaller values of radiation parameter the fall in temperature is perfectly linear. However, the situation is not same for relatively higher values of the radiation parameter.

The variation in temperature profiles with respect to Prandtl number is illustrated in Fig.9. In general it is noticed that as the Prandtl number increases the temperature decreases. At the higher values of the Prandtl number the profiles are found to be more parabolic. For sufficiently smaller values the relation between the Prandtl number and temperature field is perfectly linear.

The contribution of Schmidt number on the concentration profiles is shown in Fig. 10. It is noticed that increase in the Schmidt number contributes to the decrease in the concentration of the fluid media. It is observed that relatively for the smaller values of the Schmidt number the concentration is perfectly linear. Increase in the Schmidt number contributes to the parabolic nature of the profiles. The effect of the frequency of excitation on the concentration field is illustrated in Fig. 11. As the frequency of excitation increases the concentration is found to be decreasing.

The effect of thermal Grash of number on skin-friction is observed in Fig. 12. It is noticed that as thermal Grash of number increases, the skin-friction is found to be increasing. Further, it is noticed that the effect of thermal Grash of number remains constant as the frequency of excitation increases. The effect of Prandtl number on skin-friction is illustrated in Fig. 13. It is noticed that as the Prandtl number increases, the skin-friction on the bounding surface decreases. It is noticed that, as the frequency of excitation increases with respect to the Prandtl number, not much of significant change is noticed. However, as both the parameters increasing the skin-friction is found to have a negative effect.

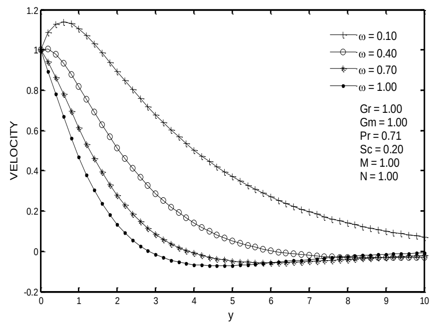

Fig. 1 Effect of frequency of excitation on velocity profiles

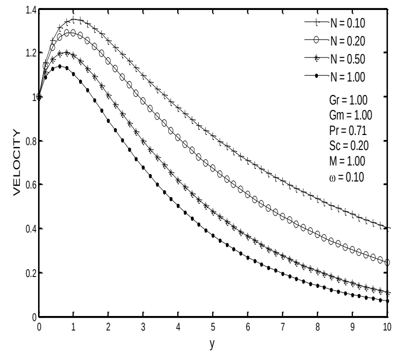

Fig. 2.Effect of radiation parameter on velocity profiles

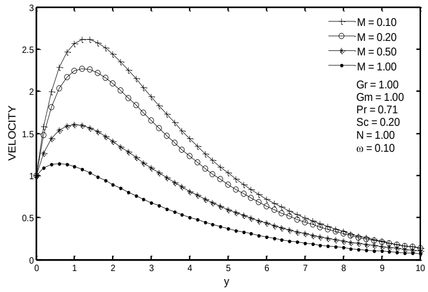

Fig.3. Effect of magnetic parameter on velocity profiles

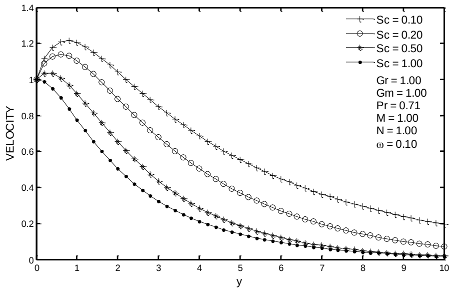

Fig. 4. Effect of Schmidt number on velocity profiles

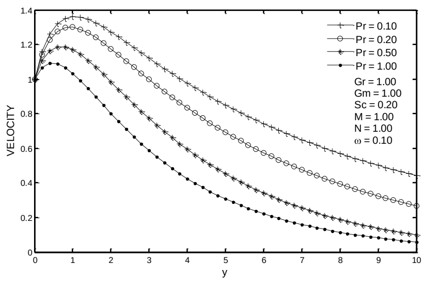

Fig. 5. Effect of Prandtl number on velocity profiles

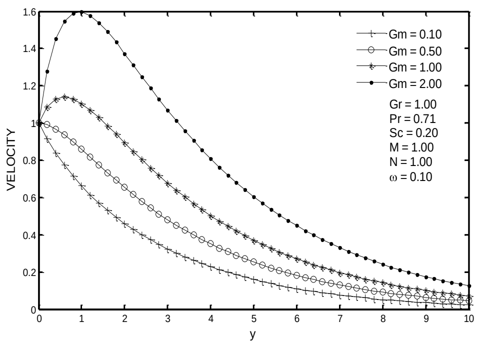

Fig. 6. Effect of solutal Grashof number on velocity profiles

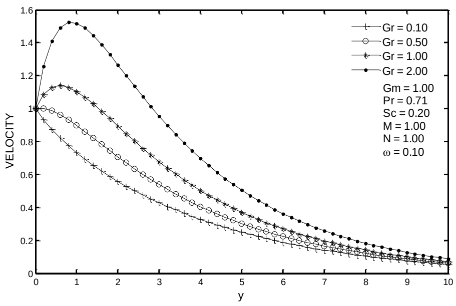

Fig. 7. Effect of thermal Grashof number on velocity profiles

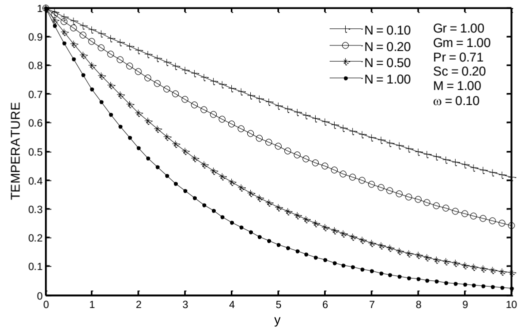

Fig. 8. Effect of radiation parameter on temperature

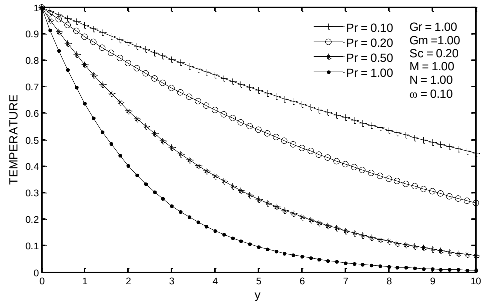

Fig. 9. Effect of Prandtl number on temperature

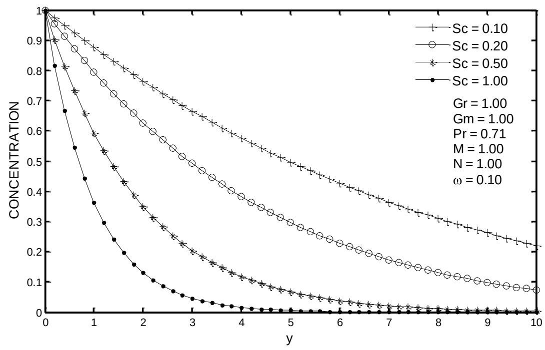

Fig. 10. Effect of Schmidt number on concentration

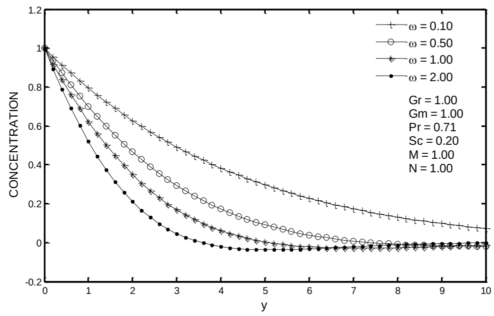

Fig. 11. Effect of frequency of excitation on concentration

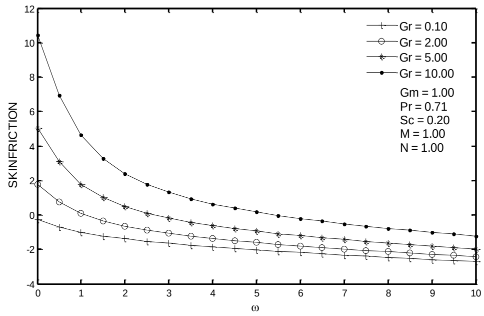

Fig. 12. Effect of thermal Grashof number on skin-friction

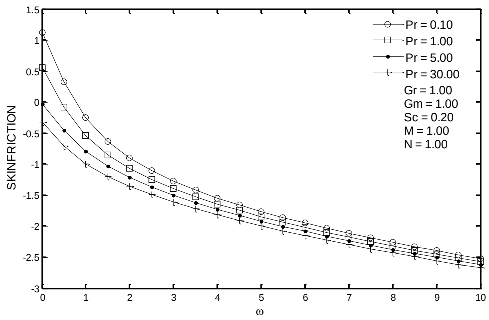

Fig. 13. Effect of Prandtl number on skin-friction

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