Available online at www.elixirpublishers.com (Elixir International Journal)

Applied Mathematics

Elixir Appl. Math. 45 (2012) 7817-7821

ISSN: 2229-712X

On Some New Applications in $\mathcal{R}^0{}_{\widetilde{\alpha\delta}}$ and $\mathcal{R}^1{}_{\widetilde{\alpha\delta}}$ Spaces via $\alpha\delta$ -Open Sets

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ARTICLE INFO

Article history:

Received: 20 February 2012;

Received in revised form:

17 March 2012;

Accepted: 9 April 2012;

Keywords

$Sober\ \text{-}\alpha\delta\mathcal{R}_0$,

$\mathcal{R}^0{}_{\widetilde{\alpha\delta}}$ *-space,*

$\mathcal{R}^1{}_{\widetilde{\alpha\delta}}$ *-space.*

ABSTRACT

The notion of $\alpha\delta$ -closed set was introduced and studied by R. Devi, V. Kokilavani and P. Basker [2]. In [5], the $\alpha\delta$ -closure and the $\alpha\delta$ -kernel are defined in terms of this weakly ultra-$\alpha\delta$ -separation and also investigate some of the properties of the $\alpha\delta$ -kernel and the $\alpha\delta$ -closure. Using this concept we introduce $Sober\ \text{-}\alpha\delta\mathcal{R}_0$ -spaces, $\mathcal{R}^0{}_{\widetilde{\alpha\delta}}$ and $\mathcal{R}^1{}_{\widetilde{\alpha\delta}}$ spaces are also defined.

1. Introduction and Preliminaries

Closedness are basic concept for the study and investigation in general topological spaces. This concept has been generalized and studied by many authors from different points of views. In particular, Njastad [6] and Velicko [7] introduced α -open sets and δ -closed sets respectively. R.Devi et al.[1] introduced α -generalized closed (briefly αg -closed) sets. More recently R.Devi, V.Kokilavani and P.Basker. [2] has introduced and studied the notion of $\alpha\delta$ -closed sets which is implied by that of δ -closed sets. In [5], the $\alpha\delta$ -closure and the $\alpha\delta$ -kernel are defined in terms of this weakly ultra-$\alpha\delta$ -separation and also investigate some of the properties of the $\alpha\delta$ -kernel and the $\alpha\delta$ -closure. In this paper we introduce $Sober$ -$\alpha\delta\mathcal{R}_0$ spaces, $\mathcal{R}^0{}_{\widetilde{\alpha\delta}}$ and $\mathcal{R}^1{}_{\widetilde{\alpha\delta}}$ spaces are also defined.

Throughout this present paper, spaces *X* and *Y* always mean topological spaces. Let *X* be a topological space and *A*, a subset of *X*. The closure of *A* and the interior of *A* are denoted by $cl(A)$ and $int(A)$, respectively. A subset *A* is said to be regular open (resp. regular closed) if $A = int(cl(A))$ (resp. $A = cl(int(A))$, The δ -interior [7] of a subset *A* of *X* is the union of all regular open sets of *X* contained in *A* and is denoted by $Int_\delta(A)$. The subset *A* is called δ -open [7] if $A = Int_\delta(A)$, *i.e.*, a set is δ -open if it is the union of regular open sets. The complement of a δ -open set is called δ -closed. Alternatively, a set $A \subset (X, \tau)$ is called δ -closed [7] if $A = cl_\delta(A)$, where $cl_\delta(A) = \left\{\frac{x}{x} \in U \in \tau \Rightarrow int(cl(A)) \cap A \neq \varphi\right\}$. The family of all δ -open (resp. δ -closed) sets in X is denoted by $\delta O(X)$ (resp. $\delta C(X)$). A subset A of X is called α -open [6] if $A \subset int\left(cl\left(int(A)\right)\right)$ and the complement of a α -open are called α -closed. The intersection of all α -closed sets containing *A* is called the α -closure of *A* and is denoted by $\alpha cl(A)$, Dually, α -interior of *A* is defined to be the union of all α -open sets contained in *A* and is denoted by $\alpha int\ (A)$. We recall the following definition used in sequel.

Definition 1.1. A subset A of a space X is said to be

(a) An α -generalized closed [1] (αg -closed) set if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α -open in (X, τ).

(b) A $\alpha\delta$ -closed set [2] if $cl_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is αg -open in (X, τ).

(c) The intersection of all $\alpha\delta$ -open subsets of (X, τ) containing A is called the $\alpha\delta$ -kernel of A (briefly, $\alpha\delta^{\sim Ker}(A)$) .i.e.,$\alpha\delta^{\sim Ker}(A) = \cap\{G \in \alpha\delta O(X, \tau) : A \subseteq G\}$.

(d) Let $x \in X$. Then $\alpha\delta$ -kernel of x is denoted by $\alpha\delta^{\sim Ker}(\{x\}) = \cap\{G \in \alpha\delta O(X, \tau) : x \in G\}$.

(e) A point $x \in A$ is said to be $\alpha\delta$ -Interior point of A if A is a $\alpha\delta$ -nbhd of x . The set of all $\alpha\delta$ -Interior point of A is called the $\alpha\delta$ -Interior of A and is denoted by $\alpha\delta_{Int}(A)$. [4]

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(f) we define the $\alpha\delta$ -closure of A as follows $\alpha\delta_{Cl}(A) = \bigcap \{F : F \text{ is } \alpha\delta - closed \text{ in } X, \quad A \subset X\}$. [4]

2. Sober-$\alpha\delta\mathcal{R}_0$ spaces

Definition 2.1. A topological space (X,τ) is said to be $Sober$ -$\alpha\delta\mathcal{R}_0$ if $\bigcap_{x\in X} \alpha\delta_{Cl}(\{x\}) = \varphi$.

Theorem 2.2. A topological space (X,τ) is $Sober$ -$\alpha\delta\mathcal{R}_0$ if and only if $\alpha\delta^{\sim Ker}(\{x\}) \neq X$ for every x $x \in X$

Proof. Suppose that the space (X,τ) be $Sober$ -$\alpha\delta\mathcal{R}_0$. Assume that there is a point y in X such that $\alpha\delta^{\sim Ker}(\{y\}) = X$. Then $y \notin O$ which O is some proper $\alpha\delta$ -open subset of X. This implies that $y \in \bigcap_{x\in X} \alpha\delta_{Cl}(\{x\})$.

But this is a contradiction.

Now assume that $\alpha\delta^{\sim Ker}(\{x\}) \neq X$ for every $x \in X$. If there exists a point y in X such that $y \in \bigcap_{x\in X} \alpha\delta_{Cl}(\{x\})$, then every $\alpha\delta$ -open set containing y must contain every point of X. This implies that the space X is the unique $\alpha\delta$ -open set containing y. Hence $\alpha\delta^{\sim Ker}(\{x\}) = X$ which is a contradiction. Therefore (X,τ) is $Sober$ -$\alpha\delta\mathcal{R}_0$.

Theorem 2.3. If the topological space X is $Sober$ -$\alpha\delta\mathcal{R}_0$ and Y is any topological space, then the product $X \times Y$ is $Sober$ -$\alpha\delta\mathcal{R}_0$.

Proof. By showing that $\bigcap_{(x,y)\in X\times Y} \alpha\delta_{Cl}(\{x,y\}) = \varphi$

we are done.We have: $\bigcap_{(x,y)\in X\times Y} \alpha\delta_{Cl}(\{x,y\}) \subset \bigcap_{(x,y)\in X\times Y} (\alpha\delta_{Cl}(\{x\}) \times \alpha\delta_{Cl}(\{y\}))$

$= \bigcap_{x\in X} \alpha\delta_{Cl}(\{x\}) \times \bigcap_{y\in Y} \alpha\delta_{Cl}(\{y\}) \subset \varphi \times Y = \varphi$.

3. Application of $\mathcal{R}^0{}_{\widetilde{\alpha\delta}}$ and $\mathcal{R}^1{}_{\widetilde{\alpha\delta}}$ spaces.

Definition 3.1. A topological space (X,τ) is said to be a $\mathcal{R}^0{}_{\widetilde{\alpha\delta}}$ if every $\alpha\delta$ -open set contains the $\alpha\delta$ -closure of each of its singletons.

Definition 3.2.
(a) A topological space (X,τ) is said to be $\mathcal{R}^1{}_{\widetilde{\alpha\delta}}$ if for x, y in X with $\alpha\delta_{Cl}(\{x\}) \neq \alpha\delta_{Cl}(\{y\})$, there exist disjoint $\alpha\delta$ -open sets U and V such that $\alpha\delta_{Cl}(\{x\})$ is a subset of U and $\alpha\delta_{Cl}(\{y\})$ is a subset of V.
(b) A topological space (X,τ) is $\alpha\delta$ -symmetric if for x and y in X ; $x \in \alpha\delta_{Cl}(\{y\})$ implies $y \in \alpha\delta_{Cl}(\{x\})$.

Lemma 3.3. Let (X,τ) be a topological space and $x \in X$. Then $y \in \alpha\delta^{\sim Ker}(\{x\})$ if and only if $x \in \alpha\delta_{Cl}(\{y\})$.

Proof. Suppose that $y \notin \alpha\delta^{\sim Ker}(\{x\})$. Then there exists a $\alpha\delta$ -open set V containing x such that $\notin V$. Therefore we have $x \notin \alpha\delta_{Cl}(\{y\})$. The converse is similarly shown.

Theorem 3.4. If (X,τ) is $\mathcal{R}^1{}_{\widetilde{\alpha\delta}}$, then (X,τ) is $\mathcal{R}^0{}_{\widetilde{\alpha\delta}}$.

Proof. Let U be $\alpha\delta$ -open and $x \in U$. If $y \notin U$, then since $x \notin \alpha\delta_{Cl}(\{y\})$, $\alpha\delta_{Cl}(\{x\}) \neq \alpha\delta_{Cl}(\{y\})$. Hence, there exists a $\alpha\delta$ -open V_y such that $\alpha\delta_{Cl}(\{y\}) \subset V_y$ and V_y, which implies $y \notin \alpha\delta_{Cl}(\{x\})$. Thus $\alpha\delta_{Cl}(\{x\}) \subset U$. Therefore (X,τ) is $\mathcal{R}^0{}_{\widetilde{\alpha\delta}}$.

Theorem 3.5. A topological space (X,τ) is $\mathcal{R}^1{}_{\widetilde{\alpha\delta}}$ if and only if for $x,y\in X$; $\alpha\delta^{\sim Ker}(\{x\})\neq\alpha\delta^{\sim Ker}(\{y\})$, there exist disjoint $\alpha\delta$ -open sets U and V such that $\alpha\delta_{Cl}(\{x\})\subset U$ and $\alpha\delta_{Cl}(\{y\})\subset V$.

Proof. It follows from Theorem 2.10[5].

Theorem 3.6. A topological space (X,τ) is a $\mathcal{R}^0{}_{\widetilde{\alpha\delta}}$ if and only for any x and y in X , $\alpha\delta_{Cl}(\{x\})\neq\alpha\delta_{Cl}(\{y\})$ implies $\alpha\delta_{Cl}(\{x\})\cap\alpha\delta_{Cl}(\{y\})=\varphi$.

Proof. *Necessary*. Assume (X,τ) $\mathcal{R}^0{}_{\widetilde{\alpha\delta}}$ and $x,y\in X$ such that $\alpha\delta_{Cl}(\{x\})\neq\alpha\delta_{Cl}(\{y\})$. Then, there exist $z\in\alpha\delta_{Cl}(\{x\})$ such that $z\notin\alpha\delta_{Cl}(\{y\})$ (or $z\in\alpha\delta_{Cl}(\{y\})$ such that $z\notin\alpha\delta_{Cl}(\{x\})$). There exists $V\in\alpha\delta O(X,\tau)$ such that $y\notin V$ and $\in V$; hence $x\in V$. Therefore, we have $x\notin\alpha\delta_{Cl}(\{y\})$. Thus $x\in X\square\alpha\delta\square_{\downarrow}Cl\ (\{y\})\in\alpha\delta O(X,\tau)$, which implies $\square\alpha\delta\square_{\downarrow}Cl\ (\{x\})\subset X\square\alpha\delta\square_{\downarrow}Cl\ (\{y\})$ and $\alpha\delta_{Cl}(\{x\})\cap\alpha\delta_{Cl}(\{y\})=\varphi$. The proof for otherwise is similar.
Sufficiency. Let $V\in\alpha\delta O(X,\tau)$ and let $\in V$. We will show that $\alpha\delta_{Cl}(\{x\})\subset V$. Really, let $\notin V$, *i.e.*, $y\in X\backslash V$. Then $x\neq y$ and $x\notin\alpha\delta_{Cl}(\{y\})$. This shows that $\alpha\delta_{Cl}(\{x\})\neq\alpha\delta_{Cl}(\{y\})$. By assumption, $\alpha\delta_{Cl}(\{x\})\cap\alpha\delta_{Cl}(\{y\})=\varphi$. Hence $y\notin\alpha\delta_{Cl}(\{x\})$. Therefore $\alpha\delta_{Cl}(\{x\})\subset V$.

Theorem 3.7. A topological space (X,τ) is a $\mathcal{R}^0{}_{\widetilde{\alpha\delta}}$ space if and only if for any points x and y in X , $\alpha\delta^{\sim Ker}(\{x\})\neq\alpha\delta^{\sim Ker}(\{y\})$ implies $\alpha\delta^{\sim Ker}(\{x\})\cap\alpha\delta^{\sim Ker}(\{y\})=\varphi$.

Proof. Suppose that (X,τ) is a $\mathcal{R}^0{}_{\widetilde{\alpha\delta}}$ space. Thus by Theorem 2.10[5], for any points x and y in X if $\alpha\delta^{\sim Ker}(\{x\})\neq\alpha\delta^{\sim Ker}(\{y\})$ then $\alpha\delta_{Cl}(\{x\})\neq\alpha\delta_{Cl}(\{y\})$. Now we prove that $\alpha\delta^{\sim Ker}(\{x\})\cap\alpha\delta^{\sim Ker}(\{y\})=\varphi$. Assume that $z\in\alpha\delta^{\sim Ker}(\{x\})\cap\alpha\delta^{\sim Ker}(\{y\})$. By $z\in\alpha\delta^{\sim Ker}(\{x\})$ and Lemma 3.3, it follows that $x\in\alpha\delta_{Cl}(\{z\})$. Since $x\in\alpha\delta_{Cl}(\{x\})$, by Theorem 3.6 $\alpha\delta_{Cl}(\{x\})=\alpha\delta_{Cl}(\{z\})$. Similarly, we have $\alpha\delta_{Cl}(\{y\})=\alpha\delta_{Cl}(\{z\})=\alpha\delta_{Cl}(\{x\})$. This is a contradiction. Therefore, we have $\alpha\delta^{\sim Ker}(\{x\})\cap\alpha\delta^{\sim Ker}(\{y\})=\varphi$.
Conversely , let (X,τ) be a topological space such that for any points x and y in X , $\alpha\delta^{\sim Ker}(\{x\})\neq\alpha\delta^{\sim Ker}(\{y\})$ implies $\alpha\delta^{\sim Ker}(\{x\})\cap\alpha\delta^{\sim Ker}(\{y\})=\varphi$. If $\alpha\delta_{Cl}(\{x\})\neq\alpha\delta_{Cl}(\{y\})$, then by Theorem 2.10[5], $\alpha\delta^{\sim Ker}(\{x\})\neq\alpha\delta^{\sim Ker}(\{y\})$. Therefore $\alpha\delta^{\sim Ker}(\{x\})\cap\alpha\delta^{\sim Ker}(\{y\})=\varphi$ which implies $\alpha\delta_{Cl}(\{x\})\cap\alpha\delta_{Cl}(\{y\})=\varphi$. Because $z\in\alpha\delta_{Cl}(\{x\})$ implies that $x\in\alpha\delta^{\sim Ker}(\{z\})$ and therefore $\alpha\delta^{\sim Ker}(\{x\})\cap\alpha\delta^{\sim Ker}(\{z\})\neq\varphi$. By hypothesis, therefore we have $\alpha\delta^{\sim Ker}(\{x\})=\alpha\delta^{\sim Ker}(\{z\})$. Then $z\in\alpha\delta_{Cl}(\{x\})\cap\alpha\delta_{Cl}(\{y\})$ implies that $\alpha\delta^{\sim Ker}(\{x\})=\alpha\delta^{\sim Ker}(\{z\})=\alpha\delta^{\sim Ker}(\{y\})$. This is a contradiction. Therefore, $\alpha\delta_{Cl}(\{x\})\cap\alpha\delta_{Cl}(\{y\})=\varphi$ and by Theorem 3.6 (X,τ) is a $\mathcal{R}^0{}_{\widetilde{\alpha\delta}}$ space.

Theorem 3.8. For a topological space (X,τ), the following properties are equivalent:
(a) (X,τ) is a $\mathcal{R}^0{}_{\widetilde{\alpha\delta}}$ space;
(b) For any nonempty set A and $G\in\alpha\delta O(X,\tau)$ such that $A\cap G\neq\varphi$, there exists $F\in\alpha\delta C(X,\tau)$ such that $A\cap F\neq\varphi$; and $F\subset G$;
(c) Any $G\in\alpha\delta O(X,\tau)$, $G=\cup\left\{F\in\frac{\alpha\delta C(X,\tau)}{F}\subset G\right\}$;
(d) Any $F\in\alpha\delta C(X,\tau)$, $F=\cap\left\{G\in\frac{\alpha\delta O(X,\tau)}{F}\subset G\right\}$;
(e) For any $x\in X$; $\alpha\delta_{Cl}(\{x\})\subset\alpha\delta^{\sim Ker}(\{x\})$

Proof: $(a)\Rightarrow(b)$ Let A be a nonempty set of X and $G\in\alpha\delta O(X,\tau)$ such that $A\cap G\neq\varphi$. There exists $x\in A\cap G$. Since $x\in G\in\alpha\delta O(X,\tau)$, $\alpha\delta_{Cl}(\{x\})\subset G$. Set $F=\alpha\delta_{Cl}(\{x\})$ then $F\in\alpha\delta C(X,\tau)$, $F\subset G$ and $A\cap F\neq\varphi$.
$(b)\Rightarrow(c)$: Let $G\in\alpha\delta O(X,\tau)$, then $G\supset\cup\left\{F\in\frac{\alpha\delta C(X,\tau)}{F}\subset G\right\}$. Let x be any point of G . There exists $F\in\alpha\delta C(X,\tau)$ such that $x\in F$ and $F\subset G$. Therefore, we have $x\in F\subset\cup\left\{F\in\frac{\alpha\delta C(X,\tau)}{F}\subset G\right\}$ and hence $G=\cup\left\{F\in\frac{\alpha\delta C(X,\tau)}{F}\subset G\right\}$.
$(c)\Rightarrow(d)$ This is obvious.

$(d) \Rightarrow (e)$ Let x be any point of X and $y \notin \alpha\delta^{\sim Ker}(\{x\})$. There exists $V \in \alpha\delta O(X,\tau)$ such that $x \in V$ and $\notin V$; hence $\alpha\delta_{Cl}(\{y\}) \cap V = \varphi$. By (d) $(\cap\{G \in \alpha\delta O(X,\tau) / \alpha\delta_{Cl}(\{y\}) \subset G\} \cap V = \varphi$ and there exists $G \in \alpha\delta O(X,\tau)$ such that $x \notin G$ and $\alpha\delta_{Cl}(\{y\}) \subset G$. Therefore, $\alpha\delta_{Cl}(\{x\}) \cap G = \varphi$ and $y \notin \alpha\delta_{Cl}(\{x\})$. Consequently, we obtain $\alpha\delta_{Cl}(\{x\}) \subset \alpha\delta^{-Ker}(\{x\})$.
$(e) \Rightarrow (a)$ Let $G \in \alpha\delta O(X,\tau)$ and $x \in G$. Let $y \in \alpha\delta^{\sim Ker}(\{x\})$ then $x \in \alpha\delta_{Cl}(\{y\})$ and $y \in G$. This implies that $\alpha\delta^{\sim Ker}(\{x\}) \subset G$. Therefore, we obtain $x \in \alpha\delta_{Cl}(\{x\}) \subset \alpha\delta^{\sim Ker}(\{x\}) \subset G$. This shows that (X,τ) is a $\mathcal{R}^0{}_{\widetilde{\alpha\delta}}$ space.

Corollary 3.9. For a topological space (X,τ), the following properties are equivalent:
(a) (X,τ) is a $\mathcal{R}^0{}_{\widetilde{\alpha\delta}}$ space
(b) $\alpha\delta_{Cl}(\{x\}) = \alpha\delta^{\sim Ker}(\{x\})$ for all $x \in X$.

Proof. $(a) \Rightarrow (b)$ Suppose that (X,τ) is a $\mathcal{R}^0{}_{\widetilde{\alpha\delta}}$ space. By Theorem 3.8, $\alpha\delta_{Cl}(\{x\}) \subset \alpha\delta^{\sim Ker}(\{x\})$ for each $x \in X$. Let $y \in \alpha\delta^{\sim Ker}(\{x\})$ then $x \in \alpha\delta_{Cl}(\{y\})$ and by Theorem 3.6 $\alpha\delta_{Cl}(\{x\}) = \alpha\delta_{Cl}(\{y\})$. Therefore, $y \in \alpha\delta_{Cl}(\{x\})$ and hence $\alpha\delta^{\sim Ker}(\{x\}) \subset \alpha\delta_{Cl}(\{x\})$. This shows that $\alpha\delta_{Cl}(\{x\}) = \alpha\delta^{\sim Ker}(\{x\})$.
$(b) \Rightarrow (a)$ This is obvious by Theorem 3.8.

Theorem 3.10. For a topological space (X,τ), the following properties are equivalent:
(a) (X,τ) is a $\mathcal{R}^0{}_{\widetilde{\alpha\delta}}$ space.
(b) $x \in \alpha\delta_{Cl}(\{y\})$ if and only if $y \in \alpha\delta_{Cl}(\{x\})$.

Proof. $(a) \Rightarrow (b)$ Assume X is $\mathcal{R}^0{}_{\widetilde{\alpha\delta}}$. Let $x \in \alpha\delta_{Cl}(\{y\})$ and D be any $\alpha\delta$ -open set such that $y \in D$. Now by hypothesis, $x \in D$. Therefore, every $\alpha\delta$ -open set which contains y contains x . Hence $y \in \alpha\delta_{Cl}(\{x\})$.
$(b) \Rightarrow (a)$ Let U be a $\alpha\delta$ -open set and $x \in U$. If $y \notin U$, then $x \notin \alpha\delta_{Cl}(\{y\})$ and hence $y \notin \alpha\delta_{Cl}(\{x\})$. This implies that $\alpha\delta_{Cl}(\{x\}) \subset U$. Hence (X,τ) is $\mathcal{R}^0{}_{\widetilde{\alpha\delta}}$.
By Theorem 3.10 and Definition 3.2(b), we have:

Remark 3.11. For a topological space (X,τ), the following properties are equivalent:
(a) (X,τ) is a $\mathcal{R}^0{}_{\widetilde{\alpha\delta}}$ space.
(b) (X,τ) is a $\alpha\delta$ -symmetric.

Theorem 3.12. For a topological space (X,τ), the following properties are equivalent:
(a) (X,τ) is a $\mathcal{R}^0{}_{\widetilde{\alpha\delta}}$ space.
(b) If F is $\alpha\delta$ -closed, then $F = \alpha\delta^{\sim Ker}(F)$
(c) If F is $\alpha\delta$ -closed and $x \in F$, then $\alpha\delta^{\sim Ker}(\{x\}) \subset F$.
(d) If $x \in X$, then $\alpha\delta^{\sim Ker}(\{x\}) \subset \alpha\delta_{Cl}(\{x\})$.

Proof. $(a) \Rightarrow (b)$ This obviously follows from Theorem 3.8.
$(b) \Rightarrow (c)$ In general, $A \subset B$ implies $\alpha\delta^{\sim Ker}(A) \subset \alpha\delta^{\sim Ker}(B)$. Therefore, it follows from (b) that $\alpha\delta^{\sim Ker}(\{x\}) \subset \alpha\delta^{\sim Ker}(F) = F$.
$(c) \Rightarrow (d)$ Since $x \in \alpha\delta_{Cl}(\{x\})$ and $\alpha\delta_{Cl}(\{x\})$ is $\alpha\delta$ -closed, by (c) $\alpha\delta^{\sim Ker}(\{x\}) \subset \alpha\delta_{Cl}(\{x\})$.
$(d) \Rightarrow (a)$ We show the implication by using Theorem 3.10. Let $x \in \alpha\delta_{Cl}(\{y\})$. Then by Lemma 3.3 $y \in \alpha\delta^{\sim Ker}(\{x\})$. Since $x \in \alpha\delta_{Cl}(\{x\})$ and $\alpha\delta_{Cl}(\{x\})$ is $\alpha\delta$ -closed, by (d) we obtain $y \in \alpha\delta^{\sim Ker}(\{x\}) \subset \alpha\delta_{Cl}(\{x\})$. Therefore $x \in \alpha\delta_{Cl}(\{y\})$ implies $y \in \alpha\delta_{Cl}(\{x\})$. The converse is obvious and (X,τ) is $\mathcal{R}^0{}_{\widetilde{\alpha\delta}}$.

Lemma 3.13. Let (X,τ) be a topological space and let x and y be any two points in X such that every net in X $\alpha\delta$ -converging to y $\alpha\delta$ -converges to x . Then $x \in \alpha\delta_{Cl}(\{y\})$.

Proof. Suppose that $x_n = y$ for each $n \in N$. Then $\{x_n\}_{n \in N}$ is a net in $\alpha\delta_{Cl}(\{y\})$. By the fact that $\{x_n\}_{n \in N}$ $\alpha\delta$ -converges to y , then $\{x_n\}_{n \in N}$ $\alpha\delta$ -converges to x and this means that $x \in \alpha\delta_{Cl}(\{y\})$.

Theorem 3.14. For a topological space (X,τ), the following properties are equivalent:
(a) (X,τ) is a $\mathcal{R}^0{}_{\widetilde{\alpha\delta}}$ space.
(b) If $x, y \in X$ then $y \in \alpha\delta_{Cl}(\{x\})$ if and only if every net in X $\alpha\delta$ -converging to y $\alpha\delta$ -converging to x .

Proof. $(a) \Rightarrow (b)$ Let $x, y \in X$ such that $y \in \alpha\delta_{Cl}(\{x\})$. Let $\{x_\alpha\}_{\alpha\in\Lambda}$ be a net in X such that $\{x_\alpha\}_{\alpha\in\Lambda}$ $\alpha\delta$ -converges to y . Since $y \in \alpha\delta_{Cl}(\{x\})$, by Theorem 3.6 we have $\alpha\delta_{Cl}(\{x\}) = \alpha\delta_{Cl}(\{y\})$. Therefore $x \in \alpha\delta_{Cl}(\{y\})$. This means that $\{x_\alpha\}_{\alpha\in\Lambda}$ $\alpha\delta$ -converges to x . Conversely, let $x, y \in X$ such that every net in X $\alpha\delta$ -converging to y $\alpha\delta$ -converges to x . Then $x \in \alpha\delta_{Cl}(\{y\})$. by Lemma 3.13. By Theorem 3.6, we have $\alpha\delta_{Cl}(\{x\}) = \alpha\delta_{Cl}(\{y\})$. Therefore $y \in \alpha\delta_{Cl}(\{x\})$.

$(b) \Rightarrow (a)$ Assume that x and y are any two points of X such that $\alpha\delta_{Cl}(\{x\}) \cap \alpha\delta_{Cl}(\{y\}) \neq \varphi$. Let $z \in \alpha\delta_{Cl}(\{x\}) \cap \alpha\delta_{Cl}(\{y\})$. So there exists a net $\{x_\alpha\}_{\alpha\in\Lambda}$ in $\alpha\delta_{Cl}(\{x\})$ such that $\{x_\alpha\}_{\alpha\in\Lambda}$ $\alpha\delta$ -converges to z . Since $z \in \alpha\delta_{Cl}(\{y\})$ then $\{x_\alpha\}_{\alpha\in\Lambda}$ $\alpha\delta$ -converges to y . It follows that $y \in \alpha\delta_{Cl}(\{x\})$. By the same token we obtain $x \in \alpha\delta_{Cl}(\{y\})$. Therefore $\alpha\delta_{Cl}(\{x\}) = \alpha\delta_{Cl}(\{y\})$ and by Theorem 3.6 (X, τ) is a $\mathcal{R}^0{}_{\widetilde{\alpha\delta}}$.

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