



Critical analysis of Bose–Einstein and Fermi–Dirac statistics

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ABSTRACT

The modern critical analysis of Bose–Einstein and Fermi–Dirac statistics is proposed. Methodological basis of analysis is the unity of formal logic and rational dialectics. It is shown that Bose–Einstein and Fermi–Dirac statistics – as consequence of Bose's erroneous method – contain logical errors. The main error is that Bose–Einstein and Fermi–Dirac distribution functions contain chemical potential. It is proved that chemical potential is identically equal to zero because the concept of chemical potential is contrary to the concept of temperature. There exist the only one correct statistics in statistical physics: Gibbs statistics.

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Introduction

Rationalization of thought and science is the dialectical imperative of our time. Rationalization of science cannot be achieved without valid criticism of the theories created by the classics of science. As is recently shown for the first time in the work [1], the critical analysis of the generally accepted foundations of theoretical physics (i.e. Newtonian mechanics, Maxwell electrodynamics, thermodynamics, statistical mechanics and physical kinetics, the theory of relativity, quantum mechanics) results in the following statements. Theoretical physics of the 20th century is not essence science (theory) but phenomenon science (theory). It means that theoretical physics is an unwieldy science (since its foundations were created with the inductive method); having primitive non-universal foundations; not having a clear purpose; containing a set of delusions, logic errors, and vagueness (vagueness often cannot even be realized and formulated in the standard physical concepts since physics does not contain many universal concepts; furthermore, vagueness often results from the “thoughtless application of mathematics” (L. Boltzmann)). Therefore, physical theories and fields of physics defy both natural unification and development. In the light of this statements, Bose–Einstein (B–E) and Fermi–Dirac (F–D) statistics – as the important part of modern physics – are not exceptions because they do not represent a consequence of the probability theory. In order to correctly analyze scientific works one should use universal criterion of truth. This criterion is formulated as follows: a physical theory must not contradict the system of the universal (i.e. general-scientific) concepts and laws. The system of the universal concepts and laws represents the unity of formal logic and dialectics since this unity is a science of most general laws of development of the Nature, human society, and thought. This unity is correct methodological basis of science. The purpose of the present work is to proposed critical analysis Bose's work [2], Bose–Einstein (B–E) and Fermi–Dirac (F–D) statistics (as corollary of Bose's work) within the framework of formal logic and probability theory.

1. Analysis

As is known, in 1924 Bose proposed the method of derivation of Planck formula. The method is characterized by the peculiarity that the quantum-statistical description of heat radiation is reached without consideration of interaction between radiation and substance: “the hypothesis of light quanta in a combination with statistical mechanics (in the form in what it has been adapted by Planck for needs of the quantum theory) is sufficient ground for derivation of the law independently of the classical theory” (S. Bose). Bose's method can be interpreted as follows. One considers the isolated gas of photons putted in volume. The phase space of one photon is divided into S -layers (i.e. “elementary regions of energy” – the monochromatic layers defined by the relationship $|E_m - E_n|$ where E_m and E_n are values of energy of the molecule emitting and absorbing photons), $0 \leq S < \infty$. And each S -layer is divided into phase space cells – states of photon gas. One postulates that:

- existence of phase space cells (states of photon gas) of the layer $|E_m - E_n|$ does not depend on existence of photon gas;
- “empty phase space cell” (i.e. “vacuum state of photon gas”) not containing any photon can exist;
- energy of monochromatic photon gas is a discrete random quantity;
- energy state of photon gas in a cell is characterized by quantum number – number of monochromatic photons (i.e. there is “secondary quantization” of energy of gas);
- this quantum number takes values from 0 up to ∞ ;
- the space of permissible states of photon contains the “empty phase space cell”.

Quantum-statistical task is to find dependence of probability of state on energy of monochromatic photon gas under condition of conservation of full energy of photon gas. This task is solved with the help combinatory method and Stirling's formula. The found solution represents Gibbs quantum canonical distribution. As a result of calculation of average energy of the monochromatic photon gas in cell, one obtains Planck function, i.e. “Bose's distribution”. Product of average energy of the monochromatic photon gas in cell and number of cells in S -

layer gives Planck's formula. Einstein characterized Bose's work as follows: "Bose's derivation is elegant but its essence remains foggy". In my opinion, the essence of this method is foggy because Bose's reasoning contains logical errors. The main logic errors are as follows [3]:

1. One considers the isolated gas of photons. In this case, energy of monochromatic photon gas cannot be random quantity.
2. Quantum-statistical description of heat radiation is made without taking into consideration the probability of quantum states of the molecule emitting and absorbing radiation. Therefore, the parameter T of Bose's distribution is treated as temperature of photon gas.
3. The method is based on concept "empty phase space cell". In accordance with definition, the phase space (set of phase space cells) of physical object (for example, photon) represents set of available (accessible, permissible) states of this object. Since the physical object (photon) cannot be in unavailable (inaccessible, impermissible) state, this state represents the "empty phase space cell". If the "phase space of one photon" is interpreted as space of elementary events at the quantum-statistical description of photon gas, the concept of probability that photon gas is in the "empty phase space cell" loses sense.
4. Bose included "empty phase space cell" (i.e. inaccessible, impermissible state) in space of elementary events (i.e. set of accessible, permissible states). Obviously, such inclusion is equivalent to replacement of the isolated system "photon gas" by the isolated system "molecule + molecular gas + photon gas" since cells (states) of S -layer are born by molecule and are defined by the relation $|E_m - E_n|$. In this case, the concept of probability that photon gas is in the "empty phase space cell" (i.e. there are no photons in the state $|E_m - E_n|$) has sense. Owing to it, Bose's logical error was happy: Bose derived Planck formula.

Thus, Bose's reasoning concerning the theory of photon gas are open to objections because these reasoning contain formal-logical errors. Planck, Einstein, Fermi, Dirac, and others could not comprehend these errors. Till now, these errors exist in the modern theoretical physics and are manifested in concepts "Bose-Einstein statistics", "Fermi-Dirac statistics", "Dirac physical vacuum" [4-7], etc.

The formal-logical analysis of Bose-Einstein and Fermi-Dirac statistics – as consequence of Bose's method – leads to the following main results [4, 5]. Firstly, in accordance with the definition, Bose-Einstein (B-E) and Fermi-Dirac (F-D) distribution functions $f_{(B-E)}^s$, $f_{(F-D)}^s$ are the average values of the random quantity:

$$f^s \equiv \varepsilon^s / \varepsilon_1^s, \quad \varepsilon^s \equiv \sum_r \varepsilon_r^s p_r^s,$$

$$p_r^s = p_0^s \exp [-(\alpha + \beta \varepsilon_1^s) r],$$

where

$$r = 0, 1, K \quad (\text{B-E}),$$

$$r = 0, 1 \quad (\text{F-D}),$$

f^s is the average number of the noninteracting monoenergetic identical quantum particles in the S -layer cell; ε_1^s is energy of one particle of kind S ; p_r^s is the generalized Gibbs quantum canonical distribution, i.e. is the probability that energy takes on the value

$$\varepsilon_r^s = \varepsilon_1^s r \equiv (\alpha + \beta \varepsilon_1^s) r / \beta;$$

$1/\beta \equiv T$ is temperature; $\alpha \equiv -\beta\mu$ is degeneration parameter; μ is chemical potential. Secondly, in accordance

with the logic law of identity, $p_r^s \equiv p_r^s$,

$\varepsilon_r^s = \varepsilon_1^s r \equiv (\alpha + \beta \varepsilon_1^s) r / \beta$. Then $\alpha \equiv 0$. Consequently, $\mu \equiv 0$.

Thus, Bose-Einstein and Fermi-Dirac statistics represent logical errors.

Discussion

If the principle of the unity of formal logic and of rational dialectics is a correct methodological basis of science, then the concept of random quantity must be a starting-point of any physical-statistical theory [3-8]. In this case, distribution of probabilities gives correct and complete physical-statistical description of the physical system. "The insufficient understanding of this circumstance is a root of those difficulties which one should overcome now" (A. Einstein). As is known, in the case of statistical theory of heat phenomena, energy is a random quantity. However, Planck's, Einstein's, Bose's, and Fermi's works on the theory quantum gas and Boltzmann distribution (used by Planck, Einstein, and Bose) are not in accordance with this argument [3-8].

Many years later, Bose recollected: "I did not imagine that I did something new. I not so understood statistics to understand how much my approach differed from the approach which Boltzmann could have proposed on the basis of his statistics. Instead of imagining light quanta in the form of particles, I spoke about these states" [50]. Einstein characterized Bose's work as follows: "Bose's derivation is elegant but its essence remains foggy". Bose's idea and method rendered essential influence on Einstein's, Fermi's, and Dirac's works: "The derivation of Planck formula, proposed by Bose, is a great achievement. The method used by him gives also the quantum theory of ideal gas... since light quantum in essence differs from one-atom molecule only in the respect that the rest mass of quantum is vanishing small. The analogy between gas of quanta and gas of molecules should be full" (A. Einstein). This analogy and "foggy essence of Bose's method" resulted in the erroneous theories of molecular quantum gas, "Bose-Einstein statistics", "Fermi-Dirac statistics", and the erroneous concepts "chemical potential", "secondary quantization", "physical vacuum" [3-8].

Conclusion

Thus, the modern critical analysis of the generally accepted foundations of theory of quantum gas (Bose-Einstein and Fermi-Dirac statistics) leads to the following main statements:

1. Planck's, Einstein's, Bose's, and Fermi's works on the theory of quantum gas contain logical errors.
2. The concepts "Bose-Einstein statistics" and "Fermi-Dirac statistics" represent logical errors because they contain the erroneous concept of chemical potential.
3. Chemical potential is identically equal to zero.
4. Quantum gas obeys "Gibbs statistics": equilibrium quantum gas is described by Gibbs quantum canonical distribution which does not contain chemical potential. There exist the only one correct statistics in statistical physics: Gibbs statistics.

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