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A study on intuitionistic fuzzy subsemiring of a semiring

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ABSTRACT

In this paper, we made an attempt to study the algebraic nature of intuitionistic fuzzy subsemiring of a semiring.

Introduction

There are many concepts of universal algebras generalizing an associative ring (*R; + ; .*). Some of them in particular, nearrings and several kinds of semirings have been proven very useful. An algebra (R ; +, .) is said to be a semiring if (R ; +) and (R ; .) are semigroups satisfying a. (b+c) = a. b+a. c and (b+c) .a = b. a+c. a for all a, b and c in R. A semiring R is said to be additively commutative if a+b = b+a for all a, b in R. A semiring R may have an identity 1, defined by 1. a = a = a. 1 and a zero 0, defined by 0+a = a = a+0 and a.0 = 0 = 0.a for all a in R. After the introduction of fuzzy sets by L.A.Zadeh[22], several researchers explored on the generalization of the concept of fuzzy sets. The concept of intuitionistic fuzzy subset was introduced by K.T.Atanassov[1,2], as a generalization of the notion of fuzzy set. The notion of fuzzy subnearrings and ideals was introduced by S.Abou Zaid[18]. In this paper, we introduce the some theorems in intuitionistic fuzzy subsemiring of a semiring.

1. Preliminaries:

1.1 Definition: Let X be a non–empty set. A fuzzy subset A of X is a function A: X → [0, 1].

1.2 Definition: Let R be a semiring. A fuzzy subset A of R is said to be a fuzzy subsemiring (FSSR) of R if it satisfies the following conditions:

(ii) $\mu_A(x + y) \geq \min\{\mu_A(x), \mu_A(y)\}$,

(i) $\mu_A(xy) \geq \min\{ \mu_A(x), \mu_A(y) \}$, for all x and y in R.

1.3 Definition: Let R be a semiring. A fuzzy subset A of R is said to be an anti-fuzzy subsemiring (AFSSR) of R if it satisfies the following conditions:

(i) $\mu_A(x + y) \leq \max\{ \mu_A(x), \mu_A(y) \}$,

(ii) $\mu_A(xy) \leq \max\{ \mu_A(x), \mu_A(y) \}$, for all x and y in R.

1.4 Definition [5]: An intuitionistic fuzzy subset (IFS) A in X is defined as an object of the form $A =\{\langle x, \mu_A(x), \nu_A(x)\rangle / x\in X\}$, where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x\in X$ respectively and for every $x \in X$ satisfying $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

1.1 Example: Let X= {a, b, c} be a set. Then A= {⟨a, 0.52, 0.34⟩, ⟨ b, 0.14, 0.71⟩, ⟨ c, 0.25, 0. 34⟩ } is an intuitionistic fuzzy subset of X.

1.5 Definition: If A is a intuitionistic fuzzy subset of X , then the complement of A, denoted A^c is the intuitionistic fuzzy set of X, given by $A^c(x) = \{< x, \nu_A(x) , \mu_A(x)> / x \in X \}$, for all $x \in X$.

1.2 Example: Let A ={ ⟨ a, 0.7 , 0.1 ⟩, ⟨ b, 0.6 , 0.2 ⟩, ⟨ c, 0.2 , 0.3⟩ } is a fuzzy subset of X ={a, b, c}. The complement of A is A^c ={⟨ a, 0.1 , 0.7⟩,⟨ b, 0.2 , 0.6 ⟩, ⟨ c, 0.3 , 0.2 ⟩ }.

1.6 Definition: Let A and B be any two intuitionistic fuzzy subsets of a set X. We define the following operations:

(i) $A \cap B = \{ \langle x, \min\{ \mu_A(x) , \mu_B(x)\} , \max\{ \nu_A(x) , \nu_B(x)\} \rangle \}$, for all $x \in X$.

(ii) $A \cup B = \{ \langle x, \max\{ \mu_A(x) , \mu_B(x)\} , \min\{ \nu_A(x) , \nu_B(x)\} \rangle \}$, for all $x \in X$.

(iii) $\Box A = \{ \langle x, \mu_A(x) , 1 - \mu_A(x) \rangle / x \in X \}$, for all x in X.

(iv) $\Diamond A = \{ \langle x, 1 - \nu_A(x) , \nu_A(x) \rangle / x \in X \}$, for all x in X.

1.7 Definition: Let R be a semiring. An intuitionistic fuzzy subset A of R is said to be an intuitionistic fuzzy subsemiring (IFSSR) of R if it satisfies the following conditions:

(i) $\mu_A(x + y) \geq \min \{\mu_A(x), \mu_A(y)\}$,

(ii) $\mu_A(xy) \geq \min\{ \mu_A(x), \mu_A(y) \}$,

(iii) $\nu_A(x + y) \leq \max \{ \nu_A(x), \nu_A(y) \}$,

(iv) $\nu_A(xy) \leq \max\{ \nu_A(x), \nu_A(y) \}$, for all x and y in R.

1.8 Definition: Let A and B be intuitionistic fuzzy subsets of sets G and H, respectively. The product of A and B, denoted by AxB, is defined as $AxB = \{ \langle (x, y), \mu_{AxB}(x,y), \nu_{AxB}(x,y) \rangle$ / for all x in G and y in H }, where $\mu_{AxB}(x, y) = \min\{ \mu_A(x), \mu_B(y) \}$ and $\nu_{AxB}(x, y) = \max\{ \nu_A(x), \nu_B(y) \}$.

1.9 Definition: Let A be an intuitionistic fuzzy subset in a set S, the strongest intuitionistic fuzzy relation on S, that is a

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intuitionistic fuzzy relation on A is V given by $\mu_V(x, y) = \min\{ \mu_A(x), \mu_A(y) \}$ and $\nu_V(x, y) = \max\{ \nu_A(x), \nu_A(y) \}$, for all x and y in S.

1.10 Definition: Let (R, +, ·) and (R^l, +, ·) be any two semirings. Let $f : R \to R^l$ be any function and A be an intuitionistic fuzzy subsemiring in R, V be an intuitionistic fuzzy subsemiring in $f(R) = R^l$, defined by $\mu_V(y) = \sup_{x \in f^{-1}(y)} \mu_A(x)$ and $\nu_V(y) = \inf_{x \in f^{-1}(y)} \nu_A(x)$, for all x in R and y in R^l. Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

1.11 Definition: Let A be an intuitionistic fuzzy subsemiring of a semiring (R, +, ·) and a in R. Then the pseudo intuitionistic fuzzy coset $(aA)^p$ is defined by $((a\mu_A)^p)(x) = p(a)\mu_A(x)$ and $((a\nu_A)^p)(x) = p(a)\nu_A(x)$, for every x in R and for some p in P.

2. Properties of intuitionistic fuzzy subsemiring of a semiring r

2.1 Theorem: Intersection of any two intuitionistic fuzzy subsemiring of a semiring R is a intuitionistic fuzzy subsemiring of R.

Proof: Let A and B be any two intuitionistic fuzzy subsemirings of a semiring R and x and y in R. Let $A=(x,\mu_A(x),\nu_A(x))/ x\in R\}$ and $B=\{(x, \mu_B(x), \nu_B(x)) / x\in R \}$ and also le $C= A \cap B = \{ (x, \mu_C(x) ,\nu_C(x)) / x\in R\}$,where $\min\{\mu_A(x),\mu_B(x)\}=\mu_C(x)$ and $\max \{ \nu_A(x) , \nu_B(x) \} = \nu_C(x)$ .Now, $\mu_C (x + y) = \min \{ \mu_A(x+y) , \mu_B(x+y) \}\geq \min \{ \min \{ \mu_A(x , \mu_A(y) \} , \min \{ \mu_B(x) , \mu_B(y) \} \} = \min \{ \min \{ \mu_A(x) , \mu_B(x) \}, \min\{\mu_A(y),\mu_B(y)\}\} = \min \{ \mu_C(x), \mu_C(y)\}$. Therefore, $\mu_C(x +y) \geq \min \{ \mu_C(x), \mu_C(y) \}$, for all x and y in R. And, $\mu_C(xy)=\min\{\mu_A(xy),\mu_B(xy)\}\geq\min\{ \min\{ \mu_A(x), \mu_A(y) \}, \min \{ \mu_B(x), \mu_B(y)\} = \min\{ \min \{ \mu_A(x), \mu_B(x) \}, \min \{ \mu_A(y), \mu_B(y)\}\}= \min\{ \mu_C(x), \mu_C(y)\}$. Therefore, $\mu_C(xy) \geq \min\{ \mu_C(x), \mu_C(y)\}$, for all x and y in R. Now, $\nu_C(x + y) = \max \{ \nu_A(x + y), \nu_B(x + y) \} \leq \max\{ \max \{ \nu_A(x), \nu_A(y) \}, \max \{ \nu_B(x), \nu_B(y)\}\} = \max\{ \max\{ \nu_A(x), \nu_B(x) \}, \max \{ \nu_A(y), \nu_B(y)\}\} = \max \{ \nu_C(x), \nu_C(y)\}$. Therefore, $\nu_C(x + y) \leq \max\{\nu_C(x), \nu_C(y)\}$, for all x and y in R. And, $\nu_C(xy) = \max \{ \nu_A(xy), \nu_B(xy)\}\leq \max\{\max\{\nu_A(x), \nu_A(y) \}, \max \{ \nu_B(x), \nu_B(y)\}\}= \max\{ \max \{ \nu_A(x), \nu_B(x) \}, \max \{ \nu_A(y), \nu_B(y)\}\} = \max\{ \nu_C(x), \nu_C(y) \}$.Therefore, $\nu_C(xy) \leq \max\{ \nu_C(x), \nu_C(y) \}$, for all x and y in R. Therefore C is an intuitionistic fuzzy subsemiring of R. Hence the intersection of any two intuitionistic fuzzy subsemirings of a semiring R is an intuitionistic fuzzy subsemiring of R.

2.2 Theorem: The intersection of a family of intuitionistic fuzzy subsemirings of semiring R is an intuitionistic fuzzy subsemiring of R.

Proof: Let $\{ V_i : i\in I\}$ be a family of intuitionistic fuzzy subsemirings of a semiring R and let $A=\bigcap_{i\in I} V_i$. Let x and y in R. Then, $\mu (x + y) = \inf_{i\in I} \mu_{Vi} (x + y) \geq \inf_{i\in I} \min\{\mu_{Vi}(x), \mu_{Vi}(y) \}= \min \{ \inf_{i\in I} \mu_{Vi}(x), \inf_{i\in I} \mu_{Vi}(y) \}= \min \{ \mu_A(x), \mu_A(y) \}$. Therefore, $\mu_A(x + y) \geq \min\{\mu_A(x),\mu_A(y)\}$, for all x and y in R. And, $\mu_A(xy) = \inf_{i\in I} \mu_{Vi}(xy) \geq \inf_{i\in I} \min\{ \mu_{Vi}(x) , \mu_{Vi}(y) \} = \min\{ \inf_{i\in I} \mu_{Vi}(x), \inf_{i\in I} \mu_{Vi}(y)\} = \min\{ \mu_A(x), \mu_A(y)\}$. Therefore, $\mu_A(xy) \geq \min\{ \mu_A(x), \mu_A(y) \}$, for all x and y in R. Now, $\nu_A(x + y) = \sup_{i\in I} \nu_{Vi}(x + y) \leq \sup_{i\in I} \max \{ \nu_{Vi}(x), \nu_{Vi}(y) \}= \max \{ \sup_{i\in I} \nu_{Vi}(x), \sup_{i\in I} \nu_{Vi}(y) \}= \max\{ \nu_A(x), \nu_A(y) \}$. Therefore, $\nu_A(x + y) \leq \max\{ \nu_A(x), \nu_A(y) \}$, for all x and y in R. And, $\nu_A(xy) = \sup_{i\in I} \nu_{Vi}(xy) \leq \sup_{i\in I} \max\{ \nu_{Vi}(x), \nu_{Vi}(y) \}= \max\{ \sup_{i\in I} \nu_{Vi}(x), \sup_{i\in I} \nu_{Vi}(y) \}= \max\{ \nu_A(x), \nu_A(y) \}$. Therefore, $\nu_A(xy) \leq \max\{ \nu_A(x), \nu_A(y) \}$, for all x and y in R. That is, A is an intuitionistic fuzzy subsemiring of a semiring R. Hence, the intersection of a family of intuitionistic fuzzy subsemirings of R is an intuitionistic fuzzy subsemiring of R.

2.3 Theorem: If A and B are any two intuitionistic fuzzy subsemirings of the semirings R_1 and R_2 respectively, then AxB is an intuitionistic fuzzy subsemiring of $R_1 x R_2$.

Proof: Let A and B be two intuitionistic fuzzy subsemirings of the semirings R_1 and R_2 respectively. Let x_1 and x_2 be in R_1, y_1 and y_2 be in R_2. Then (x_1, y_1) and (x_2, y_2) are in $R_1 x R_2$. Now, $\mu_{AxB} [(x_1,y_1) + (x_2, y_2)]=\mu_{AxB} (x_1 + x_2, y_1+ y_2) = \min \{ \mu_A(x_1+ x_2), \mu_B(y_1+ y_2) \}\geq \min\{ \min\{ \mu_A(x_1), \mu_A(x_2) \}, \min\{\mu_B(y_1), \mu_B(y_2)\}\}= \min\{ \min \{ \mu_A(x_1), \mu_B(y_1) \}, \min \{ \mu_A(x_2), \mu_B(y_2) \}\}= \min\{ \mu_{AxB} (x_1, y_1), \mu_{AxB} (x_2, y_2) \}$. Therefore, $\mu_{AxB} [(x_1, y_1) + (x_2, y_2)] \geq \min \{ \mu_{AxB} (x_1, y_1), \mu_{AxB} (x_2, y_2) \}$.Also, $\mu_{AxB} [(x_1, y_1)(x_2, y_2)] = \mu_{AxB} (x_1x_2, y_1y_2) = \min \{ \mu_A(x_1x_2), \mu_B(y_1y_2) \}\geq \min \{ \min \{ \mu_A(x_1), \mu_A(x_2) \}, \min\{ \mu_B(y_1), \mu_B(y_2) \}\}= \min\{ \min \{ \mu_A(x_1), \mu_B(y_1) \}, \min\{\mu_A(x_2), \mu_B(y_2) \}\}= \min\{\mu_{AxB} (x_1, y_1), \mu_{AxB} (x_2, y_2) \}$.Therefore, $\mu_{AxB} [(x_1, y_1)(x_2, y_2)] \geq \min\{\mu_{AxB} (x_1, y_1), \mu_{AxB} (x_2, y_2)\}$. Now, $\nu_{AxB} [(x_1, y_1)+(x_2, y_2)]=\nu_{AxB} (x_1 + x_2, y_1+ y_2) = \max\{\nu_A(x_1+x_2), \nu_B(y_1+ y_2)\}\leq \max\{ \max\{ \nu_A(x_1), \nu_A(x_2) \}, \max\{\nu_B(y_1), \nu_B(y_2) \}\}= \max\{ \max\{\nu_A(x_1), \nu_B(y_1)\}, \max\{ \nu_A(x_2), \nu_B(y_2) \}\}= \max\{ \nu_{AxB} (x_1, y_1), \nu_{AxB} (x_2, y_2) \}$. Therefore, $\nu_{AxB} [(x_1, y_1) + (x_2, y_2)] \leq \max\{ \nu_{AxB} (x_1, y_1), \nu_{AxB} (x_2, y_2) \}$. Also, $\nu_{AxB} [(x_1, y_1)(x_2, y_2)] = \nu_{AxB} (x_1x_2, y_1y_2)= \max \{ \nu_A(x_1x_2), \nu_B(y_1y_2) \}\leq \max \{ \max \{\nu_A(x_1),\nu_A(x_2)\},\max\{\nu_B(y_1),\nu_B(y_2)\}\}=\max\{\max\{\nu_A(x_1),\nu_B(y_1)\}, \max\{\nu_A(x_2), \nu_B(y_2)\}= \max\{\nu_{AxB}(x_1,y_1),\nu_{AxB} (x_2, y_2)\}$. Therefore, $\nu_{AxB} [(x_1,y_1)(x_2,y_2)]\leq\max\{\nu_{AxB} (x_1, y_1), \nu_{AxB} (x_2, y_2) \}$. Hence AxB is an intuitionistic fuzzy subsemiring of semiring of $R_1 x R_2$.

2.4 Theorem: Let A be an intuitionistic fuzzy subset of a semiring R and V be the strongest intuitionistic fuzzy relation of R. Then A is an intuitionistic fuzzy subsemiring of R if and only if V is an intuitionistic fuzzy subsemiring of RxR.

Proof: Suppose that A is an intuitionistic fuzzy subsemiring of a semiring R. Then for any $x=(x_1,x_2)$ and $y=(y_1, y_2)$ are in RxR. We have, $\mu_V (x+y) = \mu_V [(x_1, x_2) + (y_1, y_2)] = \mu_V(x_1+ y_1 , x_2 + y_2) = \min \{ \mu_A(x_1 + y_1), \mu_A(x_2+ y_2)\}\geq \min \{ \min \{ \mu_A(x_1), \mu_A(y_1) \}, \min \{ \mu_A(x_2), \mu_A(y_2) \}\} = \min \{ \min \{ \mu_A(x_1), \mu_A(x_2) \}, \min \{\mu_A(y_1), \mu_A(y_2)\}\} = \min \{ \mu_V (x_1, x_2), \mu_V (y_1, y_2) \}= \min \{ \mu_V (x), \mu_V (y) \}$. Therefore, $\mu_V (x + y) \geq \min \{ \mu_V (x), \mu_V (y) \}$, for all x and y in RxR. And, $\mu_V (xy) = \mu_V [(x_1, x_2) (y_1, y_2)] = \mu_V(x_1y_1 , x_2y_2) = \min \{ \mu_A(x_1y_1), \mu_A(x_2y_2) \}\geq \min \{ \min\{ \mu_A(x_1), \mu_A(y_1) \}, \min\{ \mu_A(x_2), \mu_A(y_2) \}\}= \min\{ \min \{ \mu_A(x_1), \mu_A(x_2) \}, \min\{ \mu_A(y_1), \mu_A(y_2) \} \} = \min\{\mu_V(x_1,x_2),\mu_V(y_1,y_2)\}= \min\{\mu_V (x),\mu_V(y)\}$.Therefore, $\mu_V(xy)\geq \min\{\mu_V (x), \mu_V(y) \}$, for all x and y in RxR.We have, $\nu_V (x+ y) = \nu_V [(x_1, x_2)+(y_1, y_2)]= \nu_V(x_1+ y_1 ,x_2+ y_2)= \max \{\nu_A(x_1+ y_1),\nu_A(x_2+ y_2)\}\leq \max \{\max \{\nu_A(x_1), \nu_A(y_1) \}, \max \{ \nu_A(x_2), \nu_A(y_2) \} \}= \max\{\max \{ \nu_A(x_1), \nu_A(x_2)\}, \max \{ \nu_A(y_1), \nu_A(y_2) \} \}= \max\{ \nu_V (x_1, x_2), \nu_V (y_1, y_2) \}= \max\{ \nu_V (x), \nu_V (y)\}$.Therefore, $\nu_V (x + y) \leq \max\{ \nu_V (x), \nu_V (y) \}$, for all x and y in RxR.And, $\nu_V(xy) = \nu_V[(x_1, x_2)(y_1, y_2)] = \nu_V(x_1y_1, x_2y_2) = \max\{\nu_A(x_1y_1), \nu_A(x_2y_2)\} \le \max\{\max\{\nu_A(x_1), \nu_A(y_1)\}, \max\{\nu_A(x_2), \nu_A(y_2)\}\} = \max\{\max\{\nu_A(x_1), \nu_A(x_2)\}, \max\{\nu_A(y_1), \nu_A(y_2)\}\} = \max\{\nu_V(x_1, x_2), \nu_V(y_1, y_2)\} = \max\{\nu_V(x), \nu_V(y)\}$. Therefore, $\nu_V(xy) \le \max\{\nu_V(x), \nu_V(y)\}$, for all x and y in RxR. This proves that V is an intuitionistic fuzzy subsemiring of RxR.

Conversely assume that V is an intuitionistic fuzzy subsemiring of RxR, then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in RxR, we have $\min\{\mu_A(x_1 + y_1), \mu_A(x_2 + y_2)\} = \mu_V(x_1 + y_1, x_2 + y_2) = \mu_V[(x_1, x_2) + (y_1, y_2)] = \mu_V(x + y) \ge \min\{\mu_V(x), \mu_V(y)\} = \min\{\mu_V(x_1, x_2), \mu_V(y_1, y_2)\} = \min\{\min\{\mu_A(x_1), \mu_A(x_2)\}, \min\{\mu_A(y_1), \mu_A(y_2)\}\}$. If $\mu_A(x_1 + y_1) \le \mu_A(x_2 + y_2)$, $\mu_A(x_1) \le \mu_A(x_2)$, $\mu_A(y_1) \le \mu_A(y_2)$, we get, $\mu_A(x_1 + y_1) \ge \min\{\mu_A(x_1), \mu_A(y_1)\}$, for all x_1 and y_1 in R. And, $\min\{\mu_A(x_1y_1), \mu_A(x_2y_2)\} = \mu_V(x_1y_1, x_2y_2) = \mu_V[(x_1, x_2)(y_1, y_2)] = \mu_V(x y) \ge \min\{\mu_V(x), \mu_V(y)\} = \min\{\mu_V(x_1, x_2), \mu_V(y_1, y_2)\} = \min\{\min\{\mu_A(x_1), \mu_A(x_2)\}, \min\{\mu_A(y_1), \mu_A(y_2)\}\}$. If $\mu_A(x_1y_1) \le \mu_A(x_2y_2)$, $\mu_A(x_1) \le \mu_A(x_2)$, $\mu_A(y_1) \le \mu_A(y_2)$, we get $\mu_A(x_1y_1) \ge \min\{\mu_A(x_1), \mu_A(y_1)\}$, for all x_1 and y_1 in R. We have $\max\{\nu_A(x_1 + y_1), \nu_A(x_2 + y_2)\} = \nu_V(x_1 + y_1, x_2 + y_2) = \nu_V[(x_1, x_2) + (y_1, y_2)] = \nu_V(x + y) \le \max\{\nu_V(x), \nu_V(y)\} = \max\{\nu_V(x_1, x_2), \nu_V(y_1, y_2)\} = \max\{\max\{\nu_A(x_1), \nu_A(x_2)\}, \max\{\nu_A(y_1), \nu_A(y_2)\}\}$. If $\nu_A(x_1 + y_1) \ge \nu_A(x_2 + y_2)$, $\nu_A(x_1) \ge \nu_A(x_2)$, $\nu_A(y_1) \ge \nu_A(y_2)$, we get, $\nu_A(x_1 + y_1) \le \max\{\nu_A(x_1), \nu_A(y_1)\}$, for all x_1 and y_1 in R. And, $\max\{\nu_A(x_1y_1), \nu_A(x_2y_2)\} = \nu_V(x_1y_1, x_2y_2) = \nu_V[(x_1, x_2)(y_1, y_2)] = \nu_V(xy) \le \max\{\nu_V(x), \nu_V(y)\} = \max\{\nu_V(x_1, x_2), \nu_V(y_1, y_2)\} = \max\{\max\{\nu_A(x_1), \nu_A(x_2)\}, \max\{\nu_A(y_1), \nu_A(y_2)\}\}$. If $\nu_A(x_1y_1) \ge \nu_A(x_2y_2)$, $\nu_A(x_1) \ge \nu_A(x_2)$, $\nu_A(y_1) \ge \nu_A(y_2)$, we get $\nu_A(x_1y_1) \le \max\{\nu_A(x_1), \nu_A(y_1)\}$, for all x_1 and y_1 in R.
Therefore A is an intuitionistic fuzzy subsemiring of R.

2.5 Theorem: If A is an intuitionistic fuzzy subsemiring of a semiring $(R, +, \cdot)$, then $H = \{x / x \in R: \mu_A(x) = 1, \nu_A(x) = 0\}$ is either empty or is a subsemiring of R.

Proof: If no element satisfies this condition, then H is empty. If x and y in H, then $\mu_A(x + y) \ge \min\{\mu_A(x), \mu_A(y)\} = \min\{1, 1\} = 1$. Therefore, $\mu_A(x + y) = 1$. And $\mu_A(xy) \ge \min\{\mu_A(x), \mu_A(y)\} = \min\{1, 1\} = 1$. Therefore, $\mu_A(xy) = 1$. Now, $\nu_A(x+y) \le \max\{\nu_A(x), \nu_A(y)\} = \max\{0, 0\} = 0$. Therefore, $\nu_A(x + y) = 0$. And $\nu_A(xy) \le \max\{\nu_A(x), \nu_A(y)\} = \max\{0, 0\} = 0$. Therefore, $\nu_A(xy) = 0$. We get x+y , xy in H. Therefore, H is a subsemiring of R. Hence H is either empty or is a subsemiring of R.

2.6 Theorem: If A be an intuitionistic fuzzy subsemiring of a semiring $(R, +, \cdot)$, then

(i) if $\mu_A(x+y) = 0$, then either $\mu_A(x) = 0$ or $\mu_A(y) = 0$, for all x and y in R.

(ii) if $\mu_A(x+ y) = 1$, then either $\mu_A(x) = 1$ or $\mu_A(y) = 1$, for all x and y in R.

Proof: Let x and y in R. (i) By the definition $\mu_A(x-y) \ge \min\{\mu_A(x), \mu_A(y)\}$,
which implies that $0 \ge \min\{\mu_A(x), \mu_A(y)\}$. Therefore, either $\mu_A(x) = 0$ or $\mu_A(y) = 0$.

(ii) By the definition $\mu_A(x+y) \le \max\{\mu_A(x), \mu_A(y)\}$, which implies that $1 \le \max\{\mu_A(x), \mu_A(y)\}$. Therefore, either $\mu_A(x) = 1$ or $\mu_A(y) = 1$.

2.7 Theorem: If A is an intuitionistic fuzzy subsemiring of a semiring $(R, +, \cdot)$, then $H = \{\langle x, \mu_A(x)\rangle : 0 < \mu_A(x) \le 1$ and $\nu_A(x) = 0\}$ is either empty or a fuzzy subsemiring of R.

Proof: If no element satisfies this condition, then H is empty. If x and y satisfies this condition, then $\nu_A(x+y) \le \max\{\nu_A(x), \nu_A(y)\} = \max\{0, 0\} = 0$. Therefore, $\nu_A(x+y) = 0$, for all x and y in R. And, $\nu_A(xy) \le \max\{\nu_A(x), \nu_A(y)\} = \max\{0, 0\} = 0$. Therefore, $\nu_A(xy) = 0$, for all x and y in R. And, $\mu_A(x+y) \ge \min\{\mu_A(x), \mu_A(y)\}$. Therefore, $\mu_A(x+y) \ge \min\{\mu_A(x), \mu_A(y)\}$, for all x and y in R. And, $\mu_A(xy) \ge \min\{\mu_A(x), \mu_A(y)\}$. Therefore, $\mu_A(xy) \ge \min\{\mu_A(x), \mu_A(y)\}$, for all x and y in R. Hence H is a fuzzy subsemiring of R. Therefore, H is either empty or a fuzzy subsemiring of R.

2.8 Theorem: If A is an intuitionistic fuzzy subsemiring of a semiring $(R, +, \cdot)$ then $H = \{\langle x, \mu_A(x)\rangle : 0 < \mu_A(x) \le 1\}$ is either empty or a fuzzy subsemiring of R.

Proof: If no element satisfies this condition, then H is empty. If x and y satisfies this condition, then $\mu_A(x+y) \ge \min\{\mu_A(x), \mu_A(y)\}$. Therefore, $\mu_A(x+y) \ge \min\{\mu_A(x), \mu_A(y)\}$, for all x and y in R. And $\mu_A(xy) \ge \min\{\mu_A(x), \mu_A(y)\}$. Therefore, $\mu_A(xy) \ge \min\{\mu_A(x), \mu_A(y)\}$, for all x and y in R. Therefore, H is either empty or a fuzzy subsemiring of R.

2.9 Theorem: If A is an intuitionistic fuzzy subsemiring of a semiring $(R, +, \cdot)$, then $H = \{\langle x, \nu_A(x)\rangle : 0 < \nu_A(x) \le 1\}$ is either empty or an anti-fuzzy subsemiring of R.

Proof: If no element satisfies this condition, then H is empty. If x and y satisfies this condition, then $\nu_A(x+y) \le \max\{\nu_A(x), \nu_A(y)\}$. Therefore, $\nu_A(x+y) \le \max\{\nu_A(x), \nu_A(y)\}$, for all x and y in R. And $\nu_A(xy) \le \max\{\nu_A(x), \nu_A(y)\}$. Therefore, $\nu_A(xy) \le \max\{\nu_A(x), \nu_A(y)\}$, for all x and y in R. Hence H is either empty or an anti-fuzzy subsemiring of R.

2.10 Theorem: If A is an intuitionistic fuzzy subsemiring of a semiring $(R, +, \cdot)$, then □A is an intuitionistic fuzzy subsemiring of R.

Proof: Let A be an intuitionistic fuzzy subsemiring of a semiring R. Consider $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle\}$, for all x in R, we take $□A = B = \{\langle x, \mu_B(x), \nu_B(x)\rangle\}$, where $\mu_B(x) = \mu_A(x)$, $\nu_B(x) = 1 - \mu_A(x)$. Clearly, $\mu_B(x+y) \ge \min\{\mu_B(x), \mu_B(y)\}$, for all x and y in R and $\mu_B(xy) \ge \min\{\mu_B(x), \mu_B(y)\}$, for all x and y in R. Since A is an intuitionistic fuzzy subsemiring of R, we have $\mu_A(x+y) \ge \min\{\mu_A(x), \mu_A(y)\}$, for all x and y in R, which implies that $1 - \nu_B(x+y) \ge \min\{(1 - \nu_B(x)), (1 - \nu_B(y))\}$, which implies that $\nu_B(x+y) \le 1 - \min\{(1 - \nu_B(x)), (1 - \nu_B(y))\} = \max\{\nu_B(x), \nu_B(y)\}$. Therefore, $\nu_B(x+y) \le \max\{\nu_B(x), \nu_B(y)\}$, for all x and y in R. And $\mu_A(xy) \ge \min\{\mu_A(x), \mu_A(y)\}$, for all x and y in R, which implies that $1 - \nu_B(xy) \ge \min\{(1 - \nu_B(x)), (1 - \nu_B(y))\}$ which implies that $\nu_B(xy) \le 1 - \min\{(1 - \nu_B(x)), (1 - \nu_B(y))\} = \max\{\nu_B(x), \nu_B(y)\}$. Therefore, $\nu_B(xy) \le \max\{\nu_B(x), \nu_B(y)\}$, for all x and y in R. Hence B = □A is an intuitionistic fuzzy subsemiring of a semiring R.

Remark: The converse of the above theorem is not true. It is shown by the following example:

Consider the semiring $Z_5 = \{0, 1, 2, 3, 4\}$ with addition modulo 5 and multiplication modulo 5 operations. Then $A = \{\langle 0, 0.7, 0.2\rangle, \langle 1, 0.5, 0.1\rangle, \langle 2, 0.5, 0.4\rangle, \langle 3, 0.5, 0.1\rangle, \langle 4, 0.5, 0.4\rangle\}$ is not an intuitionistic fuzzy subsemiring of Z_5, but $□A = \{\langle 0, 0.7, 0.3\rangle, \langle 1, 0.5, 0.5\rangle, \langle 2, 0.5, 0.5\rangle, \langle 3, 0.5, 0.5\rangle, \langle 4, 0.5, 0.5\rangle\}$ is an intuitionistic fuzzy subsemiring of Z_5.

2.11 Theorem: If A is an intuitionistic fuzzy subsemiring of a semiring $(R, +, \cdot)$, then ◊A is an intuitionistic fuzzy subsemiring of R.

Proof: Let A be an intuitionistic fuzzy subsemiring of a semiring R. That is $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle\}$, for all x in R. Let

$\Diamond A = B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle \}$, where $\mu_B(x) = 1-\nu_A(x)$, $\nu_B(x) = \nu_A(x)$. Clearly, $\nu_B(x+y) \le \max\{ \nu_B(x), \nu_B(y) \}$, for all x and y in R and $\nu_B(xy) \le \max\{ \nu_B(x), \nu_B(y) \}$, for all x and y in R. Since A is an intuitionistic fuzzy subsemiring of R, we have $\nu_A(x+y) \le \max\{ \nu_A(x), \nu_A(y) \}$, for all x and y in R, which implies that $1-\mu_B(x+y) \le \max \{ (1- \mu_B(x)) , (1- \mu_B(y)) \}$,which implies that $\mu_B(x+y) \ge 1- \max \{ (1- \mu_B(x)), (1- \mu_B(y)) \}= \min \{ \mu_B(x), \mu_B(y) \}$. Therefore, $\mu_B(x+y) \ge \min\{\mu_B(x),\mu_B(y)\}$, for all x and y in R. And $\nu_A(xy) \le \max \{ \nu_A(x), \nu_A(y) \}$, for all x and y in R, which implies that $1-\mu_B(xy) \le \max \{ (1-\mu_B(x)) , (1-\mu_B(y)) \}$, which implies that $\mu_B(xy) \ge 1- \max \{ (1-\mu_B(x)), (1-\mu_B(y)) \}= \min \{ \mu_B(x), \mu_B(y) \}$. Therefore, $\mu_B(xy) \ge \min \{ \mu_B(x), \mu_B(y) \}$, for all x and y in R. Hence $B = \Diamond A$ is an intuitionistic fuzzy subsemiring of a semiring R.

Remark: The converse of the above theorem is not true. It is shown by the following example:

Consider the semiring $Z_5 = \{ 0, 1, 2, 3, 4 \}$ with addition modulo 5 and multiplication modulo 5 operations. Then $A =\{\langle 0, 0.5, 0.1\rangle, \langle 1, 0.6, 0.4\rangle, \langle 2, 0.5, 0.4\rangle, \langle 3, 0.6, 0.4\rangle, \langle 4, 0.5, 0.4\rangle \}$ is not an intuitionistic fuzzy subsemiring of Z_5, but $\Diamond A =\{ \langle 0, 0.9, 0.1 \rangle, \langle 1, 0.6, 0.4\rangle, \langle 2, 0.6, 0.4\rangle, \langle 3, 0.6, 0.4\rangle, \langle 4, 0.6, 0.4\rangle \}$ is an intuitionistic fuzzy subsemiring of Z_5.

2.12 Theorem: Let (R, +, .) be a semiring and A be a non empty subset of R. Then A is a subsemiring of R if and only if $B = \langle \chi_A, \overline{\chi_A} \rangle$ is an intuitionistic fuzzy subsemiring of R, where χ_A is the characteristic function.

Proof: Let (R, +, .) be a semiring and A be a non empty subset of R. First let A be a subsemiring of R.Take x and y in R. Case (i): If x and y in A, then x+y, xy in A, since A is a subsemiring of R, $\chi_A(x) = \chi_A(y) = \chi_A(x+y) = \chi_A(xy) =1$ and $\overline{\chi_A}(x) = \overline{\chi_A}(y) = \overline{\chi_A}(x+y) = \overline{\chi_A}(xy) = 0$. So, $\chi_A(x+ y) \ge \min\{ \chi_A(x), \chi_A(y) \}$, for all x and y in R, $\chi_A(xy) \ge \min\{ \chi_A(x), \chi_A(y)\}$, for all x and y in R. So, $\overline{\chi_A}(x+y) \le \max\{ \overline{\chi_A}(x), \overline{\chi_A}(y) \}$, for all x and y in R, $\overline{\chi_A}(xy) \le \max \{ \overline{\chi_A}(x), \overline{\chi_A}(y) \}$, for all x and y in R.

Case (ii): If x in A, y not in A (or x not in A, y in A), then x+y, xy may or may not be in A, $\chi_A(x) =1$, $\chi_A(y) = 0$ (or $\chi_A(x) =0$, $\chi_A(y) =1$), $\chi_A(x+y) = \chi_A(xy) =1$ (or 0) and $\overline{\chi_A}(x) = 0$, $\overline{\chi_A}(y) =1$ (or $\overline{\chi_A}(x) =1$, $\overline{\chi_A}(y) =0$), $\overline{\chi_A}(x+y) = \overline{\chi_A}(xy)= 0$ (or 1). Clearly $\chi_A(x+y) \ge \min\{ \chi_A(x), \chi_A(y)\}$, for all x and y in R, $\chi_A(xy) \ge \min\{ \chi_A(x), \chi_A(y) \}$, for all x and y in R, and $\overline{\chi_A}(x + y) \le \max \{ \overline{\chi_A}(x), \overline{\chi_A}(y) \}$, for all x and y in R. $\overline{\chi_A}(xy) \le \max \{ \overline{\chi_A}(x), \overline{\chi_A}(y) \}$, for all x and y in R.

Case (iii): If x and y not in A, then x+y, xy may or may not be in A, $\chi_A(x)= \chi_A(y)= 0$, $\chi_A(x+y) = \chi_A(xy) =1$ or 0 and $\overline{\chi_A}(x) = \overline{\chi_A}(y) =1$, $\overline{\chi_A}(x+y) = \overline{\chi_A}(xy)= 0$ or 1. Clearly $\chi_A(x+ y) \ge \min \{ \chi_A(x), \chi_A(y) \}$, for all x and y in R and $\chi_A(xy) \ge \min \{ \chi_A(x), \chi_A(y) \}$, for all x and y in R, and $\overline{\chi_A}(x + y) \le \max \{ \overline{\chi_A}(x), \overline{\chi_A}(y) \}$, for all x and y in R $\overline{\chi_A}(xy) \le \max \{ \overline{\chi_A}(x), \overline{\chi_A}(y) \}$, for all x and y in R. So in all the three cases, we have B is an intuitionistic fuzzy subsemiring of a semiring R. Conversely, let x and y in A, since A is a non empty subset of R, so, $\chi_A(x)= \chi_A(y)= 1$, $\overline{\chi_A}(x) = \overline{\chi_A}(y) = 0$. Since $B= \langle \chi_A, \overline{\chi_A} \rangle$ is an intuitionistic fuzzy subsemiring of R, we have $\chi_A(x + y) \ge \min\{ \chi_A(x), \chi_A(y)\}=\min\{1,1\}= 1$, $\chi_A(xy) \ge \min\{ \chi_A(x), \chi_A(y) \}= \min\{1,1\}=1$. Therefore $\chi_A(x+y)= \chi_A(xy)=1$. And, $\overline{\chi_A}(x+ y) \le \max\{ \overline{\chi_A}(x), \overline{\chi_A}(y) \}= \max\{0, 0 \} = 0$, $\overline{\chi_A}(xy) \le \max \{ \overline{\chi_A}(x), \overline{\chi_A}(y) \} = \max \{ 0, 0 \} = 0$. Therefore $\overline{\chi_A}(x + y) = \overline{\chi_A}(xy) = 0$. Hence x + y and xy in A, so A is a subsemiring of R.

In the following Theorem ◦ is the composition operation of functions:

2.13 Theorem: Let A be an intuitionistic fuzzy subsemiring of a semiring H and f is an isomorphism from a semiring R onto H. Then $A\circ f$ is an intuitionistic fuzzy subsemiring of R.

Proof: Let x and y in R and A be an intuitionistic fuzzy subsemiring of a semiring H.

Then we have, $\mu_A\circ f)(x+y)=\mu_A(f(x+y))=\mu_A(f(x)+ f(y)) \ge \min \{ \mu_A(f(x)), \mu_A(f(y))\} \ge \min\{(\mu_A\circ f)(x),(\mu_A\circ f)(y)\}$, which implies that $(\mu_A\circ f)(x+y) \ge \min\{(\mu_A\circ f)(x),(\mu_A\circ f) (y) \}$. And $(\mu_A\circ f)(xy) = \mu_A(f(xy)) = \mu_A(f(x)f(y)) \ge \min \{ \mu_A(f(x)), \mu_A(f(y)) \} \ge \min\{(\mu_A\circ f)(x),(\mu_A\circ f)(y)\}$, which implies that $(\mu_A\circ f)(xy) \ge \min\{(\mu_A\circ f)(x), (\mu_A\circ f)(y) \}$. Then we have, $(\nu_A\circ f)(x+y)=\nu_A(f(x+y))=\nu_A(f(x)+f(y)) \le \max \{ \nu_A(f(x)), \nu_A(f(y)) \} \le \max\{(\nu_A\circ f)(x),(\nu_A\circ f)(y)\}$, which implies that $(\nu_A\circ f)(x+y) \le \max\{(\nu_A\circ f)(x), (\nu_A\circ f)(y) \}$. And$(\nu_A\circ f)(xy) = \nu_A(f(xy)) = \nu_A(f(x)f(y)) \le \max \{\nu_A(f(x)),\nu_A(f(y))\} \le \max \{(\nu_A\circ f)(x), (\nu_A\circ f)(y) \}$, which implies that $(\nu_A\circ f)(xy) \le \max \{ (\nu_A\circ f)(x), (\nu_A\circ f)(y) \}$. Therefore ($A\circ f$) is an intuitionistic fuzzy subsemiring of a semiring R.

2.14 Theorem: Let A be an intuitionistic fuzzy subsemiring of a semiring H and f is an anti-isomorphism from a semiring R onto H. Then $A\circ f$ is an intuitionistic fuzzy subsemiring of R.

Proof: Let x and y in R and A be an intuitionistic fuzzy subsemiring of a semiring H.

Then we have, $(\mu_A\circ f)(x+ y) = \mu_A(f(x+y)) = \mu_A(f(y)+f(x)) \ge \min \{ \mu_A(f(x)), \mu_A(f(y)) \} \ge \min \{ (\mu_A\circ f)(x), (\mu_A\circ f)(y) \}$, which implies that $(\mu_A\circ f)(x+y) \ge \min \{ (\mu_A\circ f)(x), (\mu_A\circ f)(y) \}$. $(\mu_A\circ f)(xy) = \mu_A(f(xy)) = \mu_A(f(y)f(x)) \ge \min\{\mu_A(f(x)), \mu_A(f(y))\} \ge \min \{(\mu_A\circ f)(x), (\mu_A\circ f)(y) \}$, which implies that $(\mu_A\circ f)(xy) \ge \min \{(\mu_A\circ f)(x),(\mu_A\circ f)(y)\}$. Then we have, $(\nu_A\circ f)(x+y) = \nu_A(f(x+y)) = \nu_A(f(y)+f(x)) \le \max \{ \nu_A(f(x)), \nu_A(f(y)) \} \le \max \{ (\nu_A\circ f)(x), (\nu_A\circ f)(y) \}$, which implies that $(\nu_A\circ f)(x+y) \le \max \{ (\nu_A\circ f)(x), (\nu_A\circ f)(y) \}$.$(\nu_A\circ f)(xy) = \nu_A(f(xy)) = \nu_A(f(y)f(x)) \le \max \{ \nu_A(f(x)), \nu_A(f(y)) \} \le \max \{ (\nu_A\circ f)(x), (\nu_A\circ f)(y) \}$, which implies that $(\nu_A\circ f)(xy) \le \max \{ (\nu_A\circ f) (x), (\nu_A\circ f) (y) \}$. Therefore $A\circ f$ is an intuitionistic fuzzy subsemiring of a semiring R.

2.15 Theorem: Let A be an intuitionistic fuzzy subsemiring of a semiring (R, +, .), then the pseudo intuitionistic fuzzy coset $(aA)^p$ is an intuitionistic fuzzy subsemiring of a semiring R, for every a in R.

Proof: Let A be an intuitionistic fuzzy subsemiring of a semiring R. For every x and y in R, we have, $((a\mu_A)^p)(x + y) = p(a)\mu_A(x+ y) \ge p(a) \min \{ (\mu_A(x), \mu_A(y)\}= \min \{ p(a)\mu_A(x),$

$p(a)\mu_A(y)\}$= min{ ($(a\mu_A)^p$)(x), ($(a\mu_A)^p$)(y) }. Therefore, $((a\mu_A)^p)(x+y) \geq \min\{((a\mu_A)^p)(x),((a\mu_A)^p)(y)\}$. Now, $((a\mu_A)^p)(xy) = p(a)\mu_A(xy) \geq p(a)\min\{\mu_A(x),\mu_A(y)\} = \min\{p(a)\mu_A(x), p(a)\mu_A(y)\} = \min\{((a\mu_A)^p)(x), ((a\mu_A)^p)(y)\}$. Therefore, $((a\mu_A)^p)(xy) \geq \min\{((a\mu_A)^p)(x), ((a\mu_A)^p)(y)\}$. For every x and y in R, we have, $((a\nu_A)^p)(x+y)=p(a)\nu_A(x+y)\leq p(a)\max\{(\nu_A(x),\nu_A(y)\}=\max\{p(a)\nu_A(x), p(a)\nu_A(y)\} = \max\{((a\nu_A)^p)(x), ((a\nu_A)^p)(y)\}$. Therefore, $((a\nu_A)^p)(x+y) \leq \max\{((a\nu_A)^p)(x), ((a\nu_A)^p)(y)\}$. Now, $((a\nu_A)^p)(xy) = p(a)\nu_A(xy) \leq p(a)\max\{\nu_A(x), \nu_A(y)\}=\max\{p(a)\nu_A(x), p(a)\nu_A(y)\}=\max\{((a\nu_A)^p)(x), ((a\nu_A)^p)(y)\}$. Therefore, $((a\nu_A)^p)(xy) \leq \max\{((a\nu_A)^p)(x), ((a\nu_A)^p)(y)\}$. Hence $(aA)^p$ is an intuitionistic fuzzy subsemiring of a semiring R.

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