



## EOQ model with package cost

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### ABSTRACT

Today the world is at the risk of environmental disarray. The main reason is population explosion. The result of the uncontrollable population, forces us to maintain the inventory of all products at a higher level. The suppliers are comparatively lesser than buyers. To retain the confidence of the buyers, the suppliers seek logistics and packaging for the deliverance of the goods to the customers at the right time and in the right manner respectively. This paper deals with an Economic Order Quantity model (EOQ) which associates the package cost along with the costs of ordering, purchasing and holding. The need for packaging and the impact of its inclusion in the classical EOQ model together with transportation cost are discussed.

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### Introduction

The Economic Order Quantity (EOQ) model has been extensively used in inventory management for a long time. The economic order quantity (EOQ) model of (Harris,1913) is the foundation of modern-day inventory models. Following his style, many researchers, out of their keen interest have contributed a great deal to the development of EOQ model, (Taft, 1918; Wilson, 1934; Hadley, 1963; Zipkin, 2000; Axsaster, 2000; Roach,2005). Many scholars have modified the classical EOQ models by introducing new costs like shortage cost and backorder cost, in addition to the purchasing cost, setup cost, holding costs which are the components of the classical EOQ model. EOQ model with the inclusion of shortage cost was developed by (Cárdenas-Barron, 2001) and then Ronald (2004) derived shortage model with and without derivatives. Jason Changa (2005) examined the concept of backorders. Teunter and Dekker (2008) put forward the EOQ model with backorders in a simple way. Later the concept of shortage and backorder were integrated with inflation, discount, deteriorating and imperfect items. (Kuo, 2006)

In general, the need for transportation begins, when the retailers and the buyers are far apart from each other. Langley (1980) discussed the need for the inclusion of transportation cost to the inventory model, Russell (1989) and Krajewski (1991) derived the EOQ model with the inclusion of transportation cost of a single item. Carter (1996) substantiated the importance of transportation cost in his work.

Jaber (2011) explicated the requirement of transportation along with the social cost that results from the emission of pollutants. He has added the notion of transportation cost to the classical EOQ model. The cost of transportation is expressed as  $C_t(Q) + C_e(Q) + C_w(Q)$ , where  $C_t(Q)$  is the Transportation cost per cycle (delivery and collection of returned items),  $C_e(Q)$  is the Emission cost from transportation,  $C_w(Q)$  is the cost of Disposal of waste produced by the inventory system per cycle.

The items that are transported, must reach the buyers in good condition, for which packaging is essential. The cost of packaging is included along with the Enviro-EOQ model (Jaber, 2011)

This paper elucidates about packaging; comprises the formulation of an EOQ model with the inclusion of package cost; discusses the ways and means of reducing the social cost from packaging; presents a numerical example; concludes the proposed work.

### Packaging

Packaging is the science, art, technology and a coordinated system of enclosing or protecting products for distribution. The benefits of packaging are as follows (Davis 2006)

- to provide protection from physical damage, contamination and deterioration;
- to give sales appeal;
- to ensure the product identity in a easily recognizable manner;
- to give information about the product;
- to optimize distribution and storage costs;
- to provide consumer convenience and safety.

Thus packaging preserves the products, reserves the decorum of the producers and sustains the consumption of the customers.

An extensive range of materials are used for packaging ,that includes metal, glass, wood, paper or pulp-based materials, plastics or combination of more than one material as composites. There are three broad categories of packaging:

- Primary packaging, which is normally in contact with the goods and taken home by consumers.
- Secondary packaging, which covers the larger packaging such as boxes, used to carry quantities of primary packaged goods.
- Tertiary packaging, which refers to the packaging that is used to assist transport of large quantities of goods, such as wooden pallets and plastic wrapping.

Therefore tertiary packaging is employed at times, where the items are carried between the vendor and the buyer. As technology develops, the packaging industry also makes use of modern techniques. To make packaging easier and to make the material used for packaging reliable, different tools are used which resulted in the production of non-biodegradable materials. One such example is plastic. Plastic materials for packaging have seen a dramatic increase in the last two decades and over the past 50 years synthetic polymers have been replacing more

traditional materials such as paper, glass and metals in many packaging applications. This is because of their low cost, low density, resistance to corrosion, desirable physical (e.g. barrier and optical) and mechanical properties and ease of processing. Most plastics are made almost entirely from chemicals derived from crude oil (Carthy, 1993). A large variety of plastics are available from different grades of thermoplastics and thermosetting polymers.

These non-biodegradable materials when exposed to the environment emit toxic pollutants. Plastics, are petroleum-based, contain toxins that can leach into water supplies and pollutes the environment. The production of plastic requires large amount of chemicals, the constant exposure to heat melts plastic, emits gases into the atmosphere by the process of out gassing. Once the goods are delivered, the materials used for packaging are expelled to the environment as waste. This waste accounts for the proportion of waste produced per lot Q as in the case of Jaber (2011).

**Notations and Assumptions**

The following notations and assumptions are used throughout to develop the EOQ model.

**Notations**

- Monetary units ( mu ) (e.g.£,€, \$ )
- A order cost (mu)
- c unit purchase cost (mu / unit)
- h holding cost (mu / unit/ year)
- P labour cost for packing per parcel (mu)
- L the cost of material used for packing per parcel (mu)
- a fixed cost per trip (mu)
- b variable cost per unit transported per distance travelled (mu / unit/km)
- d distance travelled (from supplier to buyer, km)
- α proportion of demand returned (0 < α < 1)
- D demand rate (units/year)
- β social cost from vehicle emission (mu/h)
- v average velocity (km/h)
- γ cost to dispose waste to the environment (mu/unit)
- θ proportion of waste produced per lot Q.
- m number of parcels.

**Assumptions**

The assumptions of this model are that

- (1) The units that are transported are finally packed in parcels.
- (2) The packaging cost is incurred for each parcel.
- (3) The packaging cost per parcel includes the labour cost and the material cost used for packaging.
- (4) The demand, ordering cost, proportion of the demand returned are constant per cycle.
- (5) The proportion of waste produced per lot Q incorporates the waste produced due to the disposal of the materials used for packaging.

**Model Development**

In classical EOQ model the cost per cycle is the sum of the ordering cost, purchasing cost and holding cost. Jaber has worked out a new EOQ model with the inclusion of the cost of transportation. Consider the situation where the buyer and the seller are far away from each other. The items that are required by the buyer are transported to their consign via vehicles. The items may be perishable, deteriorating, or non perishable. In order to categorise and to conserve the items and its quality when transported, packaging is essentially employed. The materials used for packaging varies according to the nature of

the item to be packed. The labour cost for packing also differs as per the techniques used in packaging. The units that are transported to the buyer are packed in parcels.

EOQ cost per cycle

$$C(Q) = A + cQ + \frac{hQ^2}{2D}$$

Transportation cost per cycle (delivery and collection of returned items)

$$C_t(Q) = 2a + bdQ + bd\alpha Q$$

Emission cost from transportation and package per cycle.

$$C_e(Q) = 2\beta \frac{d}{v}$$

Waste produced by the inventory system per cycle.

$$C_w(Q) = \gamma_o + \gamma Q (\theta + \alpha)$$

Packaging cost per parcel includes both the labour costs and the material costs. Packaging cost per parcel = P+L

The total cost of packaging per cycle

$$C_p(Q) = (P+L)m$$

Total cost per unit of time

$$\Psi(Q) = \frac{C(Q) + C_t(Q) + C_e(Q) + C_w(Q) + C_p(Q)}{T} \quad \text{where } T = \frac{Q}{D}$$

$$= \frac{AD}{Q} + cD + \frac{hQ}{2} + \frac{2aD}{Q} + bdD * (1+\alpha) + \frac{2\beta dD}{vQ} + \frac{D\gamma_o}{Q} + \gamma D(\theta + \alpha) + (P+L)m \frac{D}{Q}$$

$$\frac{\partial \Psi(Q)}{\partial Q} = \frac{\partial}{\partial Q} \left( \frac{AD}{Q} + cD + \frac{hQ}{2} + \frac{2aD}{Q} + bdD * (1+\alpha) + \frac{2\beta dD}{vQ} + \frac{D\gamma_o}{Q} + \gamma D(\theta + \alpha) + (P+L)m \frac{D}{Q} \right)$$

The objective is to determine the optimal quantity. The necessary condition is

$$\frac{\partial \Psi(Q)}{\partial Q} = 0$$

The optimal solution is

$$Q = \sqrt{\frac{2D(A + 2a + (P+L)m + 2\beta \frac{d}{v} + \gamma_o)}{h}}$$

**Steps to reduce the social cost**

In spite of the advantages of packaging, it also has certain shortcomings. Packaging waste forms a significant part of municipal solid waste and as such has caused increasing environmental concerns. The waste from packaging constitutes nearly 67 million tonnes of all municipal solid waste in Europe. (Klingbeil, 2000). In the United Kingdom, 3.2 million tonnes (Wasteline, 2002) of household waste produced annually, and packaging waste, which equates to over 12% of the total household waste produced. A question that arises in the minds of the people is, how the effect of pollutants can be reduced. The answer to this existing question is, proper planning with apt style of execution. Svetan (2003) considered a generalised concurrent engineering representation to identify the requirements on production and inventory so that they could be more responsive. Bonney (2011) extended these ideas by developing activity and research matrices to identify the do's and don'ts within a concurrent engineering context expressed over the product life cycle. Environmental needs and inventory can be examined

within the research matrix context. Some of the suggestions to control the social costs are as follows,

Gupta (1996) in his paper suggested some steps to manage the environment which were later condensed into two steps viz., assessment and prevention. The cost that results from packaging can be reduced by following these steps. Each firm must organise a committee comprising the experts of science and environment to assess the causes of the social cost, but it is not possible for every firm to do it. So the legislative bodies are implemented which tabulate the rules and regulations to be followed by all the firms in reducing the effects of social cost from vehicle emission and disposal of materials used for packaging.

Prevention is the next step to control the social cost. The social cost that arises as the result of packaging can be prevented by using biodegradable packaging materials. The non-biodegradable packaging material like plastic can be subjected to recycling. Recycling conserves resources embodied in waste plastic. Currently, a large proportion of the packaging is recycled within industry, particularly since the introduction of the UK Packaging Waste Regulations. In 2001, obligated UK industries had a packaging recycling rate of 42% against the minimum material specific target of 15% set within the regulations for all materials (IEMA, 2003).

Another way to reduce the social cost is by the use of bioplastic materials for packaging. Above all the suggestions, the most important is self-realisation. If the people of this world have awareness about the environmental sanitation then this world will be in good physical shape devoid of pollution.

#### Numerical Example

To illustrate the result obtained in this paper, a numerical example is built up. Consider an inventory system with the following characteristics.

$D = 10, A = 5, c = 3, h = 1, m = 1, P = 3, L = 2, a = 5, b = 0.5, d = 250,$   
 $\beta = 0.5, \gamma_0 = 1, v = 180.$

The optimal solution

$$Q = \sqrt{\frac{2D(A + 2a + (P + L)m + 2\beta\frac{d}{v} + \gamma_0)}{h}}$$

$$= 15.86$$

#### Conclusion

This paper concludes that, though packaging affects the environment, it is inevitable. Many environmental problems arise from the misuse of products but, others are a consequence of the type of society that we have created. In general the sole reason for the gradation of the transparency of our environment is the recklessness and irresponsibility of each individual. The steps to make our society a better one are discussed in this paper. This paper also presents an Economic Order Quantity which includes the cost of packaging. The optimal ordering quantity that is determined depends on the social cost which is the output of the package waste. If the social cost is reduced then the optimal ordering quantity also gets reduced.

#### Reference

Axsater, S., 2000, *Inventory Control*. Kluwer Academic Publishers, Boston.  
 Carter, J.R., Ferrin B.G., 1996. Transportation costs and inventory management: Why transportation costs matter, *Production and Inventory Management Journal* 37 (3), 58–62.  
 Carthy S.P., 1993. *Biodegradable Polymers for Packaging in Biotechnological Polymers*. Conference Proceedings, Lancaster,

PA, pp. 214–222.

Clardenas-Barron, L.E., 2001. The economic production quantity.

Davis G, Song, J.H., 2006. Biodegradable packaging based on raw materials from crops and their impact on waste management. *Industrial Crops and Products* 23, 147–161.

Gilbert, J., 2001. Confusion Over Catering Wastes. *Composting News*, vol. 5, issue 4. The Composting Association, UK, Summer, pp. 1–3.

Gupta, S.K., 1996. Recycling - an environmentally friendly and income generating activity towards sustainable solid waste management, *Resources, Conservation and Recycling*. 17 299–309.

Harris F.W., 1913. How many parts to make at once, factory?, *Mag Manage*. 10, 135–136 (p. 152).

Hadley, G., Whitin, T.M., 1963. *Analysis of Inventory Systems*. Prentice-Hall Englewood Cliffs.

IEMA, May 2003. *Producer Responsibility and the Packaging Regulations: Five Years On*, vol. 3. The Institute of Environmental Management and Assessment, UK.

Jason Changa, S.K., Jones, P.C. Chuanga, Hsiao-Jung Chen., 2005. Short comments on technical note—The EOQ and EPQ models with shortages derived without derivatives. *Int. J. Production Economics* 97 (241–243).

Kim, M., Lee, S., 2002. Characteristics of crosslinked potato starch and starch-filled linear low-density polyethylene films. *Carbohydr. Polym.* 50, 331–337.

Kuo-Lung Hou, 2006. An inventory model for deteriorating items with stock-dependent consumption rate and shortages under inflation and time discounting. *European Journal of Operational Research* 168, 463–474.

Langley, J.C., 1980. The inclusion of transportation costs in inventory models: some considerations. *Journal of Business Logistics* 2 (1) 106–125.

Maurice Bonney, Mohamad Y. Jaber, September 2011. Environmentally responsible inventory models: Non-classical models for a non-classical era Original Research Article. *International Journal of Production Economics*, Volume 133, Issue 1, Pages 43–53.

Maurice Bonney, Svetan Ratchev, Idir Moualek., 2003. The changing relationship between production and inventory examined in a concurrent engineering context. *Int. J. Production Economics* 81–82, 243–254.

Roach B., 2005. Origin of the economic order quantity formula; transcription or transformation?. *Manage Decis.* 43, 1262–1268.

Ronald, R.J., Yang, G.K., Chu, P., 2004. Technical note—The EOQ and EPQ models with shortages derived without derivatives. *International Journal of Production Economics* 92 (2), 181–184.

Russell, R.M., 1989. Optimal purchase and transportation cost lot sizing for a single item, *Decision Sciences Institute 1989 Proceedings*, Atlanta, pp. 1109–1111.

Russell, R.M., Krajewski, L.J., 1991. Optimal purchase and transportation cost lot sizing for a single item, *Decision Sciences* 22 (4), 940–951.

Taft E.W., 1918. The most economical production lot. *The Iron Age* 101 1410–1412.

Teunter R., Dekker, R., 2008. An easy derivation of the order level optimality condition for inventory systems with backordering. *Int. J. Prod. Econ.* 114 (1), 201–204.

Wasteline., 2002. *Information Sheet on Packaging*. Wastewatch, UK.

Whitin, T.M., 1953. *The Theory of Inventory Management*. Princeton University Press, Princeton, NJ.

Wilson R.H., 1934. A scientific routine for stock control. *Harvard Business Review* 13 , 116–128.

Woolsey, G., 1990. A Requiem for the EOQ: an editorial. *Hospital Material*.

Zipkin P.H., 2000. *Foundations of Inventory Management*. McGraw-Hill, Singapore.