Available online at www.elixirpublishers.com (Elixir International Journal)

Pollution

Elixir Pollution 46 (2012) 8475-8478

ISSN: 2229-712X

Maximum air concentrations from Non-Gaussian Plume Model

Khaled S. M. Essa[1], M. Embaby[1], A. N. MINA[2], Hany S. Hamdy[2] and Mamdouh Higazy[3]
[1]Mathematics & Theoretical Physics Department, NRC, AEA P.O. 13759, Cairo-Egypt.
[2]Department of Theoretical Physics, Beni-Suef University.
[3]Egyptian Environmental Affairs Agency, Maadi.

ARTICLE INFO
Article history:
Received: 29 February 2012;
Received in revised form:
15 May 2012;
Accepted: 24 May 2012;

Keywords
Maximum Ground Level,
Herimitized Advection Diffusion,
Critical Wind Speed.

ABSTRACT
The maximum ground level air concentration from elevated point source over simple terrain is estimated using previous work by Essa et al. (2007). The eddy diffusivities in linear forms are used. The critical of the wind speed, plume height and downwind distance are estimated. The results of the maximum concentrations are applied using meteorological data of a plant stack located in, Inshas, Egypt.

Introduction

Pasquill and Smith (1983) obtained the worst case results when the ratio of the vertical to lateral dispersion coefficients didn't depend on downwind distance. Simulation of photochemical smog in the Melbourne air shed: worse case study was studied by Hess (1989). A simple prediction formula for maximum ground level concentrations from coning plumes was investigated by Laless et al. (1977). The effect of the plume rise and wind speed on extreme value of air pollutant concentration was studied by Essa et al. (2006). The hermitized concentration from advection diffusion equation using linear form of the eddy diffusivities was obtained by Essa et al. (2007). Modeling extreme concentrations from a source in a turbulent flow over a rough wall was found by Zheng et al. (2007).

In this paper, the worse case of the hermitized ground level concentrations at centerline is estimated and the result is extended to a plume which is trapped between the ground and elevated inversion layer. The critical of the wind speed, plume height and downwind distance are estimated. The results of the maximum concentrations are applied using meteorological data of a plant stack located in, Inshas, Egypt.

Maximum Concentration

An analytical solution of the mathematical model for hermitized atmospheric dispersion of a pollutant was obtained in the steady-state form of advection-diffusion equation with linearly varying eddy diffusivities from a continuously emitting point source (Essa et al. (2007)). The ground level concentration along the plume centerline for hermitized advection diffusion equation from a height source is obtained as follows:

$$C(x,0,0)=\frac{Q}{U\pi\sqrt{\beta\gamma}x^2}\left[1+\frac{2\alpha H^2}{x^2}\right]^{-\left(\frac{1}{\alpha}+1\right)} \qquad (1)$$

where Q is the emission rate, U is the wind speed at stack height, H is the effective height of the emission which equals (h_s+ Δh); h_s is the stack height and Δh is the plume rise, and α, β and γ are the turbulence parameters depend on the stability.

To obtain the maximum concentration, differentiate equation (1) with respect to "x", and equal the result with zero, the maximum downwind distance is obtained as follows:

$$x_{\max}=\sqrt{2}H \qquad (2)$$

In order to get the maximum value, estimate $\partial^2 C/\partial x^2$, and substitute with equation (2), as follows:

$$\partial^2 C/\partial x^2=\frac{-Q(1+\alpha)^{-(\frac{1}{\alpha}+3)}}{U\pi\sqrt{\beta\gamma}H^4} \quad at\ x_{\max}=\sqrt{2}H \qquad (3)$$

By substituting the values in Eqn (3), a negative value for the second derivative is obtained; then, the downwind distance and the concentrations are maximum values.

The maximum ground concentration along the plume centerline is obtained as follows:

$$C_{\max}(x,0,0)=\frac{Q}{2U\pi\sqrt{\beta\gamma}H^2}[1+\alpha]^{-\left(\frac{1}{\alpha}+1\right)} \qquad (4)$$

Since the plume rise is $\Delta h=3w_oD/U$ (Briggs, 1969), where w_o is the exit velocity of the plume and D is the inside diameter of the stack. The effective height H is written as:

$H=h_s+3w_oD/U=h_s+A/U$ (5)

where $A=3w_oD$, A is a number which depends on the exit velocity and diameter of the stack.

In unstable conditions, the empirical relations for turbulence parameters based on convective velocity w_* (Arya, 1999) is estimated as:

$\alpha=\beta=0.31(w_*/U)^2$; $\gamma=0.16(w_*/U)^2$ (6)

where w_* is the scale convective velocity. Also the intensity parameters α, β and γ for stable conditions depend on the friction velocity u_* (Sharan et al. 1998) are estimated as:

$\alpha=\beta=3.61(u_*/U)^2$; $\gamma=0.16(u_*/U)^2$ (7)

To obtain the maximum wind speed, Differentiate equation (1) with respect to "U", and equal the result with zero. The result with the positive value is estimated as follows:

Tele:
E-mail addresses: mohamedksm56@yahoo.com

$$U_{maximum}=\left|-\frac{3A}{h_s}\right| \quad (8)$$

Then the maximum effective height is given as follows:

$H_{maximum}=h_s+A/U=h_s+Ah_s/3A=4h_s/3$ (9)

Substituting from equations (8), (9) and (7) respectively in equation (4), the maximum ground concentration along the plume centerline in stable conditions is given as follows:

$$C_{stable,maximum}(x,0,0)=\frac{1.1102QA}{\pi u_*^2 h_s^3}[1+\alpha]^{-\left(\frac{1}{\alpha}+1\right)} \quad (10)$$

Also, by substituting from equations (8), (9) and (6) respectively in equation (4), the maximum ground concentration along the plume centerline in unstable conditions is obtained as follows:

$$C_{unstable,maximum}(x,0,0)=\frac{3.7885QA}{\pi \omega_*^2 h_s^3}[1+\alpha]^{-\left(\frac{1}{\alpha}+1\right)} \quad (11)$$

Worst Trapped Concentration

If one considers that the plume is trapped under the stack of a point source, the ground level concentration along the plume centerline is written as (Ragland 1976):

$$C_i(x,0,0)=\frac{Q}{U\pi\sqrt{\beta\gamma}x^2}\left[1+\frac{4\alpha(H-2Z_i)^2}{x^2}\right]^{-\left(\frac{1}{\alpha}+1\right)} \quad (12)$$

where Z_i is the height of inversion layer. The maximum concentration is happened if the plume rise is just up to the inversion layer; hence, Eqn (12) is obtained as:

$$C_i(x,0,0)=\frac{Q}{U\pi\sqrt{\beta\gamma}x^2}\left[1+\frac{4\alpha Z_i^2}{x^2}\right]^{-\left(\frac{1}{\alpha}+1\right)} \quad (13)$$

Where $Z_i=h_s+A/U$, Taking $\partial C_i/\partial x=0$, the maximum downwind distance is calculated as follows:

$$x_{max}=2Z_i \quad (14)$$

Then the maximum ground level concentration along centerline due to inversion layer is

$$C_{i,maximum}(x,0,0)=\frac{Q}{4U\pi\sqrt{\beta\gamma}Z_i^2}[1+\alpha]^{-\left(\frac{1}{\alpha}+1\right)} \quad (15)$$

The above formula is just half of the previous result estimated in (Eqn (4)), where Z_i equals the effective height H if the plume rise is just up the inversion layer. The results of the critical wind speed and effect height have values as same as for non-Gaussian. Notice that the maximum concentration depends on the height of inversion layer and the effective height.

Case study

It is useful to apply the derived normalized concentration C/Q on a plant stack located in Inshas Cairo, Egypt. A continuous Ventilation system is provided with the plant to the surrounded areas. The total ventilation rate which is emitted from the plant stack of 43 m height, 1 m internal diameter, and exist velocity 4 m/s was 39965 m^3/hr.

The calculated values of u_*, $x_{maximum}$ and normalized concentrations C/Q for neutral, stable and unstable conditions is presented in Tables (1), (2) and (3) respectively. The last column in the three Tables is given in 48 hours where, the usual continuous operation time of the plant. The variations of the normalized concentrations are shown in the right column of these Tables.

For the maximum case concentrations, the trapping type plume is obtained by multiplying the concentrations of Eqn. (4) by a factor of half. The ratio between trapping and non-Gaussian concentration is estimated as follows:

$$\frac{C_{i_{max}}}{C_{max}}=\frac{H^2}{2Z_i^2} \quad (16)$$

Table (1) Wind speed, friction velocity, and the concentration at the axis of the plume in neutral classes.

U (m/s)	u* (m/s)	C/Q *10 $^{-6}$ sec/m3
5.27	0.33	77.17
5.31	0.33	77.17
5.34	0.34	72.70
6.37	0.4	52.53
5.17	0.32	82.07
4.45	0.28	107.20
5.1	0.32	82.07
4.81	0.3	93.38
5.3	0.33	77.17
4.86	0.31	87.45
5.36	0.34	72.70
5.19	0.33	77.17
5.41	0.34	72.70
5.54	0.35	68.61
5.2	0.33	77.17
5.61	0.35	68.61
5.79	0.36	64.85
6.27	0.39	55.26
5.93	0.37	61.39
6.01	0.38	58.20
5.41	0.34	72.70
5.75	0.36	64.85
5.26	0.33	77.17

Table (2) Wind speed, friction velocity, and the concentration at the axis of the plume in stable classes.

u (m/s)	u* (m/s)	C /Q *10 ^{6} sec/m ^{3}
4.43	0.32	82.07
3.81	0.27	115.29
4	0.29	99.93
4.92	0.35	68.61
3.7	0.27	115.29
3.57	0.26	124.32
3.64	0.26	124.32
3.45	0.25	134.47
3.6	0.26	124.32
3.8	0.27	115.29
3.99	0.29	99.93
3.89	0.28	107.20
3.75	0.27	115.29
3.98	0.29	99.93
3.47	0.25	107.20
4.06	0.29	99.93
4.3	0.31	87.45
4.31	0.31	87.45
4.02	0.29	99.93
4.11	0.3	93.38
3.94	0.28	107.20
3.86	0.28	107.20
3.67	0.26	124.32

Table (3) Wind speed, friction velocity, and the concentration at the axis of the plume in unstable classes.

U (m/s)	w_* (m/s)	C /Q *10^7 sec/m^3
5.5	1.33	162.13
4.7	1.28	175.04
5.0	1.29	172.34
6.1	1.37	152.80
4.6	1.27	177.81
4.4	1.25	183.55
4.5	1.26	180.64
4.3	1.24	186.52
4.5	1.26	180.64
4.7	1.27	177.81
5.0	1.29	172.34
4.8	1.28	175.04
4.7	1.27	177.81
5.0	1.29	172.34
4.3	1.25	183.55
5.1	1.3	169.70
5.4	1.32	164.60
5.4	1.32	164.60
5.0	1.29	172.34
5.1	1.3	169.70
4.9	1.29	172.34
4.8	1.28	175.04
4.6	1.26	180.64

Fig. (1) Shows that the polynomial of degree two is fitted well with data in unstable, neutral and stable conditions respectively. The maximum value concentration has maximum value in stable condition; the most values of the maximum concentration is happened in stable condition and minimum values is obtained in unstable condition. The values of the maximum normalized concentration are $134.47*10^{-6}$ sec/m^3 at x=81.3m, $107.20*10^{-6}$ sec/m^3 at x=81.6m and $186.52*10^{-7}$ sec/m^3 at x=81.3m in stable, neutral, and unstable conditions respectively.

The fitting equations in neutral, stable, and unstable conditions are obtained as follows:

$(C_{max}/Q)_{neutral} = -70.25\ (x_{max})^2 + 11422.5\ x_{max} - 464231$ (17)

$(C_{max}/Q)_{stable} = -76.6282\ (x_{max})^2 + 12449.3\ x_{max} - 505525$ (18)

$(C_{max}/Q)_{unstable} = -9.39673\ (x_{max})^2 + 1522.26\ x_{max} - 61474.8$ (19)

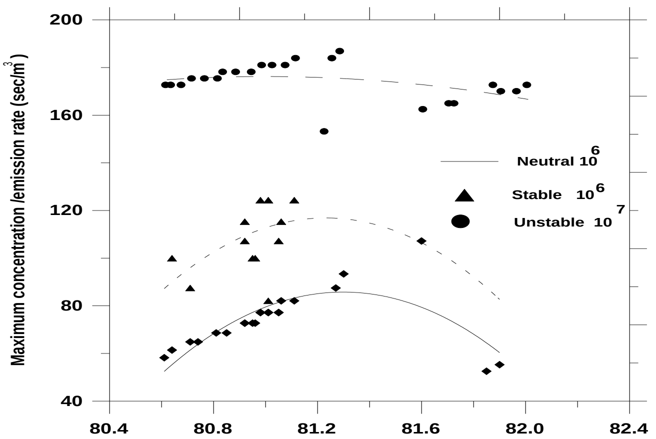

Fig. 1. Variation of the maximum concentration over emission rate (C_{max}/Q) with the maximum downwind distances (x_{max}) in all stabilities.

The ratio of maximum ground-level trapping to non-Gaussian maximum concentration with H/Z_i is shown in Fig. (2), the dimensionless of ground-level trapping to non-Gaussian concentration in stable condition is larger than neutral and unstable conditions respectively. The fitting of the ratio increases in stable condition from 0.05 to 0.08 but in neutral and unstable conditions the ratio is little bit which increases from 0.0 to 0.005.

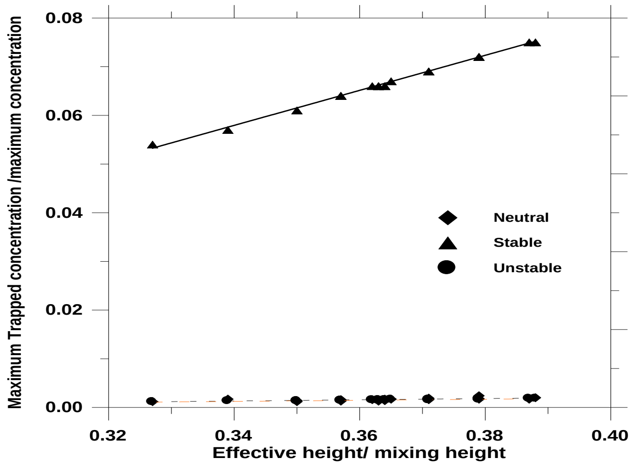

Fig. 2. The variation the ratio of ground-level trapping to non-Gaussian concentration via non-dimensional distance.

The maximum ground-level normalized concentration, wind speed, effective height and downwind distance are tabulated in Table 1 through atmospheric stability conditions. The maximum normalized concentrations have the largest value in stable condition, then neutral condition and the minimum value of maximum concentration is happened in unstable case.

Table 1. Maximum normalized concentration for non Gaussian plumes model.

Stability	C/Q sec/m^3	$U_{maximum}$ (m/s)	$H_{maximum}$ (m)	$X_{maximum}$(m)
Neutral	$107.20*10^{-6}$	4.45	57.3	81.03
Stable	$134.47*10^{-6}$	3.45	57.3	81.03
Unstable	$186.52*10^{-7}$	4.3	57.3	81.03

Conclusions

The maximum concentration has maximum value in stable, then neutral and unstable conditions respectively. The fitting of maximum concentration is found as a polynomial of order two degrees which is fitted well with data in neutral, stable and unstable conditions. The values of the worst normalized concentration are $134.47*10^{-6}$ sec/m^3 at x=81.3m, $107.20*10^{-6}$ sec/m^3 at x=81.6m and $186.52*10^{-7}$ sec/m^3 at x=81.3m in stable, neutral, and unstable conditions respectively.

The maximum concentration for trapping plume is obtained by multiplying the maximum ground level concentration along the plume centerline by a factor half.

The dimensionless of ground-level trapping to non-Gaussian concentration in stable condition is larger than in neutral and unstable conditions. The fitting of the ratio increases in stable condition from 0.05 to 0.08 but in neutral and unstable conditions the ratio is a little bit which increases from 0.0 to 0.005

References

Arya S. P. (1999) Air pollution meteorology and dispersion. Oxford University Press, New York.

Hess G. D. (1967) Simulation of photochemical smog in the Melbourne air shed: worse case study. Atmospheric Environment, Vol 23, issue 3, 1989,661-669.

Khaled S. M. Essa, Fawzia Mubarak , and Sawsan, E.M. Elsaid (2006) Effect of the plume rise and wind speed on Extreme Value of air pollutant concentration, Meteorology and Atmospheric Physics, 93, 247-253, (2006).

Khaled S. M. Essa, and Maha S. El-Otaify (2007) Mathematical model for hermitized atmospheric dispersion in low winds with eddy diffusivities linear functions down wind distance, Meteorology and Atmospheric Physics, 96, 265-275.

Laless D. P., Karras G. S. (1967) Simple prediction formula for maximum ground level concentrations from coning plumes. Atmospheric Environment, Vol 11, issue 11, 1977,1117-1119.

Pasquill F. and Smith F.B. (1983) Atmospheric diffusion. Ellis Horwood Ltd, Halstead press, Chichester.

Ragland K. W. (1976) Worst –case ambient air concentrations from point sources using the Gaussian plume model. Atmospheric Environment, 10,371-374.

Sharan M., Yadav A. K. (1998) Simulation of diffusion experiments under light wind, stable conditions by a variable K-theory model. Atmospheric Environment 32, 3481-3492.

Zheng Tong xie, Paul Hayden, Alan G. Robins, peter R. Voke (2007) Modeling extreme concentrations from a source in a turbulent flow over a rough wall. Atmospheric Environ-ment, Vol 41, issue 16, 3395-3406.

Briggs GA (1969) Plume rise. USAEC Critical Review Series, TID- 25075 National Technial Information Service, Springfield, VA